Dynamics of Premixed Flames Subjected to Helical Disturbances

Vishal Acharya and Tim Lieuwen
Ben T. Zinn Aerospace Combustion Laboratory
Daniel Guggenheim School of Aerospace Engineering, Georgia Institute of Technology

Introduction

Motivation

Lean, premixed combustion used for reducing Unfavorable emissions is prone to combustion instability

- Unfavorable feedback between unsteady heat release rate oscillations and natural acoustic modes causes high amplitude pressure oscillations, damages hardware, increases costs and emissions.
- Key mechanism
  - **Velocity oscillations → Focus**
  - Fuel/air ratio oscillations

Helical Flow Disturbances: Spatial Modal Decomposition

$\hat{u}_i'(r,\theta,z,\omega)$

$$\hat{B}_{i,m}(r,z,\omega) = \frac{1}{2\pi}\int_{\theta=0}^{\theta=2\pi} \hat{u}_i'(r,\theta,z,\omega)e^{-im\theta}d\theta$$

$$\hat{u}_i'(r,\theta,z,\omega) = \sum_{m=-\infty}^{\infty} \hat{B}_{i,m}(r,z,\omega)e^{im\theta}$$

***m*<0 – co-swirling (counter-clockwise)**
***m*>0 – counter-swirling (clockwise)**

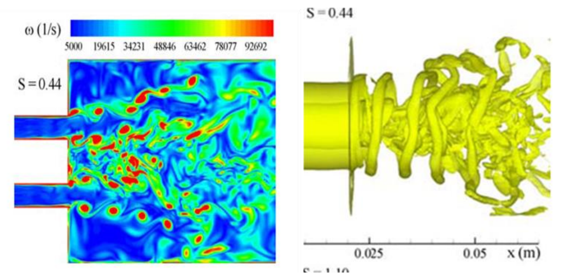
Swirl Combustors

Swirling flows aid in flame stabilization

- High turbulence content (turbulent flame speed enhancement)
- Recirculation features of flow

Swirling flows possess complex unsteady features such as helical instabilities.

- Helical mode decomposition (integer modes)
- Flow instabilities are vortical in nature
- Helical modes are excited during instabilities



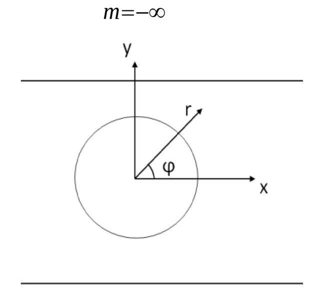
Reproduced from simulation by Huang & Yang (2005)

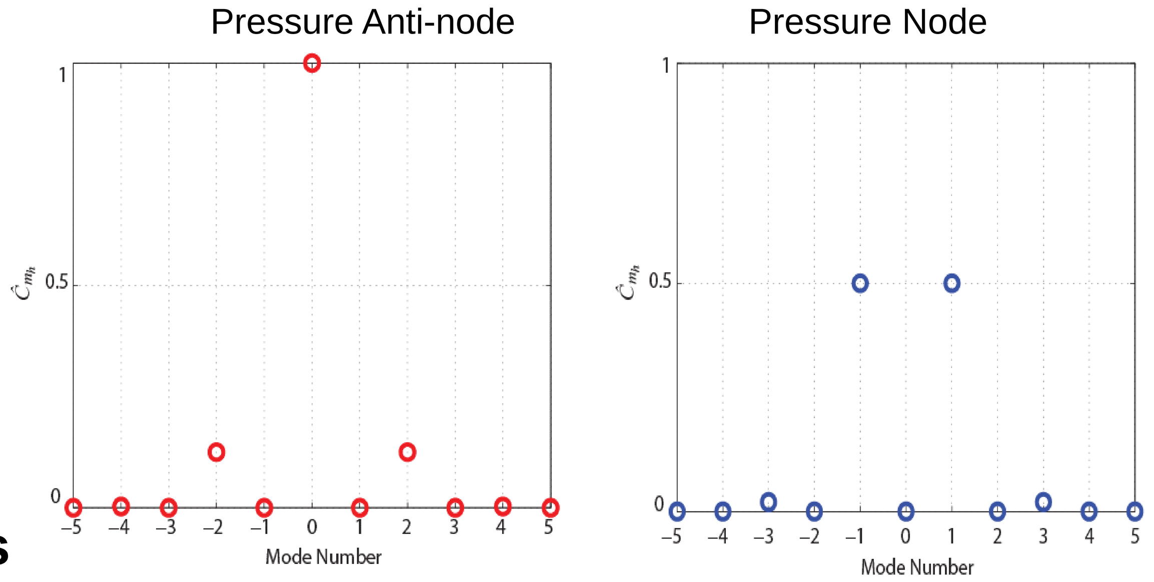
Objectives

- **Acoustics excites helical disturbances which then excite the flame**

Acoustic waveform

$$f(x) = \sum_{m=-\infty}^{\infty} \hat{C}_m(kr)e^{im\varphi}$$





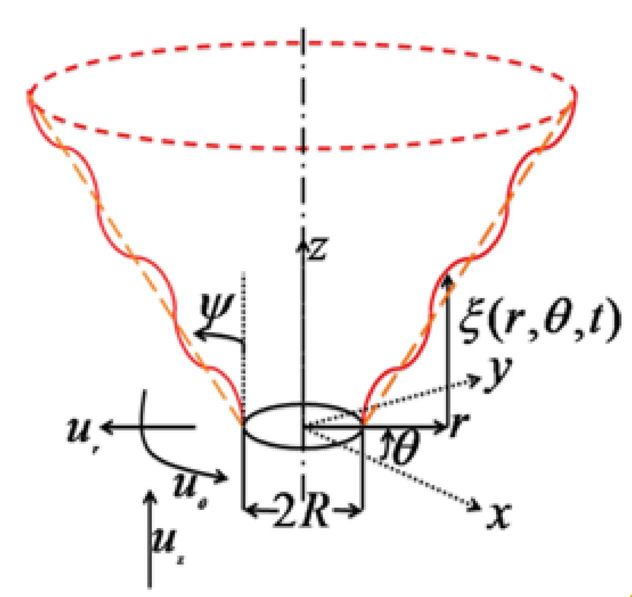
- **Experiments**
  - Asymmetric helical modes dominate flame response.
  - How conclusive are these observations?
    - Most conclusions from planar diagnostics
    - Out of plane physics unavailable
  - Motivates a 3D model for flame response
- **Modeling objectives**
  1. Determine how different helical modes affect flame response.
  2. Determine importance of mean flame shape.
  3. Do these answers change for high amplitude effects?

Analytic Flame Description

Thin-flame assumption (level-set)
Inverted conical flame

$$\frac{\partial G}{\partial t} + \vec{u}\cdot\vec{\nabla}G = s_L\left|\vec{\nabla}G\right|$$
$$G\big|_{flame} \equiv 0$$
$$\Rightarrow G(r,\theta,z,t) = z - \xi(r,\theta,t)$$

$$\frac{\partial \xi}{\partial t} + u_r\frac{\partial \xi}{\partial r} + \frac{u_\theta}{r}\frac{\partial \xi}{\partial \theta} + s_L\left[\left(\frac{\partial \xi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \xi}{\partial \theta}\right)^2 + 1\right]^{1/2} = u_z$$



Linear Dynamics

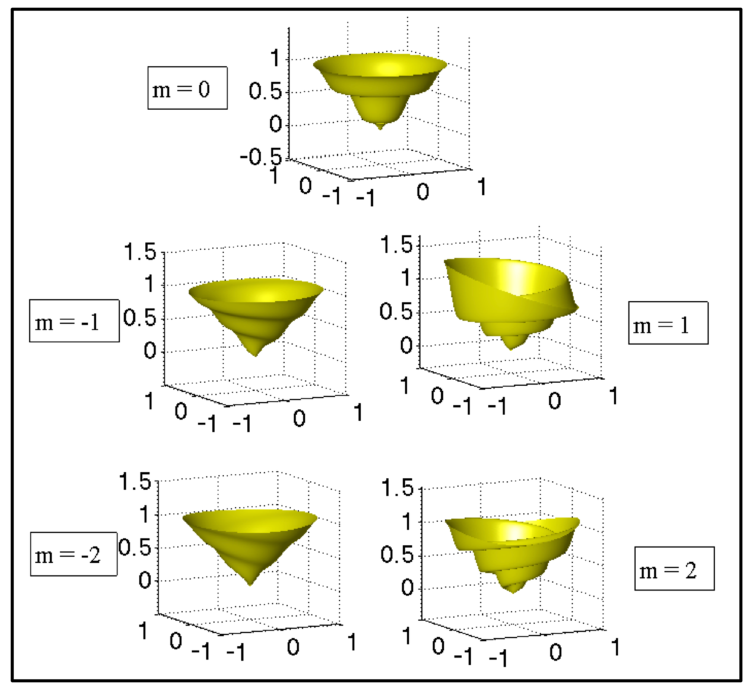
$$\frac{\partial \xi}{\partial t} + u_r\frac{\partial \xi}{\partial r} + \frac{u_\theta}{r}\frac{\partial \xi}{\partial \theta} + s_L\left[\left(\frac{\partial \xi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \xi}{\partial \theta}\right)^2 + 1\right]^{1/2} = u_z$$

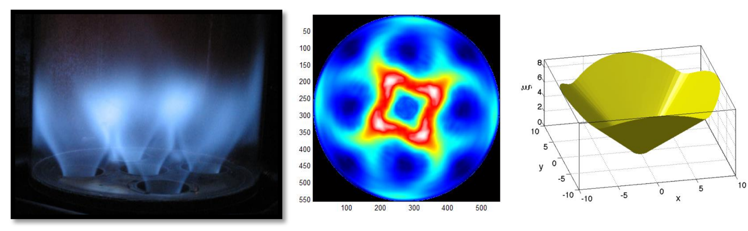
$$u_i = u_{i,0} + \varepsilon u_{i,1}$$
$$\xi = \xi_0 + \varepsilon \xi_1$$

$$u_{r,0}\frac{\partial \xi_0}{\partial r} + s_L\sqrt{1+\left(\frac{\partial \xi_0}{\partial r}\right)^2} = u_{z,0}$$

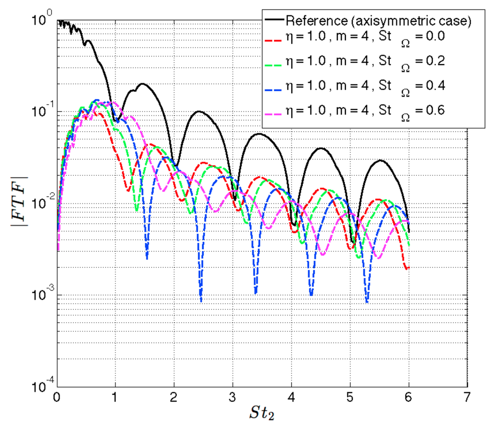
$$\frac{\partial \xi_1}{\partial t} + \left(\vec{u}_{t,0}\cdot\vec{\nabla}\xi_1\right) = u_{z,1} - u_{r,1}\frac{\partial \xi_0}{\partial r}$$

- Local flame response is non-zero for all helical flow disturbance modes.
- Non-axisymmetric modes ($m \neq 0$) dominate local flame response in general.
- Global flame response (spatially integrated total heat release) is affected **only** by the **symmetric** mode ($m = 0$).
  - Flame filters all other modes, even the dominant flow modes !
  - **Only for axisymmetric mean flames**
  - Motivates a study on mean flame asymmetry effects (flame-flame interactions).
    - Flame-flame interactions
- Azimuthal modes ($n \neq 0$) of mean flame shape interact with asymmetric helical modes ($m \neq 0$) of flow disturbance when ($m + n = 0$)
  - Flame response due to strong asymmetric helical mode amplitude are further magnified by the "magnitude" of flame asymmetry





5-flame combustor showing flame-flame interactions (courtesy of Dom Santavicca)



Non-Linear Dynamics

$$\frac{\partial \xi}{\partial t} + u_r\frac{\partial \xi}{\partial r} + \frac{u_\theta}{r}\frac{\partial \xi}{\partial \theta} + s_L\left[\left(\frac{\partial \xi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \xi}{\partial \theta}\right)^2 + 1\right]^{1/2} = u_z$$

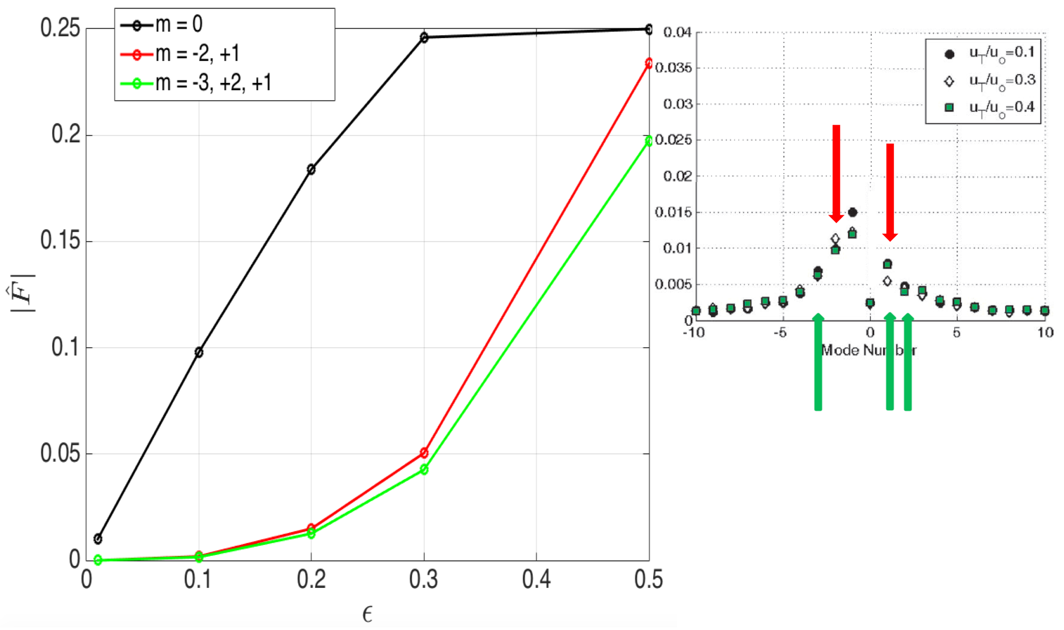
$$u_i = u_{i,0} + \varepsilon u_{i,1}$$
$$\xi = \xi_0 + \varepsilon\xi_1 + \varepsilon^2\xi_2 + \varepsilon^3\xi_3$$

$$u_{r,0}\frac{\partial \xi_0}{\partial r} + s_L\sqrt{1+\left(\frac{\partial \xi_0}{\partial r}\right)^2} = u_{z,0}$$

$$\frac{\partial \xi_1}{\partial t} + \left(\vec{u}_{t,0}\cdot\vec{\nabla}\xi_1\right) = u_{z,1} - u_{r,1}\frac{\partial \xi_0}{\partial r}$$
$$\frac{\partial \xi_2}{\partial t} + \left(\vec{u}_{t,0}\cdot\vec{\nabla}\xi_2\right) = -\left(u_{r,1}\frac{\partial \xi_1}{\partial r} + u_{\theta,1}\frac{1}{r}\frac{\partial \xi_1}{\partial \theta} + \frac{s_L}{2}\frac{\xi_{1,r}^2 + \left(\xi_{1,\theta}^2\right)/r^2}{\sqrt{1+\xi_{0,r}^2}}\right)$$
$$\frac{\partial \xi_3}{\partial t} + \left(\vec{u}_{t,0}\cdot\vec{\nabla}\xi_3\right) = -\left(u_{r,1}\frac{\partial \xi_2}{\partial r} + u_{\theta,1}\frac{1}{r}\frac{\partial \xi_2}{\partial \theta} + 2s_L\frac{\xi_{1,r}\xi_{2,r} + \left(\xi_{1,\theta}\xi_{2,\theta}\right)/r^2}{\sqrt{1+\xi_{0,r}^2}}\right)$$

- Axisymmetric flames filter out helical modes
  - Only for low amplitudes
  - High amplitude?
  - Requires non-linear analysis
- Helical mode effects
  - Linear (symmetric mode only)
  - Non-linear (**Triadic interaction)**

$$\sum_{k=1}^{3}\alpha_k m_k = 0 \quad \text{where} \quad \sum_{k=1}^{3}\alpha_k = 3$$



3 mode interaction $\rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 1; m_1 = m, m_2 = n \neq m, m_3 = -n-m$
2 mode interaction $\rightarrow \alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 0; m_1 = 2m, m_2 = -m$

Conclusions

Helical flow disturbances

- Linear low-amplitude
  - Axisymmetric mean flame → Only symmetric mode contributes=> Linear system stability affected only by m=0
  - Non-Axisymmetric mean flame (flame-flame interactions) → Asymmetric helical modes (m≠0) of flow disturbances interact with asymmetric modes of mean flame (n ≠0) when m+n=0
- Non-Linear high-amplitude
  - Axisymmetric mean flame → Triadic interactions (3[rd] order) => Limit cycle affected by asymmetric helical modes
  - Non-Axisymmetric mean flame (flame-flame interactions) → Interaction between asymmetric helical modes of flow disturbance and asymmetric modes of mean flame shape

ACKNOWLEDGEMENTS
This research was partially supported by Department of Energy under contracts DEFG26-07NT43069 (contract monitor Mark Freeman) and DE-NT0005054 (contract monitor Richard Wenglarz), National Science Foundation under contract CBET-1235779 (contract monitor Ruey-Hung Chen) and NSF XSEDE (TeraGrid) under contracts TG-CTS130016 and TG-DMS130001.

REFERENCES/PUBLICATIONS
- **Vishal Acharya**, Shreekrishna, Dong-Hyuk Shin and Tim Lieuwen, "Swirl Effects on Harmonically Excited Premixed Flame Kinematics", *Combustion and Flame,* Vol. 159, Issue 3, March 2012, pp.1139-1150.
- **Vishal Acharya,** Dong-Hyuk Shin and Tim Lieuwen, "Premixed Flame Excited by Helical Disturbances: Flame Wrinkling and Heat Release Oscillations", *Journal of Propulsion and Power,* 2013, Vol. 29, No. 6, 2013, pp. 1282-1291, doi: 10.2514/1.B34883.
- Jacqueline O'Connor, **Vishal Acharya** and Tim Lieuwen, "Transverse Combustion Instabilities: Acoustic, Fluid Mechanic and Flame Processes", *Progress in Energy and Combustion Sciences*, Vol. 49, 2015, pp. 1-39.
- **Vishal Acharya** and Tim Lieuwen, "Response of Non-axisymmetric Premixed, Swirl Flames to Helical Disturbances", *ASME IGTI Turbo Expo,* Dusseldorf, Germany, Jun. 2014, Paper# GT2014-27059.
- **Vishal Acharya** and Tim Lieuwen, "Premixed Flame Response to Helical Flow Disturbances: Non-linear Effects", *9th US National Technical Meeting of the Combustion Institute*, Cincinnati, OH, USA, May 2015, Paper# 114IC-0265.