SIMILARITY SOLUTIONS
HYDRATE DISSOCIATION IN A RADIAL DOMAIN

I. Domain Description
1-D Radial system, L x H = 1000 m x 1.0 m
Discretization: 1000 cells of dimensions $\Delta r \times \Delta z = 0.02$ m x 1 m
500 cells logarithmically distributed from $r = 20$ m to $r = 1000$ m
1 cells with $\Delta r = 1.0e^{-3}$ m (outer boundary)
The fine discretization is needed to accurately capture the front in thermal dissociation; it is less important in depressurization.

II. Initial Conditions

Thermal stimulation
Pressure: $P_i = 4.6$ MPa
Temperature: $T_i = 3$ °C
Saturations: $S_H = 0.5$, $S_A = 0.5$

Depressurization
Pressure: $P_i = 9.5$ MPa
Temperature: $T_i = 12$ °C
Saturations: $S_H = 0.4$, $S_A = 0.6$

III. Boundary Conditions

At $r = R_{max}$: Constant pressure, temperature, and saturations

Thermal stimulation
At $r = 0$: $Q_H = 150$ W

Depressurization
At $r = 0$: $Q = 0.1$ kg/s (fluid mass rate, including gas and aqueous phases)

IV. Medium Properties

Hydraulic Properties
Intrinsic permeability: For thermal stimulation, $k = 10^{-12}$ m$^2$ (1 Darcy)
For depressurization, $k = 3.0 \times 10^{-13}$ m$^2$ (0.3 Darcys)
Porosity: $\phi = 0.3$
Pore compressibility: $\beta = 10^{-9}$ Pa$^{-1}$
Grain density: $\rho_R = 2600$ kg/m$^3$
Thermal Properties

Grain specific heat: \( C = 1000 \text{ J/kg/K} \)
Dry thermal conductivity: \( k_{\Theta D} = 2 \text{ W/m/K} \)
Wet thermal conductivity: \( k_{\Theta W} = 2.18 \text{ W/m/K} \)
Composite thermal conductivity model: Linear (per earlier decision)

\[
k_{\Theta} = k_{\Theta D} + \phi (S_A k_{\Theta w} + S_H k_{\Theta H} + S_I k_{\Theta I} + S_G k_{\Theta G})
\]

where the subscripts \( w, H, I, G \) denote water, hydrate, ice and gas, respectively.

**Note:** This \( k_{\Theta} \) model will result in a thermal conductivity that is about 70\% of the composite \( k_{\Theta} \) of hydrate-bearing sediments measured in lab and field experiments.

V. Relative Permeability

The treatment and description of the wettability properties is likely to be an early departure point of the various models in the study because of differences in the models and in the approach.


\[
k_{rG} = (S_G^*)^n, \quad S_G^* = (S_G - S_{irG})/(1 - S_{irA}) \\
k_{rA} = (S_A^*)^n, \quad S_A^* = (S_A - S_{irA})/(1 - S_{irA})
\]

where the subscripts \( irG, irA \) denote aqueous and gas phase irreducible saturations, respectively. For this problem:

\[
n = 3.0 \\
S_{irA} = 0.12 \\
S_{irG} = 0.02
\]

**Effect of Solid (=Hydrate) Phase on Relative Permeabilities**

We use the Original Porous Medium (OPM) model. It is based on the treatment of (a) the medium porosity as unaffected by the emergence of hydrates and/or ice (although subject to change due to changes in pressure and temperature), (b) of the intrinsic permeability of the porous media as unchanging during the evolution of the solid phases, and (c) of the fluid flow as a relative permeability issue controlled by the saturations of the various phases in the pores.

VI. Capillary Pressure

**Thermal stimulation**

For the thermal stimulation case, a \( P_{cap} = 0 \) was used.
**Depressurization**

For the depressurization problem, the van Genuchten (1981) model is used:

\[
P_{cap} = -P_0 \left[ \left( S^* \right)^{1/n} - 1 \right]^{1/n}, \quad S^* = \frac{(S_A - S_{rA})}{(S_{mxA} - S_{rA})}
\]

subject to the restriction: \(-P_{max} \leq P_{cap} \leq 0\)

Parameters: \(\lambda = 1 - 1/n = 0.45\)  
\(S_{rA} = 0.11\) (chosen slightly smaller than the corresponding parameter in the relative permeability function)  
\(1/P_0 = \alpha/\rho_w g = 8.0 \times 10^{-5} \text{ Pa}^{-1}\)  
\(P_{max} = 5.0 \times 10^6 \text{ Pa}\)  
\(S_{mxA} = 1\)

Capillary pressure is scaled to account for the effects of the solid phases. We use the simplest capillary pressure scaling, which involves adjustments of fluid saturations to reflect the changing solid matrix. We represent *active* solids (i.e. ice that may melt or hydrate that may dissociate, as opposed to solid minerals which are inert) by means of a *solid saturation*, denoted by \(S_S = S_H + S_I\), which measures the fraction of active pore space occupied by solids. The fraction of pore space available to fluid phases is \(S_A + S_G\), and we have the constraint

\[
S_A + S_G = 1 - S_S
\]

Let \(P_{cap,0}\) denote the capillary pressure function applicable to a porous medium free of solid saturation \((S_S = 0)\) with porosity \(\phi_0\). The active porosity available to fluid is then

\[
\phi = (S_A + S_G) \phi_0 \Rightarrow \frac{\phi}{\phi_0} = (S_A + S_G)
\]

We measure liquid saturation on a scale that refers to total *active* (fluid plus hydrate- and ice-filled) pore space \(\phi_0\) in the ice-free porous medium. An aqueous saturation \(S_A\) in the medium with solid saturation \(S_S\) then corresponds to a saturation

\[
S_A^* = \frac{S_A}{S_A + S_G}
\]

relative to fluid-filled pore space, and this is the value to be inserted in the r.h.s of equation (1) instead of \(S_A\).
VII. Data and Sampling Frequency

Data to be compared
Pressure, temperature, phase saturations, phase relative permeabilities, mass fraction of CH₄ in the aqueous phase, volumetric rate of CH₄ release from the hydrate, volume of CH₄ released from the hydrate

For thermal stimulation
Results to be reported at 2 days, 5 days, 10 days, 15 days, 20 days, 30 days, 45 days and 60 days.

For depressurization to P above the Q-point
Results to be reported at 2 days, 5 days, 10 days, 15 days, 20 days and 30 days (longer simulations affect the behavior of the boundary, which is no-longer infinite-acting after t = 30 days).