Fracture Design, Placement, and Sequencing in Horizontal Wells

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Outline

• Problem statement: The need for a new model
• Peridynamics Based Hydraulic Fracturing Model
• Model Verification
• Effect of Reservoir Heterogeneity
• Interaction between Hydraulic Fractures and Natural Fractures
• Propagation of multiple fractures in horizontal wells
• Conclusion
Hydraulic Fractures are Complex

To optimize stimulation design and completion strategy, we must consider:

- **Complex fracture geometry** (multiple, non-planar)
- **Fracture networks** (interaction with natural fractures)

Impact of **natural fractures**, **heterogeneities**, poroelasticity, layering, variation in the *in situ* confining stresses etc.

Peridynamics

Unifies the mechanics of **continuous** and **discontinuous media**


Material points (particle-based discretization)
Non-local interaction (bonds) inside horizon

- **Integral Form**

**Classical model**
\[ \rho [x] \ddot{u}[x, t] = \nabla \cdot \sigma[x, t] + b[x, t] \]

**Peridynamics**
\[ \rho [x] \ddot{u}[x, t] = \int_{\mathcal{H}_x} \left( T[x, t] \langle \xi \rangle - T[x', t] \langle -\xi \rangle \right) dV_{x'} + b[x, t] \]

- **Any Known Constitutive Model in Classical Theory**

Linear elastic body:
\[ T[x, t] \langle \xi \rangle = \left( \frac{3K\theta}{m} \omega x - \frac{15G}{m} \omega e^d \right) \frac{\xi}{\|\xi\|} \]
Peridynamics Based Hydraulic Fracturing Model

Peridynamics Based Poroelastic Model

\[
\frac{\partial}{\partial t} \left( \rho_0 [x] \phi [x] \right) = \int_{\mathcal{M}_x} \left( Q [x] \langle \xi \rangle - Q [x'] \langle -\xi \rangle \right) dV_{x'} + R [x] + I [x]
\]

\[
Q [x] \langle \xi \rangle = \frac{\rho_0}{\mu} \frac{4}{\pi \delta^2} \frac{\xi (K [x] - \frac{1}{2} \text{tr} (K [x]) I) \xi}{\| \xi \|^4} \left( P [x'] - P [x] \right) \quad \text{(for 2D flow)}
\]


Porous Flow

\[
\rho_m [x] \ddot{u} [x] = \int_{\mathcal{M}_x} \left( T [x] \langle \xi \rangle - T [x'] \langle -\xi \rangle \right) dV_{x'} + b [x]
\]

\[
T [x] \langle \xi \rangle = \left[ \chi_t \left( \frac{3(K - G/3)}{m} - 3\alpha P \right) \right] \epsilon (\xi) + \frac{2 \epsilon_0^2 G}{m} \left( \| \eta + \xi \| - \| \xi \| \right) \left( \left\| \eta + \xi \right\| + \| \eta \| \right)
\]


Solid Mechanics

Fracture Flow

\[
\frac{\partial}{\partial t} \left( \rho_f [x] \phi_f [x] \right) = \int_{\mathcal{M}_x} \left( Q_f [x] \langle \xi \rangle - Q_f [x'] \langle -\xi \rangle \right) dV_{x'} + R_f [x] - I [x]
\]

Dual Permeability Concept

Before fracture propagation

**4 Primary Unknowns**: position of element \((x,y,z)\) and matrix pressure

Pore Space (Pore Pressure)


After fracture propagation

**5 Primary Unknowns**: position of element \((x,y,z)\), matrix pressure, and fracture pressure

Pore Space (Pore Pressure)
Fracture Space (Fracture Pressure)
Model Verification
Biot Consolidation Problem
(Verification of Poroelastic Model)

2-D Single Fracture Propagation (Comparison with KGD Model)

- Young’s modulus (GPa) = 60
- Poisson’s ratio = 0.25
- Shmax (MPa) = 12
- Shmin (MPa) = 8
- Permeability (nD) = 10
- Porosity = 0.3
- Initial pore pressure (MPa) = 3.2
- Fluid: Water
- Injection rate (m$^3$/min/m) = 0.12
- Number of elements 200*160
Results: Fracture Half Length and Wellbore Pressure

KGD assumes constant pressure distribution along a fracture. Infinite conductivity model shows good agreement with KGD.

Infinite conductivity model shows good agreement with KGD.
Results: Stress Distribution around Fracture (Infinite Conductivity Case)
3-D Single Fracture Propagation (Comparison with PKN Model)

- Young’s modulus (GPa) = 60
- Poisson’s ratio = 0.25
- S_{vmax} (MPa) = 60
- S_{hmin} (MPa) = 40
- Permeability (nD) = 10
- Fluid: Water
- Injection rate (m^3/min/m) = 0.12
- Number of elements 100*100*20
Results: Fracture Half Length, Fracture Width, and Wellbore Pressure
Effect of Reservoir Heterogeneities
Effect of Multi-Scale Heterogeneities

Different scale heterogeneities in the reservoir

- Layer scale (m order) heterogeneity
- Sub-layer scale (cm order) heterogeneity
- Small scale heterogeneity (mm order)

How does this multi-scale heterogeneity affect fracture propagation?


- Quartz
- Clay or kerogen
Effect of Layer Boundary

In most of the hydraulic fracturing simulators, only “crossing” or “stopping” are simulated due to planar propagation assumption.

However, in many cases, fractures can show the following **characteristic propagation behaviors** near the layer interface other than “crossing” or “stopping”.

- “turning”
- “branching”
- “kinking”
Fracture Propagation in Layered Rocks

Extracting a small area near layer boundary

Important parameters:

- Horizontal-vertical stress contrast
- Young’s modulus contrast
- Weak connection between layers
- Layer Dip
- Horizontal stress contrast
- Toughness contrast

Inject water from the bottom

Horizontal stress contrast: $\sigma_H$

Vertical stress contrast: $\sigma_V$

Elastic moduli: $E_1$ and $E_2$

Toughness: $KIC_1$ and $KIC_2$
**Effect of Layer Stresses**

*(0 degrees, E2 = 10 GPa)*

- (a) $E_1 = 10$ GPa
- (b) $E_1 = 20$ GPa
- (c) $E_1 = 40$ GPa
- (d) $E_1 = 80$ GPa

*Published shale data*
Rijken and Cooke (2001)

$E$: 4.5 - 61.0 GPa
$KIC$: 0.7 - 2.16 MPa $m^{0.5}$
Turning and Branching

• “Turning” is strongly affected by principal stress difference and fracture toughness contrast.
  • A higher fracture toughness contrast
  • A smaller principal stress difference

• Young’s modulus contrast does not have a large influence on fracture turning along the layer interface.

• “Branching” occurs under the following conditions.
  • Very high Young’s modulus contrast (> 8.0)
  • Low fracture toughness contrast (< 1.0)

  e.g. upper layer = calcite vein
  \[E = 83.8 \text{ GPa}, \ KIC = 0.19 \text{ MPa m}^{0.5}\]
Effect of Dip Angle
(15 degrees, E2 = 10 GPa)

But, “Kinking” is observed in most of the cases below the turning criteria.
Effect of Dip Angle
(30 degrees, E2 = 10 GPa)

“Kinking” is observed in every case below the turning criteria. “Turning” criteria in the high dipping angle cases becomes lower than the lower dipping angle cases.

(a) E1 = 10 GPa
(b) E1 = 20 GPa
(c) E1 = 40 GPa
(d) E1 = 80 GPa
Mechanism of Kinking

The left side stress becomes higher due to smaller strain of upper layer. The fracture turns to the right side.

Sxx distribution

E=40GPa

E=10GPa

15 cm

30 cm

The left side of the fracture is difficult to deform due to the high Young's modulus.

The fracture turns as if avoiding the layer interface.
Effect of Young’s modulus contrast on “kinking”

Kinking angle increases with the Young’s modulus contrast.

(a) E1/E2=10GPa/10GPa
(b) E1/E2=20GPa/10GPa
(c) E1/E2=40GPa/10GPa
(c) E1/E2=80GPa/10GPa
Effect of fracture toughness contrast on “kinking”

Kinking angle does not depend on the fracture toughness contrast.

(a) $KIC_1/KIC_2=0.5 \text{ MPa m}^{0.5}/0.5 \text{ MPa m}^{0.5}$

(b) $KIC_1/KIC_2=1.0 \text{ MPa m}^{0.5}/0.5 \text{ MPa m}^{0.5}$

(c) $KIC_1/KIC_2=1.4 \text{ MPa m}^{0.5}/0.5 \text{ MPa m}^{0.5}$

(d) $KIC_1/KIC_2=2.0 \text{ MPa m}^{0.5}/0.5 \text{ MPa m}^{0.5}$
Effect of principal stress difference on “kinking”

Kinking angle decreases with increase in stress contrast.

(a) principal stress difference = 1 MPa

(b) principal stress difference = 6 MPa

(c) principal stress difference = 10 MPa

(d) principal stress difference = 20 MPa
Effect of layer dip angle on “kinking”

Higher layer dip angle leads to more kinking.

(a) 15deg (stress difference = 1 MPa)

(b) 15deg (stress difference = 10 MPa)

(c) 30deg (stress difference = 1 MPa)

(d) 30deg (stress difference = 10 MPa)
Effect of Bed Dip

If the upper layer is thinner?

3 different layer thickness models

0 degrees models

30 degrees models

How the criteria changes with bed dip?

0 degree: \( E_1/E_2 = 40 \text{ GPa} / 10 \text{ GPa} \)

30 degree: \( E_1/E_2 = 40 \text{ GPa} / 10 \text{ GPa} \)
Effect of Layer Thickness
(0 degrees cases)

(a) \(\Delta \sigma = 1.0 \text{ MPa } (E_1/E_2 = 40/10)\)

(b) \(\Delta \sigma = 10.0 \text{ MPa } (E_1/E_2 = 40/10)\)

More stress reduction than the reference case

E = 10.0 GPa
E = 40.0 GPa
E = 10.0 GPa

(a) layer thickness = 1.2 cm

E = 40.0 GPa
E = 10.0 GPa

(b) reference (layer thickness = 10.0 cm)
Effect of Layer Thickness
(30 degrees cases)

- Turning criteria is strongly affected by the upper layer thickness.
- However, the magnitude of kinking is not affected by the upper layer thickness.

The kinking angles are almost same regardless of the upper layer thickness.
Effect of Weak Surface

If the layer interface is damaged for some reason, how do the turning criteria change?

Investigating the same cases as the previous fully-bonded cases by setting the following shear failure criteria

- Shear coefficient = 0.6
- Cohesion = 0.0 MPa
Effect of Weak Surface
(0 degrees cases)

Criteria without the weaker surface

(a) $E_1/E_2 = 10\,\text{GPa}/10\,\text{GPa}$
(b) $E_1/E_2 = 20\,\text{GPa}/10\,\text{GPa}$
(c) $E_1/E_2 = 40\,\text{GPa}/10\,\text{GPa}$
(d) $E_1/E_2 = 80\,\text{GPa}/10\,\text{GPa}$

: Turning
: Crossing

Branching region disappear
Effect of Weak Surface (30 degrees cases)

Different type of branching appears in these case.

- Turning
- Branching

Graphical representations showing the fracture toughness contrast for different principal stress differences.

(a) $E_1/E_2 = 10 \text{ GPa}/10 \text{ GPa}$
(b) $E_1/E_2 = 20 \text{ GPa}/10 \text{ GPa}$
(c) $E_1/E_2 = 40 \text{ GPa}/10 \text{ GPa}$
(d) $E_1/E_2 = 80 \text{ GPa}/10 \text{ GPa}$
Shale reservoirs are filled with cm scale heterogeneities (sub-layers)

Investigating how multiple layers affect fracture propagation

# Effect of cm Scale Sub-Layers (Case Settings)

Principal Stress Difference = 20 MPa (Shmin = 40 MPa, Svmax = 60 Mpa)

<table>
<thead>
<tr>
<th>Case</th>
<th>Young's modulus 1 (GPa)</th>
<th>Young's modulus 2 (GPa)</th>
<th>Fracture toughness 1 (MPa m^{0.5})</th>
<th>Fracture toughness 2 (MPa m^{0.5})</th>
<th>Energy release rate contrast between layers (hard layer/soft layer)</th>
<th>Layer dip angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>low_contrast</td>
<td>10</td>
<td>20</td>
<td>0.5</td>
<td>0.707</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>middle_contrast</td>
<td>10</td>
<td>40</td>
<td>0.5</td>
<td>1.000</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>high_contrast</td>
<td>10</td>
<td>80</td>
<td>0.5</td>
<td>1.414</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>high_contrast_2</td>
<td>10</td>
<td>80</td>
<td>0.5</td>
<td>0.707</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>low_contrast_dipping</td>
<td>10</td>
<td>20</td>
<td>0.5</td>
<td>0.707</td>
<td>1.00</td>
<td>30</td>
</tr>
<tr>
<td>middle_contrast_dipping</td>
<td>10</td>
<td>40</td>
<td>0.5</td>
<td>1.000</td>
<td>1.00</td>
<td>30</td>
</tr>
<tr>
<td>high_contrast_dipping</td>
<td>10</td>
<td>80</td>
<td>0.5</td>
<td>1.414</td>
<td>1.00</td>
<td>30</td>
</tr>
<tr>
<td>high_contrast_2_dipping</td>
<td>10</td>
<td>80</td>
<td>0.5</td>
<td>0.707</td>
<td>0.25</td>
<td>30</td>
</tr>
</tbody>
</table>

The relationship among energy release rate, fracture toughness, Poisson’s ratio, and Young’s modulus in 2-D plane strain condition.

\[
G_c = \frac{K_{IC}^2 (1 - \nu^2)}{E}
\]
Effect of cm Scale Sub-Layers
(0 degrees: Low and Middle E Contrast)

Low E Contrast

E1/E2 = 20/10; KIC1/KIC2 = 0.7/0.5; Dip angle = 0

Middle E Contrast

E1/E2 = 40/10; KIC1/KIC2 = 1.0/0.5; Dip angle = 0
Effect of cm Scale Sub-Layers
(0 degrees: High E Contrast)

High E Contrast
(Gc = const)

Young's modulus

Damage

Sxx (MPa)

E1/E2 = 80/10; KIC1/KIC2 = 1.4/0.5; Dip angle = 0

High E Contrast
(Low Kic contrast)

E1/E2 = 80/10; KIC1/KIC2 = 0.7/0.5; Dip angle = 0
Effect of cm scale sub-layers (30 degrees: Low and Middle E Contrast)

E1/E2 = 20/10; KIC1/KIC2 = 0.7/0.5; Dip angle = 30

E1/E2 = 40/10; KIC1/KIC2 = 1.0/0.5; Dip angle = 30
Effect of cm scale sub-layers (30 degrees: High E Contrast)

E1/E2 = 80/10; KIC1/KIC2 = 1.4/0.5; Dip angle = 30

Overall fracture propagation direction is inclined.

E1/E2 = 80/10; KIC1/KIC2 = 0.7/0.5; Dip angle = 30
Effect of Pore Scale Heterogeneity

Core scale heterogeneity

Pore scale heterogeneity

quartz or calcite

clay or kerogen

10 cm

10 cm

1.5 mm

1.5 mm
Effect of Pore Scale Heterogeneity (Model Construction)

(1) Borrowing the shape of minerals from the original picture

(2) Applying one of these properties to each mineral group

<table>
<thead>
<tr>
<th>Mineral type</th>
<th>Young’s modulus (GPa)</th>
<th>Shear modulus (GPa)</th>
<th>Fracture toughness (MPa m^{0.5})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>95.6</td>
<td>44.3</td>
<td>2.40</td>
</tr>
<tr>
<td>Calcite</td>
<td>83.8</td>
<td>32.0</td>
<td>0.19</td>
</tr>
<tr>
<td>Clay</td>
<td>10.0</td>
<td>4</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Effect of Pore Scale Heterogeneity (Case Settings)

Case 1: Quartz + Clay
(mineral connections: fully bonded)

Case 2: Quartz + Clay
(mineral connections: damaged)

Case 3: Quartz + Calcite + Clay
(mineral connections: fully bonded)

Case 4: Quartz + Calcite + Clay
(mineral connections: damaged)
Case 1: Quartz + Clay
No Interface Damage

Fracture initially propagates in the maximum principal stress direction.

Fracture turns along the mineral interface.

Fracture bypasses the turning path, and the old path closes.

Another case where bypassing occurs.
Case 1: Quartz + Clay
No Interface Damage
Case 2: Quartz + Clay

Interface damage

Fracture basically propagates along the mineral interface from the beginning.

Many branches appear along the mineral interfaces.

Finally the shortest path remains as the main path (bypassing).

(a) after 0.16 sec

(b) after 0.18 sec

(c) after 0.20 sec
Case 2: Quartz + Clay
With Interface damage
Case 3: Quartz + Calcite + Clay

No Interface Damage

1.5 mm

(a) after 0.01 sec

(b) after 0.05 sec

(c) after 0.08 sec

(d) after 0.1 sec

SH_max

SH_min

pre-damage zone inside the calcite

pre-damage zone inside the calcite

Main path connects pre-damage zone

Main path connects pre-damage zone

Main path connects pre-damage zone along the calcite

pre-damage zone along the calcite
Case 3: Quartz + Calcite + Clay
No Interface Damage
Case 4: Quartz + Calcite + Clay Interface Damage

(a) after 0.01 sec
(b) after 0.05 sec
(c) after 0.08 sec
(d) after 0.1 sec
Case 4: Quartz + Calcite + Clay
Interface Damage
Interaction between Hydraulic and Natural Fractures
Comparison with Experimental Results (Zhou et al. 2008)

### Case Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (degree)</td>
<td>30, 60, 90</td>
</tr>
<tr>
<td>$\Delta \sigma$ (MPa)</td>
<td>3, 5, 7, 10</td>
</tr>
</tbody>
</table>

### Basic Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>5.18</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>Matrix Permeability (mD)</td>
<td>0.1</td>
</tr>
<tr>
<td>Injection rate ($m^3$/s/m)</td>
<td>$1.05 \times 10^{-9}$</td>
</tr>
<tr>
<td>Distance between well and natural fracture (cm)</td>
<td>4.0</td>
</tr>
</tbody>
</table>
Results

*Note that Deformation is exaggerated 150 times.*

Factors Affecting Interaction of HF with NF Including Poroelastic Effects

**Base Case:**

\[ \theta = 60 \text{ deg} \quad \sigma_1 = 8 \text{MPa} \quad \sigma_3 = 5 \text{MPa} \]

Change the following six parameters for sensitivity analysis:

- Rock permeability: Poroelastic effects
- Shear strength of NF (failure criteria)
- NF toughness (NF critical strain)
- Rock toughness and Young’s modulus
- Initial Natural Fracture Permeability
- Injection Rate
Effect of Permeability

Permeability = 0.001 mD

Permeability = 0.01 mD

Permeability = 0.1 mD (Base)

High leak-off  
Low effective stress (shear failure)  
Fracture turning

Pore pressure distribution

Shear Failure Elements

Shear failure elements
Effect of Shear Strength of Natural Fracture

Coefficient = 0.89
Cohesion = 0.0 MPa

Coefficient = 0.89
Cohesion = 3.2 MPa
(Base)

Coefficient = 0.89
Cohesion = 7.0 MPa

Low shear strength of NF promotes HF turning
Effect of Natural Fracture Toughness

Natural Fracture Critical Strain
- $1.183 \times 10^{-3}$ (Base) ($K_{IC}=1.74 \text{ MPa m}^{-1/2}$)
- $0.592 \times 10^{-3}$ (Base) ($K_{IC}=0.87 \text{ MPa m}^{-1/2}$)
- $0.0$ ($K_{IC}=0.0 \text{ MPa m}^{-1/2}$)

Fracture toughness of NF also controls Mode I opening.

Low NF toughness $\rightarrow$ Easier fracture turning
Effect of Matrix Toughness

Assuming \( K_{IC} \propto \sqrt{E} \) in this case

High matrix toughness \( \rightarrow \) Encouraging fracture turning
3-D interaction Behavior

2-D interaction behavior

(a) Crossing

(b) Turning

(c) Re-initiating

If the NF fills the entire pay zone, these 2-D interactions cover all the patterns of interaction.

However, if

Hydraulic Fracture

Natural Fractures
Bahorich et al. (2012) showed more complicated 3-D interaction behavior could appear in the reservoir.

What kind of parameters affect these characteristic fracture propagation behaviors?
**Case Settings**

(Common Parameters)

- **Injector Water injection point**: 1.6 m
- **Natural fracture**: 0.64 m
- **Shmax (MPa)**: 41
- **Shmin (MPa)**: 40
- **NF shear trend**: 0.5
- **NF cohesion**: 0.0

Upper and lower boundary were fixed for mimicking boundary layers.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>30.0</td>
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<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
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<tr>
<td>Shmax (MPa)</td>
<td>41</td>
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<tr>
<td>Shmin (MPa)</td>
<td>40</td>
</tr>
<tr>
<td>NF shear trend</td>
<td>0.5</td>
</tr>
<tr>
<td>NF cohesion</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Case Settings

Effect of NF height, Position, and Tensile Strength

Case 1
(full height)

Case 2
(lower half)

Case 3
(lower half + weak TS)

Case 4
(middle half)

Case 5
(lower one-third)

Hydraulic Fracture
Natural Fracture
fracture propagation direction
pay zone (0.6m)

Hydraulic Fracture
Natural Fracture
fracture propagation direction
pay zone (0.6m)

Hydraulic Fracture
Natural Fracture
fracture propagation direction
pay zone (0.6m)

Hydraulic Fracture
Natural Fracture
fracture propagation direction
pay zone (0.6m)

weaker tensile strength

weaker tensile strength

fracture propagation direction

fracture propagation direction

fracture propagation direction

fracture propagation direction
Results
(Case 1: Reference)

(a) side-top view

(b) front-top view

(c) top view
Results
(Case 2: Lower Half NF)

(a) side-top view

(b) front-top view

(c) top view
Growth of Multiple Fractures in Naturally Fractured Reservoirs

No-NF model

5 non-competing fractures
The same amount of water is injected from each injection point.

NF model

The same amount of water is injected from each injection point.
Fracture Growth from 5 Clusters

Later, the center one grows longer with narrower thickness near wellbore.

Damage Distribution

Fracture Pressure Distribution
Multiple Fracture Growth with NF

Damage Distribution

Fracture Pressure Distribution
Conclusions

• A new peridynamics based hydraulic fracturing model was developed by modifying the existing elastic formulation to include poroelasticity and coupling it with the new peridynamics formulation for fluid flow.

• This model can simulate non-planar, multiple fracture growth in arbitrarily heterogeneous reservoirs by solving deformation of the reservoir, fracturing fluid pressure and pore pressure simultaneously.

• The validity of the model was shown through comparing model results with analytical solutions (1-D consolidation problem, the KGD model, the PKN model, and the Sneddon solution) and experiments.
Conclusions

- The effects of different types of layer heterogeneity on fracture propagation were systematically investigated.
- The factors controlling characteristic fracture propagation behaviors (“turning”, “kinking”, and “branching”) near the layer interface were quantified.
- In layered systems, the mechanical property contrast between layers, the dip angle, the stress contrast and poroelastic effects all play an important role in controlling the fracture trajectory.
- It was shown that even at the micro-scale, fracture geometry can be quite complex and is determined by the geometry and distribution of mineral grains and their mechanical properties.
Conclusions

• The principal stress difference, the approach angle, the fracture toughness of the rock, the fracture toughness of the natural fracture, and the shear strength of the natural fracture when hydraulic fractures interact with natural fractures.

• The 3-D interaction study elucidated that the height of the NF, the position of the NF, and the opening resistance of the NF have a huge impact on the three-dimensional interaction behavior between a HF and a NF.
Publications


Future Work

• Improvement of Computation Efficiency
  - Integration of peridynamics models with finite element models
  - More efficient solvers, pre-conditioners

• Model Extension/Improvement
  - Non-Newtonian fluid
  - Proppant transport

• Effect of heterogeneities at different scales
  - How smaller scale propagation behavior affects the larger scale propagation behavior
Acknowledgement

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Questions?
Backup Slides
# Project Schedule and Outcomes

<table>
<thead>
<tr>
<th>Task name</th>
<th>Assigned Resources / year</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Task 1 Project Management &amp; Planning</strong></td>
<td></td>
<td>Qtr 4</td>
<td>Qtr 1</td>
<td>Qtr 2</td>
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GRA = Graduate Research Assistant; PR = Postdoctoral Research Associate, PI = Principal Investigator
# Project Schedule and Outcomes

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<tr>
<th>Milestone Title/Description</th>
<th>Planned Completion Date</th>
<th>Actual Completion Date</th>
<th>Verification Method</th>
<th>Comments (Progress toward achieving milestone, explanation of deviation from plan etc.)</th>
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Impacts

• Both the modeling and the fracturing recommendations from this work are expected to have an immediate and long-term impact and benefit.

• The developed fracturing model and procedures would be applicable to all shale oil and gas resources that are more likely to have natural fractures and consequently result in more complex fracture patterns.

• This realistic model of hydraulic fracture propagation will allow better understanding of - the effects of fracture design on the stimulated rock volume and - well performance to potentially improve fracture and well design.

• Both items above should result in significant performance improvements and cost savings thereby allowing more wells to be drilled for the same annual budget.

• Cost reductions and smaller overall footage drilled will result in more economic wells and longer economic well lives resulting in a 5 to 10% increase in the recovery of oil and gas from these unconventional plays.
State-based Peridynamic Formulation Derivation (1)

Classical model: \( \nabla \left( \frac{\rho[x]}{\mu} K[x]. \nabla \Phi[x] \right) + r[x] = 0 \)

Develop its variational problem and infer the quadratic functional

\[
I[x] = \int_B Z[\nabla \Phi[x]] dV_x - \int_B r[x] \Phi[x] dV_x
\]

\[
Z[\nabla \Phi[x]] = \frac{1}{2} \nabla \Phi[x]. \left( \frac{\rho[x]}{\mu} K[x]. \nabla \Phi[x] \right).
\]

Remove restrictions on \( Z \)

- **Remove Locality:** let \( Z \) depend on points \( x' \) finite distance away from \( x \)
- **Remove Continuity:** let \( Z \) admit discontinuities in \( \Phi \)

\[
Z \equiv \hat{Z} = \hat{Z}(\Phi(x'), \Phi(x), x', x) = \hat{Z}(\Phi', \Phi, x', x) = \hat{Z}((\Phi' - \Phi), (x' - x)) = \hat{Z}(\Phi, \xi)
\]

We want to express \( Z \) as a function of potential difference and position difference instead of partial differentiation form
State-based Peridynamic Formulation Derivation(2)

Assume peridynamic analogue of the quadratic functional

\[
\hat{I}[x] = \int_{B} \hat{Z}[\Phi[x]]dV_x - \int_{B} r[x]\Phi[x]dV_x, \quad \Phi[x]\langle\xi\rangle = \Phi[x'] - \Phi[x]
\]

Minimizing this formulation gives us peridynamic fluid flow formulation

The stationary value of \(\hat{I}[x]\) at \(\delta\hat{I}[x] = 0\) leads to peridynamic equation

Fréchet derivative:

\[
\delta \hat{Z}[\Phi[x]] = \int_{B} \nabla \hat{Z} \langle\xi\rangle \cdot \delta \Phi[\xi]dV_{x'}
\]

\[
\delta \hat{I}[x] = \int_{B} \left( \int_{B} (-\nabla \hat{Z}[x] \langle\xi\rangle + \nabla \hat{Z}[x'] \langle-\xi\rangle) dV_{x'} + r[x] \right) \delta \Phi[x]dV_x = 0
\]

\[
Q[x]\langle\xi\rangle = -\nabla \hat{Z}[x]\langle\xi\rangle
\]

Peridynamic model:

\[
\frac{\partial}{\partial t} (\rho[x]\Phi[x]) = \int_{\mathcal{H}_x} \left( Q[x]\langle\xi\rangle - Q[x']\langle-\xi\rangle \right) dV_{x'} + R[x]
\]
\[ \int_{B} \left[ \nabla \cdot \left( \frac{\rho_0}{\mu} K[x] \nabla \Phi[x] \right) \right] \delta \Phi[x] \, dV_x + \int_{B} r[x] \delta \Phi[x] \, dV_x = 0. \]

\[ \int_{B} \left[ \nabla \delta \Phi[x] \cdot \left( \frac{\rho_0}{\mu} K[x] \nabla \Phi[x] \right) \right] \, dV_x - \int_{B} r[x] \delta \Phi[x] \, dV_x = 0, \]

\[ B[\delta \Phi[x], \Phi[x]] - l[\delta \Phi[x]] = 0, \]

\[ \delta I[x] = B[\delta \Phi[x], \Phi[x]] - l[\delta \Phi[x]] = 0, \]

\[ \delta I[x] = \frac{1}{2} B[\Phi[x], \Phi[x]] - l[\Phi[x]] = \delta \left[ \frac{1}{2} B[\Phi[x], \Phi[x]] - l[\Phi[x]] \right] = 0, \]

\[ I[x] = \frac{1}{2} \int_{B} \nabla \Phi[x] \cdot \left( \frac{\rho_0}{\mu} K[x] \nabla \Phi[x] \right) \, dV_x - \int_{B} r[x] \Phi[x] \, dV_x, \]

\[ Z[\nabla \Phi[x]] = \frac{1}{2} \nabla \Phi[x] \cdot \left( \frac{\rho_0}{\mu} K[x] \nabla \Phi[x] \right). \]

Defining peridynamics way of \( l[x] \)
Same procedure as local mocel with Frechet derivative
Local Theory

$$L = L(\dot{u}, \varepsilon_y) = T - U = \frac{1}{2} \rho \ddot{u} \cdot \ddot{u} - \left\{ \frac{1}{2} \lambda (\varepsilon_{kk})^2 + G \varepsilon_y \varepsilon_y \right\}$$

Kinetic

Strain Energy Density

$$(\partial L / \partial \dot{u})_i = \sigma_{ij}$$

$$\delta \int_0^T L dt = -\int_0^T \dot{L} dV dt$$

$$\delta \int_0^T L dt = \delta \int_0^T \int_V L dV dt$$

$$= \int_0^T \int_V \frac{\partial L}{\partial \dot{u}_i} \delta \dot{u}_i + \frac{\partial L}{\partial \varepsilon_y} \delta \varepsilon_y dV dt$$

$$= \int_0^T \int_V \frac{\partial L}{\partial \dot{u}_i} \delta \dot{u}_i dV dt + \int_0^T \int_V \frac{\partial L}{\partial \varepsilon_y} \delta \varepsilon_y dV dt$$

$$= \int_0^T \left[ -\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{u}_i} \right) - \sum_{j=1}^3 \frac{\partial L}{\partial \dot{\varepsilon}_{ij}} \delta \varepsilon_{ij} \right] dV dt$$

$$\Rightarrow -\rho \ddot{u}_i - \sum_{j=1}^3 \left( \lambda \delta_{ij} \dot{\varepsilon}_{kk} + 2G \varepsilon_y \dot{\varepsilon}_{ij} \right) = 0$$

$$\Rightarrow -\rho \ddot{u}_i - \sum_{j=1}^3 \left( \lambda \delta_{ij} \dot{\varepsilon}_{kk} + 2G \varepsilon_y \dot{\varepsilon}_{ij} \right) = 0$$

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2G \varepsilon_y \dot{\varepsilon}_{ij}$$

$$\rho \ddot{u} = -\nabla \cdot \sigma$$

Peridynamics Theory

$$L = L(\dot{u}, \varepsilon_y) = T - U = \frac{1}{2} \rho \ddot{u} dV - \int_B \Psi \left( \mathbf{Y}(x) \langle \xi \rangle \right) dV_x$$

Kinetic

Strain Energy Density

Frechet Derivative

Inserting

$$\delta \int_0^T L dt = 0 \int_0^T (\delta (T - U)) dt = 0$$

$$\int_0^T \delta T dt = -\int_0^T \int_B \rho \ddot{u} \cdot \delta u dV dt$$

$$\int_0^T \delta U dt = \int_0^T \int_B \delta \Psi \left( \mathbf{Y}(x) \langle \xi \rangle \right) dV_x dt$$

Finding stationary value

$$\int_0^T \left[ -\rho \ddot{u} - \int_B \left( \nabla \Psi \left( \mathbf{Y}(x) \langle \xi \rangle \right) - \nabla \Psi \left( \mathbf{Y}(x) \langle \xi \rangle \right) \right) dV_x \right] \delta u dV dt = 0$$

$$\int_0^T \left[ \int_B \left( \nabla \Psi \left( \mathbf{Y}(x) \langle \xi \rangle \right) \right) dV_x \right] \delta u dV dt = 0$$

$$\rho \ddot{u} = \int_B \left( \mathbf{T}(x) \langle \xi \rangle - \mathbf{T}(x') \langle -\xi \rangle \right) dV_x$$
Local Theory

\[ i[x] = \int_B Z[\nabla \Phi[x]] dV_x - \int_B r[x] \Phi[x] dV_x \]

\[ Z[\nabla \Phi[x]] = \frac{1}{2} \nabla \Phi[x]. \left( \frac{\rho[x]}{\mu} K[x]. \nabla \Phi[x] \right). \]

\[ \delta I[x] = 0 \]

\[ \nabla \left( \frac{\rho[x]}{\mu} K[x]. \nabla \Phi[x] \right) + r[x] = 0 \]

Peridynamics Theory

\[ \dot{i}[x] = \int_B \dot{Z}[\Phi[x]] dV_x - \int_B r[x] \Phi[x] dV_x \]

\[ \Phi[x](\xi) = \Phi[x'] - \Phi[x] \]

\[ \delta \dot{Z} \left[ \Phi[x] \right] = \int_B \nabla \dot{Z}(\xi). \delta \Phi(\xi) dV_{x'} \]

\[ \delta \dot{i}[x] = \int_B \left( \int_B (\nabla \dot{Z}(\xi) + \nabla \dot{Z}(\xi')(-\xi)) dV_{x'} + r[x] \right) \delta \Phi[x] dV_x = 0 \]

\[ \frac{\partial}{\partial t} (\rho[x] \Phi[x]) = \int_{\partial x} \left( \dot{Q}[x](\xi) - \dot{Q}[x'](-\xi) \right) dV_{x'} + \dot{R}[x] \]
Results for a Benchmark Flow Problem

5-spot well pattern

Injector
Producer

Steady-State

Pressure contours, (Analytical)

\[ k = 100 \text{ mD} = 10^{-13} \text{ m}^2 \]

\[ \mu = 0.001 \text{ Pa.s} \]

\[ N = 100 \]

\[ Q_T(x) = 0.001 \text{ m}^3 \text{ s}^{-1} \]
Results for a Benchmark Flow Problem

Steady-State

Classical exact at x=y
Peridynamics at x=y

Pressure (MPa)

Pressure (MPa)

at y = L/2
Constitutive Equations

- **Porosity equations**

  **matrix**

\[
\phi_i^{(n+1)} = \frac{V_{pi}^{(n+1)}}{V_{bi}} = \phi_i^{(n)} \left\{ 1 - C_r \left( P_i^{(n+1)} - P_i^{(n)} \right) \right\} + \alpha \left( 1 + \theta_{m\_local\_i}^{(n)} \right) \left\{ C_r \left( P_i^{(n+1)} - P_i^{(n)} \right) + \left( \theta_{m\_local\_i}^{(n+1)} - \theta_{m\_local\_i}^{(n)} \right) \right\}
\]

  **fracture**

\[
\phi_{fi}^{(n+1)} = \frac{V_{fi}^{(n+1)}}{V_{bi}} = \theta_{local\_i}^{(n+1)} - \theta_{local\@crit\_damage}
\]

where,

\[
\theta_{local\_i}^{(n+1)} = \sum_{j=1}^{N_{seg}} \left\{ \omega_j \left\| x_j - x_i \right\| e_j^{(n+1)} dV_j \right\} = \sum_{j=1}^{N_{seg}} \left\{ \omega_j \left\| x_j - x_i \right\| \left( \left\| y_j^{(n+1)} - x_i \right\| - \left\| x_j - x_i \right\| \right) dV_j \right\}
\]

\[
\theta_{m\_local\_i}^{(n+1)} = \sum_{j=1}^{N_{seg}} \left\{ \omega_j \left\| x_j - x_i \right\| e_j^{(n+1)} dV_j \right\} = \sum_{j=1}^{N_{seg}} \left\{ \omega_j \left\| x_j - x_i \right\| \left( \min \left( \left\| y_j^{(n+1)} - x_i \right\| - \left\| x_j - x_i \right\| \right) s_{crit} \left\| x_j - x_i \right\| \right) dV_j \right\}
\]

\[
m_{local\_i} = \sum_{j=1}^{N_{seg}} \left\{ \omega_j \left\| x_j - x_i \right\|^2 dV_j \right\}
\]

- **Fracture Permeability Equation**

\[
k_f = 0 \quad \text{if } d < d_{crit}
\]

\[
= \frac{w^2}{12} = \frac{\left( 2L_{ini} \phi_f \right)^2}{12} \quad \text{if } d \geq d_{crit}
\]
Constitutive Equations

• Flow between pore and fracture in a element

\[ I(x_i, t) = \frac{q_{i_{\text{inner}}}}{V_{bi}} = \frac{k_{mi} A_i (P_{fi} - P_{mi})}{V_{bi} \mu \Delta l_i} \]

• Force Scalar State

For bonds not passing through fracture surface

\[ T[x_i, t] \langle x_j - x_i \rangle - T[x_j, t] \langle x_i - x_j \rangle = \left( (3K - 5G) \left( \frac{\theta_i \omega_i}{m_i} + \frac{\theta_j \omega_j}{m_j} \right) - 3\alpha \left( \frac{P_i}{m_i} + \frac{P_j}{m_j} \right) \right) \| x_j - x_i \| + 15G \left( \frac{\omega_i}{m_i} + \frac{\omega_j}{m_j} \right) e_i \left( \frac{y_{j(n+1)} - y_{i(n+1)}}{y_{j(n+1)} - y_{i(n+1)}} \right) \]

For bonds passing through fracture surface

\[ T[x_i, t] \langle x_j - x_i \rangle - T[x_j, t] \langle x_i - x_j \rangle = 3P_{ji} \left( \frac{\omega_i}{m_i} + \frac{\omega_j}{m_j} \right) \| x_j - x_i \| \left( \frac{y_{j(n+1)} - y_{i(n+1)}}{y_{j(n+1)} - y_{i(n+1)}} \right) \]

• Flow Scalar State

For matrix

\[ Q_m[x_i, t] \langle x_j - x_i \rangle - Q_m[x_j, t] \langle x_i - x_j \rangle = \frac{\gamma P_{mi}}{\mu} \frac{\xi (K_{mi} - 0.25 \text{trace}(K_{mi}) \mathbf{I}) \xi}{\| \xi \|^4} (P_{mj} - P_{mi}) \]

For fracture

\[ Q_f[x_i, t] \langle x_j - x_i \rangle - Q_f[x_j, t] \langle x_i - x_j \rangle = \frac{\gamma P_{ji} k_{i,j}}{2\mu} \left( \frac{\xi (P_{ji} - P_{fi})}{\| \xi \|} \right) \]
How to apply constant stress boundary condition?

Constant stress boundary condition is given as a body force in the elements which distance from the boundary is less than horizon size.

Example case: horizon size = 3 delta (boundary forces are given to three layers from the boundary)

\[
\vec{b}_{\text{boundary}} = \sum_{j=1}^{N_{\text{ghost}}} \left\{-3\tau_{\text{bounary}} \left( \frac{\omega_i}{m_i} + \frac{\omega_j}{m_j} \right) \|x_j - x_i\left\| V_j \frac{\vec{\xi}}{\|\vec{\xi}\|} \right\right\}
\]

*note that the equation above is for 3D.

(for 2D) \[
\vec{b}_{\text{boundary}} = \sum_{j=1}^{N_{\text{ghost}}} \left\{-2\tau_{\text{bounary}} \left( \frac{\omega_i}{m_i} + \frac{\omega_j}{m_j} \right) \|x_j - x_i\left\| V_j \frac{\vec{\xi}}{\|\vec{\xi}\|} \right\right\}
\]
Fracture Surface Connection

We want to limit our fracture elements in 1\textsuperscript{st} line even if 2\textsuperscript{nd} line damage exceeds a critical damage. How?

• Defining which surface has fracture in each element
• One element cannot have two independent frac. Surface
• New fracture surface must connecting pre existing fracture surface
Comparison with KGD solution
(Fracture Pressure Distribution after 80 sec)

Permeability as a function of width

Infinite Conductivity

*note that Shmin = 8 MPa, Deformation is exaggerated 2000 times.
ervor rock requires a particular boundary condition at the tip of the fracture. This condition, for the first time suggested by Zheltov and Khristianovich, and later clarified by Bear, is that, in the case of a fracture in mobile equilibrium propagating in a brittle solid, the distribution of normal pressure exerted by the fracturing fluid on the fracture walls must be such that the faces of the fracture close smoothly at the edges. The condition of smooth closing implies that
\[
\left( \frac{d w}{d f_1} \right)_{f_2=0} = 0 \quad \text{and} \quad \left( \frac{d w}{d f_2} \right)_{f_1=0} = 0, \text{ respectively}.
\]
Barenblatt proved that this ensures that the normal stress component at the tip of the fracture is finite and equal to the tensile strength of the rock. The tensile strength can be assumed to be negligible influence for large-scale fractures in the practical range of overburden pressures (see Perkins and Ketchum). Substitution of the above boundary condition in Eqs. 3 and 4 leads to
\[
\int_0^\infty p f_1 f_2 d f_1 = -\frac{2}{3} K \left( \frac{d w}{d f_1} \right)_{f_2=0} = S, \quad \text{for the linear configuration, and}
\]
\[
\int_0^\infty p f_1 f_2 d f_1 = -\frac{2}{3} K \left( \frac{d w}{d f_1} \right)_{f_2=0} = S, \quad \text{for the circular fracture.}
\]
(Note that if the tensile strength had been taken into account, Eqs. 5 and 6 would have been found to be
\[
\int_0^\infty p f_1 f_2 d f_1 = -\frac{2}{3} K \left( \frac{d w}{d f_1} \right)_{f_2=0} = S + \frac{K}{2} \sqrt{\frac{2}{\pi} \frac{G}{S}},
\]
and
\[
\int_0^\infty p f_1 f_2 d f_1 = -\frac{2}{3} K \left( \frac{d w}{d f_1} \right)_{f_2=0} = S + \frac{K}{2} \sqrt{\frac{2}{\pi} \frac{G}{S}},
\]
respectively, in which \(K = \frac{\pi E}{2(1-\nu^2)} = \text{Barenblatt's cohesion modulus. This expression is Young's modulus}\) and \(a\) the specific surface energy. Our theory thus assumes that \(2L = \frac{\sqrt{\pi} E}{\sqrt{\nu}} \) and that \(2R \geq \frac{\nu}{2} \), respectively.

**Equations for Fracture Width and Shape**

In Appendices A and B approximate solutions for the sets of equations (Eqs. 1, 3, 5 and Eqs. 2, 4, 6) are derived. For a linearly propagating fracture the maximum width at the origin amounts approximately to
\[
w_a = 2w_0 \sqrt{\frac{G}{K}}.
\]
for an average value of Poisson's ratio, \(\nu = 0.25\); and the shape of the fracture, except in a narrow wedge-like zone near the tip, is more or less elliptical:
\[
w = w_0 \left( 1 - \frac{r}{r_f} \right).
\]
Eq. 7 is valid for
\[
\frac{\sqrt{\frac{G}{K}}}{S \sqrt{L}} \ll 1, \quad \text{say} < 0.05.
\]
This means that by combining condition (9) with Eq. 7, the theory is valid for at least \(w_o \ll \frac{S}{K} \) or \(w_o \leq \frac{S}{G} \).

For instance if \(G = 10^6 \text{kgs/cm}^2\) at a depth where \(S = 200 \text{ kg/cm} \cdot \text{cm}, w_o \) must be smaller than 20 mm for \(L = 10 \text{m}\).

Under these conditions, it develops that the fluid-injection pressure with respect to the tectonic stress perpendicular to the fracture walls, \(S\), is
\[
p_w = S + 2 G w_o
\]
Because according to Eq. 7 \(w_o\) increases in proportion to \(\sqrt{L}\), it is found that \(p_w\) decreases with increasing fracture length and approaches \(S\) for large values of \(L\). Such pressure behavior is in agreement with reported field observations. A check for the validity of the assumption of laminar flow is that the Reynolds number, \(R_o\), equals \(p_d \rho / \mu\) is less than 1,000, where \(p\) is the liquid density.

For a radially propagating fracture, the maximum width at the wellbore, again for \(\nu = 0.25\), is approximately
\[
w_a = 2 \sqrt{\frac{G}{K} w_o}
\]
and the shape is parabolic except for a narrow zone near the tip:
\[
w = w_0 \sqrt{1 - \frac{r}{r_f}}.
\]
Eq. 12 is valid for
\[
\frac{\sqrt{\frac{G}{K}}}{S \sqrt{R}} \ll 1, \quad \text{again say} < 0.05, \quad \text{or} \quad w_o \leq \frac{S}{G}.
\]
The fluid pressure at the entrance of the fracture \((r = R_o)\) decreases with increasing fracture radius \(R\) according to
\[
p_e = S - \frac{S}{2G} w_0^2 w_o.
\]
In terms of the Reynolds number, laminar flow conditions are now fulfilled provided \(R_o\) equals \(Q / \rho / 2 \pi r_f \) less than 1,000. The fracturing pressure will usually behave in a laminar fashion, except in a certain area near the wellbore. As long as this area is limited to a few well radii, it will hardly invalidate the theory given.

**Effect of Formation Permeability on Fracture Dimensions**

Communication between fracture volume and the

\[
w_a = 2 \sqrt{\frac{G}{K} w_o},
\]
for an elliptical shape with the semi-major axis \(a\) and semi-minor axis \(b\); where \(a = w_a\) and \(b = w_0\).

**Fracture Width Determination for a Linear Mode of Propagation**

The behavior of a linearly propagating fracture has been considered in some detail by Khristianovich and Zheltov. A conformal mapping technique was used for finding the displacement field. We give here a simplified approach that leads to the practical formula, Eq. 7. To this end, we assume a plausible pressure distribution in the fracture and calculate from Eq. 7 the fracture shape, and from Eq. 1 the pressure distribution in such a fracture. This will show whether or not the assumption is acceptable.

Barenblatt's condition that closure must be smooth implies infinite flow resistance at the very tip of the fracture, so that pressure here must be zero. Since smooth closure the increase in fracture width is more than proportional to the distance to the tip, the pressure gradient decreases by at least the third power of the distance (see Eq. 1). Therefore rapidly becomes very small, and it is plausible to approximate the pressure distribution in the fracture by the discontinuous one*

\[
p = \frac{p_0}{2} \frac{r}{r_f} \quad \text{for} \quad 0 < r < r_f,
\]
\[
p = 0 \quad \text{for} \quad r > r_f,
\]
where we suppose, a priori, \(p_0 = 1\), Barenblatt's condition equation (Eq. 5) gives, with such a distribution,
\[
w_o = \frac{1}{2} \sqrt{\frac{G}{K} w_o} = \frac{1}{2} \sqrt{\frac{G}{K} w_o}.
\]
The fracture shape resulting from this pressure distribution follows from Eq. 7:
\[
w = \frac{2}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o} + \frac{1}{2} \frac{1}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o}.
\]
This shows that the maximum fracture width at the wellbore amounts to
\[
w_o = \frac{2}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o} + \frac{1}{2} \frac{1}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o}.
\]
For \(r = 1\) this reduces to
\[
w = \frac{2}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o},
\]
whereas a good approximation of the equilibrium condition (Eq. 1) is
\[
(p - S) = \frac{2}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o}.
\]
Combining the last two approximations leads to
\[
w_o = \frac{2}{\sqrt{\pi} \sqrt{G}} \sqrt{\frac{G}{K} w_o}.
\]
For a given pressure in the fracture in excess of the tectonic stress, the fracture width at the origin is thus to a first approximation independent of \(p_0\), i.e., it does not depend on the extent of the region of zero

**APPENDIX A**

**Fracture Width Determination for a Linear Mode of Propagation**

The behavior of a linearly propagating fracture has been considered in some detail by Khristianovich and Zheltov. A conformal mapping technique was used for finding the displacement field. We give here a

Parallel Performance

About 30 times speed up by 128 CPU
Biot Consolidation Validation

- Young's modulus (GPa) = 30
- Poisson's ratio = 0.25
- Porosity = 0.02
- Permeability (mD) = 6
- Biot coefficient = 0.6667
- Initial pore pressure (MPa) = 3.82
- Fluid: Water

Results: Pressure and Deformation

[Diagram showing pressure and deformation over time]
Shear Failure Model in Peridynamics

Disassembling original force vector state into two directions

Deleting tangential force vector state once shear failure criteria is satisfied

Natural Fracture Surface
Results
(Case 2: Lower Half NF)
Results
(Case 4: Middle Half NF)
Investigation of Fracture Propagation Behavior in 3 Layers

If the layer is sanded by the two different Young’s modulus layers, the fracture always propagate more to the softer layer at first.

However, after reaching the layer interface with the low Young’s modulus layer, the fracture propagation behavior changes with mechanical properties of each layer.

Which parameter governs the preferential fracture propagation direction?
Model Description

Layer 1: $\sigma_{H1}$ 10 cm
E1, KIC1

Layer 2: $\sigma_{H2}$ 10 cm
E2, KIC2
Injector

Layer 3: $\sigma_{H3}$ 10 cm
E3, KIC3

$\sigma_V$
30 cm
Results

Which parameter controls the preferential propagation direction?

| E 40/20/10 GPa, KIC 0.707/0.5/0.354 MPa m^{0.5} Sxx 40/40/40 MPa (Gc 11.7/11.7/11.7 J/m^2) |
| E 20/40/10 GPa, KIC 0.5/0.707/0.354 MPa m^{0.5} Sxx 40/40/40 MPa (Gc 11.7/11.7/11.7 J/m^2) |
| E 40/10/20 GPa, KIC 0.707/0.354/0.5 MPa m^{0.5} Sxx 40/40/40 MPa (Gc 11.7/11.7/11.7 J/m^2) |
| E 40/12/10 GPa, KIC 0.707/0.5/0.354 MPa m^{0.5} Sxx 40/40/40 MPa (Gc 11.7/19.5/11.7 J/m^2) |
| E 40/20/10 GPa, KIC 1.2/0.5/0.354 MPa m^{0.5} Sxx 40/40/40 MPa (Gc 23.4/11.7/11.7) |
| E 40/20/10 GPa, KIC 1.6/0.5/0.354 MPa m^{0.5} Sxx 40/40/40 MPa (Gc 33.8/11.7/11.7) |
| E 40/20/10 GPa, KIC 0.707/0.5/0.354 MPa m^{0.5} Sxx 45/40/40 MPa (Gc 11.7/11.7/11.7 J/m^2) |
| E 40/20/10 GPa, KIC 0.707/0.5/0.354 MPa m^{0.5} Sxx 50/40/40 MPa (Gc 11.7/11.7/11.7 J/m^2) |
Theoretical Consideration
(Critical Displacement)

To break a bond, how much deformation is necessary?

\[
\omega = \frac{9Gc}{4\delta^3} = \frac{9Kc^2(1-v^2)}{4\delta^3E}
\]

\[
\omega = \int_0^{\omega} \left[ \mathbf{T}[\mathbf{x}, \mathbf{t}] \mathbf{\xi} - \mathbf{T}[\mathbf{x'}, \mathbf{t}] \mathbf{\xi'} \right] d\mathbf{\eta}
\]

If \( \omega > \omega_c \), bond will break.

\[
\mathbf{T}[\mathbf{x}, \mathbf{t}] \mathbf{\xi} = \mathbf{T}[\mathbf{x}, \mathbf{t}] \mathbf{\xi} + \mathbf{T}_0 \mathbf{\xi}
\]

For the simplicity, here we neglect background vector state term and poroelastic effect.

\[
\mathbf{\xi} \mathbf{\eta} = \left[ \frac{2}{m} \left( K - \frac{G}{3} \right) \mathbf{\omega} \mathbf{\eta} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi} \right] \frac{\mathbf{\xi} + \mathbf{\eta}}{\mathbf{\xi} + \mathbf{\eta}}
\]

inserting \( \mathbf{\xi} \mathbf{\eta} = \mathbf{\xi} - \frac{\mathbf{\eta}}{3} \mathbf{\xi} \)

\[
\mathbf{\xi} \mathbf{\eta} = \left[ \frac{2}{m} \left( K - \frac{G}{3} \right) \mathbf{\omega} \mathbf{\eta} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi} \right] \frac{\mathbf{\xi} + \mathbf{\eta}}{\mathbf{\xi} + \mathbf{\eta}}
\]

\[
\mathbf{\xi} \mathbf{\eta} = \frac{2}{m} \left( 3K - 5G \right) \mathbf{\omega} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi}
\]

\[
\mathbf{\xi} \mathbf{\eta} = \frac{2}{m} \left( 3K - 5G \right) \mathbf{\omega} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi}
\]

\[
\mathbf{\xi} \mathbf{\eta} = \frac{2}{m} \left( 3K - 5G \right) \mathbf{\omega} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi}
\]

\[
\mathbf{\xi} \mathbf{\eta} = \frac{2}{m} \left( 3K - 5G \right) \mathbf{\omega} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi}
\]

\[
\mathbf{\xi} \mathbf{\eta} = \frac{2}{m} \left( 3K - 5G \right) \mathbf{\omega} + \frac{8G}{m} \mathbf{\omega} \mathbf{\eta} \mathbf{\xi}
\]

If we assume Poisson's ratio = 0.25 and \( \omega = 1.0 \) for the simplicity, the equation above becomes the function of \( E \) and delta.

\[
= 8 \left[ \frac{G}{m} (\mathbf{\xi} + \mathbf{\eta}) \right] \frac{\mathbf{\xi} + \mathbf{\eta}}{\mathbf{\xi} + \mathbf{\eta}} = 16 \left( \frac{3}{5} \frac{E}{\pi \delta^3} \right) \frac{\mathbf{\xi} + \mathbf{\eta}}{\mathbf{\xi} + \mathbf{\eta}} = 48E \frac{5 \pi \delta^3 (\mathbf{\xi} + \mathbf{\eta})}{\mathbf{\xi} + \mathbf{\eta}}
\]
The critical displacement for breaking a bond is proportional to

\[ K_{IC} = \frac{K_{IC}^2 (1 - \nu^2)}{64E^2} \]
K/E and horizontal stress control the preferential stress direction.

- **E 40/20/10 GPa**, KIC 0.707/0.5/0.354 MPa m⁰.⁵
  - Sxx 40/40/40 MPa
  - (KIC/E 0.018/0.025/0.035)

- **E 20/40/10 GPa**, KIC 0.5/0.707/0.354 MPa m⁰.⁵
  - Sxx 40/40/40 MPa
  - (KIC/E 0.025/0.018/0.035)

- **E 40/10/20 GPa**, KIC 0.707/0.354/0.5 MPa m⁰.⁵
  - Sxx 40/40/40 MPa
  - (KIC/E 0.018/0.035/0.025)

- **E 40/12/10 GPa**, KIC 0.707/0.5/0.354 MPa m⁰.⁵
  - Sxx 40/40/40 MPa
  - (KIC/E 0.025/0.018/0.035)

- **E 40/20/10 GPa**, KIC 1.2/0.5/0.354 MPa m⁰.⁵
  - Sxx 40/40/40 MPa
  - (KIC/E 0.03/0.025/0.035)

- **E 40/20/10 GPa**, KIC 1.6/0.5/0.354 MPa m⁰.⁵
  - Sxx 40/40/40 MPa
  - (KIC/E 0.04/0.025/0.035)

- **E 40/20/10 GPa**, KIC 0.707/0.5/0.354 MPa m⁰.⁵
  - Sxx 45/40/40 MPa
  - (KIC/E 0.018/0.025/0.035)

- **E 40/20/10 GPa**, KIC 0.707/0.5/0.354 MPa m⁰.⁵
  - Sxx 50/40/40 MPa
  - (KIC/E 0.018/0.025/0.035)