A Unified Viscoplastic Strain Gradient Theory and its Applications

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Background

Strain gradients are known to affect plastic deformation at small scales and in strong gradients. How gradients affect viscoplasticity is currently not well understood and constitutive models are missing.

Objective

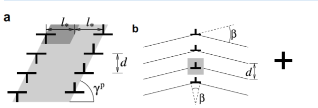
Define a unified constitutive model and implement the model as a component of a FE code.
Investigate load transients in a turbine disk.
Investigate conditions at the creep crack tip.

Methods

Implement the constitutive model as a component of a FE code to analysis (1) a conceptual turbine rotor component and (2) a crack model considering transients in loads and plastic strain gradients.

Theory: The SG-KM Model

Background



\+ *Viscoplastic Creep Hardening*

Constitutive Formulation

Total strain rate	$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^{vp}$
Elastic strain rate: $\dot{\boldsymbol{\sigma}} = \mathbf{D}\dot{\boldsymbol{\varepsilon}}^e$ *Elasticity tensor* $D_{ijkl} = \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) + \lambda\delta_{ij}\delta_{kl}$ *Lamé's coefficients:* $\lambda,\ \mu$	*Viscoplastic strain rate:* $\dot{\boldsymbol{\varepsilon}}^{vp} = \frac{\partial \phi}{\partial \boldsymbol{\sigma}}$ *Isotropic material* $\bar{\sigma} = f(\boldsymbol{\sigma}) = \sqrt{\frac{3}{2}\mathrm{s}:\mathrm{s}}$ $\dot{W}^{vp} = \bar{\sigma}\dot{\bar{\varepsilon}}^{vp} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{vp}$

Kinetic equation	$\frac{\bar{\sigma}}{\sigma_{\mathrm{ref}}} = \left(\frac{\dot{\bar{\varepsilon}}^{vp}}{\dot{\varepsilon}_0}\right)^{1/m}$
Taylor equation	$\sigma_{\mathrm{ref}} = \sigma_0 + M\alpha\mu b\sqrt{\rho}$
Dislocation density:	$\rho = \rho_S + \rho_G$
Statistically stored dislocation $\rho_S = \rho_S^+ + \rho_S^-$ - *Accumulation:* $\frac{d\rho_S^+}{d\bar{\varepsilon}^{vp}} = Mk_1\sqrt{\rho_S + \rho_G}$ - *Dynamic recovery:* $\frac{d\rho_S^-}{d\bar{\varepsilon}^{vp}} = Mk_2\rho_S$ *Strain rate sensitivity* $k_2 = k_{20}\left(\frac{\dot{\bar{\varepsilon}}^{vp}}{\dot{\varepsilon}_0}\right)^{-1/n}$	*Geometrically necessary dislocation* $\rho_G = \bar{r}\frac{\bar{\eta}^{vp}}{b}$ *Effective strain gradient* $\bar{\eta}^{vp} = \sqrt{\frac{1}{4}\eta_{ijk}^{vp}\eta_{ijk}^{vp}}$ $\eta_{ijk}^{vp} = \varepsilon_{ik,j}^{vp} + \varepsilon_{jk,i}^{vp} - \varepsilon_{ij,k}^{vp}$

FE-Implementation (ABAQUS UMAT)

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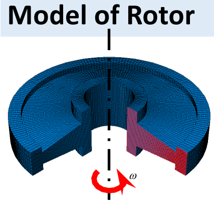
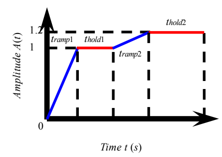
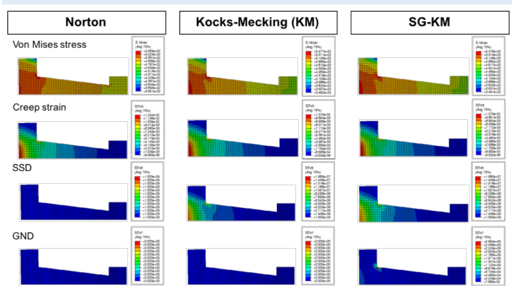
Structure

Model of Rotor

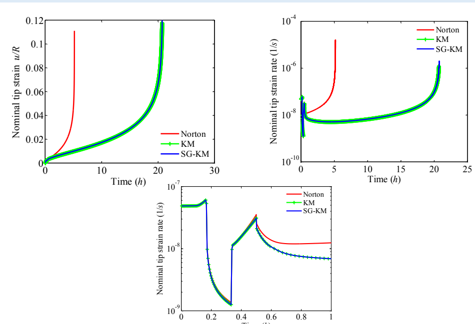




Spin Up and Steady State

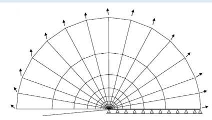


Transients in Tip Displacement & Rate



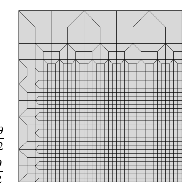
Cracks

Modified Boundary Layer Model

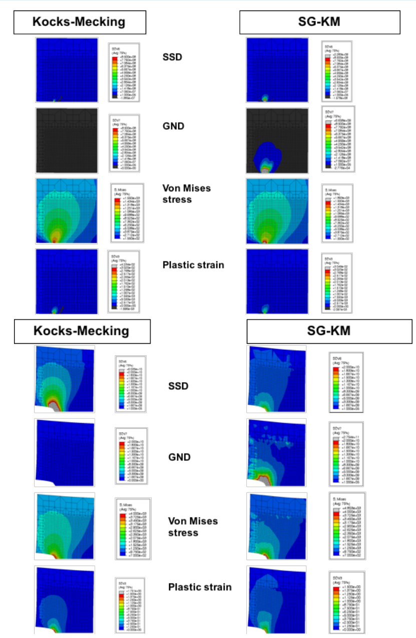


$$u_x(t) = K_I(t)\sqrt{\frac{r}{2\pi}}\frac{1+\nu}{E}\left(3-4\nu-\cos\theta\right)\cos\frac{\theta}{2}$$

$$u_y(t) = K_I(t)\sqrt{\frac{r}{2\pi}}\frac{1+\nu}{E}\left(3-4\nu-\cos\theta\right)\sin\frac{\theta}{2}$$



Crack Tip Fields



Outcomes

Define a new viscoplastic constitutive model accounting for viscoplasticity, creep and hardening under the consideration of plastic strain gradients
Implemented in FE code

Outcomes

The KM model predicts transients not captured by a conventional Norton model
SG delays creep failure but raises stresses
SG affects local conditions, not overall response

Outcomes

The SG-KM model predicts that GND (viscoplastic strain gradients) dominate over SSD (viscoplastic strains) and that the relevance of GND increases over time
SG-KM predicts higher stresses than KM theory

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