Geomechanical Performance of Hydrate-bearing Sediments in Offshore Environments

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Objectives

- **Motivation**: Quantify the impact of hydrate dissociation on the strength and elastic properties of hydrate-bearing sediments

- **The Task**: Estimate the changes of elastic moduli in order to populate the grid blocks for coupled TOUGH Fx/HYDRATE- FLAC3D simulations
  - **Inputs**: initial elastic moduli, variable pore pressure and hydrate saturation
  - **Tool**: Pore-scale quasi-static equilibrium model
  - **Method**: Verification of the model against available experiments and numerical simulations
Approach

• Saturated sediment is modeled as a granular medium
• The skeleton can be either unconsolidated or cemented
• Some of the pores are filled with gas-hydrates
• Consequences of hydrate dissociation:
  – Increased pore pressure and reduced effective stress
  – Decrease of the skeleton strength by losing hydrate support
  – Possibly, thermal contraction/expansion
Grain Pack Properties

- 3D random heterogeneous grain packing
  - Spherical grains, differ in radii and mechanical properties
  - Each grain is homogeneous, isotropic, and linearly elastic
- To be implemented
  - Adhesion or cement at grain-to-grain contacts
  - Different grain shapes
Contact-Mechanics Model

- Hertz-Mindlin theory of elastic interaction of a grain pair
  - Small deformations localized around a small neighborhood of the contact area
  - Planar circular contact surface
  - Forces and moments at the contact
Normal Contact: Hertzian Model

Contact force:

\[ P_{ij} = \frac{4}{3} E^*_ij \sqrt{R^*_ij h^3_{ij}} \]

Elastic strain energy:

\[ U_{ij} = \frac{8}{15} E^*_ij \sqrt{R^*_ij h^5_{ij}} \]

where

\[ \frac{1}{E^*_ij} = \frac{1-\nu^2_i}{E_i} + \frac{1-\nu^2_j}{E_j} \]

\[ \frac{1}{R^*_ij} = \frac{1}{R_i} + \frac{1}{R_j} \]

\[ h_{ij} = R_i + R_j - \|r_{ij}\| \]

\[ r_{ij} = r_i - r_j \]
Frictional Contact: Mindlin’s Theory

- Linear and rotational displacements introduce tangential tractions and moments
  - Tangential stiffness depends on normal pressure
  - Tangential tractions are path-dependent
  - Partial/complete slip can occur
  - Mindlin theory: normal tractions are not affected by the tangential components

- To eliminate some of these difficulties, we consider:
  - Pre-stressed pack
  - Small deformations
  - Static friction (no slip)
Effective Properties via Simulations

• A grain pack is enclosed in a semi-rigid container
• Boundary conditions = wall displacements
• Macroscopic stress is generated by contact forces
• Quasi-static model: equilibrium configurations
• Equilibrium = minimum total elastic energy
• Conjugate Gradient minimization algorithm
  – Functional to minimize = total energy of the pack
  – Dynamic list of contacts
• Effective moduli using Hooke’s law
Example of Simulation

Parameters of the pack:
- Radii of 0.07-0.13 mm
- Moduli normally distributed
- Mean values: \( E = 100 \text{ GPa}, \nu = 0.15 \)
- Number of grains: 306 (small pack); 2,740 (large)

\( \phi = 43.83\% \quad N = 4.16 \)
Example of Simulation

\[ \phi = 22.61\% \quad \text{Mean Coordination No.} = 8.38 \]
Modeling Challenges

Difficulties are in the modeling of grain pairs and packs

• Grain pairs
  – Nonlinearity of force-displacement relations
  – Stress depends on deformation history
  – Slip, partial or complete

• Grain packs
  – Complex contact geometry
  – Number of contacts, orientation and stiffness vary during the deformation
  – Deformation hysteresis even with fully elastic contacts
Creating a Stable Pack: Rearrangement vs Deformation

- The minimum coordination number for a stable pack is 3 (with gravity)
- Pack produced by D.E.M. simulations is unstable, i.e. not in equilibrium
- Our algorithm eliminates most unstable structures, by mere rearrangements (no grain deformations)
- As stresses increase, the pack gets more stable
Non-Linear Response

Local failure in a small pack (306 grains)

- Four consecutive configurations, similar macroscopic strains
- Force chains plotted are the top 10% contact forces
- Line width is scaled with the force magnitude
- Abrupt change from $2 \rightarrow 3$; brown grain is forced through a constriction
Non-Linear Response

• At a particular combination of contact forces, some grains experience large, irreversible (inelastic) displacements:
  – Macroscopic stress is reduced by rearrangements of grain clusters
  – Local phenomenon, affecting only a small neighborhood of each rearranged cluster
  – More pronounced in smaller packs
  – Local failure, followed by stiffening of the pack
  – Similar to strain-hardening in metals
Loading/Unloading Hysteresis

- Elastic response of a single contact
- Inelastic behavior of a grain assembly

Triaxial compression test:
\[ \phi = 35.8\sim30.8\% ; \]
\[ N = 7.2\sim8.3 \]
Stiffness vs. Compaction

- Bulk modulus $K$ increases with pack density
- Density increases as porosity $\phi$ decreases and coordination number $N$ increases

\[ \sigma_{\text{Mean}} = \frac{(\sigma_X + \sigma_Y + \sigma_Z)}{3} \text{ (MPa)} \]

\[ \varepsilon_{\text{Vol}} = \varepsilon_X + \varepsilon_Y + \varepsilon_Z \text{ (-)} \]

Hydrostatic compression tests:
Left: Simulations;
Right: Experiment Vesic & Clough, 1968)
Conclusions

• Granular media exhibit non-linear, path-dependent behavior, even for a frictionless contact model

• Because of grain rearrangement, macroscopic deformation is possible with little deformation of grains

• Hysteretic effects are more pronounced when grain ‘jumps’ occur

• Introduction of frictional contacts increases the path-dependency (hysteretic effects)
Summary: Phase I

- A grain-scale model of rock
  - Hertz-Mindlin contact mechanics
  - Stable equilibrium grain packs
  - Simulation of loading-unloading hysteresis
  - Matching published laboratory measurements
  - Efficient numerical procedure
Summary

• Grain-scale model provides a tool to estimate effective moduli
  – Irreversibility of deformations is captured
  – Stiffening ($K \uparrow$) with compaction is evident
  – Values of $K$ match physical experiments
  – Poisson’s ratio high, due lack of friction, cement, and simplified grain shapes
  – Efficient algorithm based on conjugate gradient method
Next...

• Use the already developed model to
  – Incorporate gas-hydrates and investigate
    ➢ Solid skeleton support
    ➢ Pore pressure and effective stress changes
  – Perform ensemble-averaging and investigate sample size effects

• Enhance the existing model by adding
  – Cementation/adhesion between grains
  – Failure criterion for cement
  – More complex grain shapes
  – Large strains
Thank You!