Adaptive Homogenization for Upscaling Heterogeneous Porous Media

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1. Introduction

The multiscale nature of the flow and transport problem in porous media is rising attention in practical applications:
- Petroleum reservoir recovery evaluations
- Nuclear waste disposal systems
- CO2 sequestration
- Groundwater remediation.

We propose an adaptive multiscale approach to improve the efficiency and the accuracy of numerical computations by combining upsampling and domain decomposition methods. We use the Enhanced Velocity Mixed Element Method (EVMFEM) as a domain decomposition approach to couple the coarse and fine subdomains [3].

2. Motivation

- Direct numerical computation is computationally prohibitive, since
  - Heterogeneity of porous media
  - Complexity of dynamic systems
  - Capture fine scale features
  - Higher resolution only near well bore: polymer injection, gas injection
  - Non-linear reactions with large variation in reaction rates
  - Effective parameter computation precision

3. Model formulation

Phase Mass Conservation
\[ \sum_{\Omega} \nabla \cdot \left( \phi \mathbf{u} - \mathbf{f}(\mathbf{c}) \right) = \sum_{\partial \Omega} \mathbf{q} \cdot \mathbf{n} \]

Component Mass Conservation
\[ \frac{\partial c_i}{\partial t} + \nabla \cdot (\phi c_i \mathbf{u}) = \nabla \cdot \mathbf{F}_i + \frac{\partial c_i}{\partial t} \]

Boundary and Initial conditions
\[ \mathbf{u} \cdot \mathbf{n} = 0, \quad c_i \cdot \mathbf{n} = 0 \quad \text{on} \quad \partial \Omega \times J \]
\[ c = c_0, \quad p = p^0 \quad \text{at} \quad \Omega \times \{ t = 0 \} \]

4. Adaptivity criteria

- Criteria 1 (gradient in time):
  \[ \Omega_I = \{ x \mid \nabla \cdot \left( \phi \mathbf{u}_h - \mathbf{f}(\mathbf{c}_h) \right) > \| \mathbf{q}_h \| \} \]

- Criteria 2 (gradient in space): We define
  \[ I_{\text{adaptive}}(x) = \{ y \mid E_y \cap \Omega_{\text{adaptive}} \neq \emptyset \} \]

The criteria can then be defined as,
\[ \Omega_I = \{ \max_{E_y \in I_{\text{adaptive}}(x)} \| \mathbf{q}_h \| \} \]

5. Numerical Experiments

5.1 Homogeneous permeability

5.2 Benchmark datasets: SPE 10

6. Conclusions

We developed an adaptive multiscale scheme using local numerical homogenization and EV MFEM for upsampling single phase flow and transport in a heterogeneous porous media. The numerical results on different layers of SPE10 also indicate that an upsampling based solely upon numerical homogenization is in good agreement with the fine scale solution for a Gaussian or periodic permeability distribution. However, for highly channelized or layered permeability distributions the results deviate substantially. The adaptive multiscale scheme, on the other hand, is in good agreement for Gaussian, periodic, and layered permeability distributions and is approximately faster than the fine scale simulation for a tracer slug injection of 50 days in all the above numerical tests. It is important to note that a tracer slug injection is specifically chosen to test adaptivity since there are two transient regions at the front and back of the injected slug. The accuracy of the adaptive multiscale approach presented here, compared to the fine scale solution, is dependent on the nature of physical process and validity of the adaptivity criteria in capturing fine scale physics. An optimal tolerance for the adaptivity criteria is chosen to reduce computational cost without substantial loss in solution accuracy.

References