Estimating Current Densities in Equilibrium Magnetohydrodynamic Generator Channels

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Magnetohydrodynamic Power Generation

- **Magnetohydrodynamic Generators (MHDG)** offer thermodynamic efficiency improvements as a topping cycle to traditional steam cycle power generation.
- Due to thermal boundary layer effects, a conductivity gap forms in a thin layer near electrodes
- Current must cross this gap and does so in a dense “arc”.
- Modeling and detection of these arcs is necessary for controlling the phenomenon.

Current Reconstruction from External Magnetic Fields

Reconstruction of currents from induced fields has many applications.

- Diagnostics for batteries and fuel cells
- Explosive shock detection relies on passing explosive products through a strong magnetic field
- Arc detection in vacuum arc remelters

Simulation Based Estimation

Current reconstruction is typically done by employing the Biot-Savart Law

- This relies on the solution of *integral equations*, which typically involves special assumptions of geometry or material parameters
- Instead, we solve a *differential equations model*
- Requires the minimization of a discrepancy function using Newton’s method to explore parameter space
- Requires no special assumptions of geometry or material

Static Maxwell System

- Assume: generator is in equilibrium
- Variables: $\mathbf{e}$ electric field, $\mathbf{b}$ magnetic flux density
- Data: ρ_c charge density, $\mathbf{j}$ current density.

$\mathbf{e} - \nabla\psi = 0$	Faraday’s Law
$\nabla\cdot\epsilon\mathbf{e} = \rho_c$	Gauss’ Law
$\nabla\times\mu^{-1}\nabla\times\mathbf{a} + \nabla\lambda = \mathbf{j}$	Ampère’s Law
$\nabla\cdot\mathbf{a} = 0$	Coulomb Gauge
$\mathbf{j} = \sigma(\mathbf{e} + \mathbf{u}\times\mathbf{b}) + \beta\lvert\mathbf{b}\rvert^{-1}\mathbf{j}\times\mathbf{b}$	Ohm’s Law

Mimetic Finite Differences

- **Mimetic Finite Differences (MFD)** generalize Yee-Scheme type staggered differences to general meshes.
- It is amenable to Lagrangian frame discretizations and complex domain geometry.
- **MFD** Discretizations obey classical duality relationships on appropriate spaces.

$$\int_G \nabla\phi\cdot\mathbf{u} = -\int_G \phi\nabla\cdot\mathbf{u} \qquad \phi\in H_0^1, \mathbf{u}\in\mathbf{H}^{\nabla\cdot}$$

$$\int_G \nabla\times\mathbf{u}\cdot\mathbf{v} = \int_G \mathbf{u}\cdot\nabla\times\mathbf{v} \qquad \mathbf{u},\mathbf{v}\in\mathbf{H}_0^{\nabla\times}$$

- The **MFD** discretizes the exterior calculus and preserves important range conditions:

$$\mathrm{Range}(\nabla) = \mathrm{Kernel}(\nabla\times) \qquad \mathrm{Range}(\nabla\times) = \mathrm{Kernel}(\nabla\cdot)$$

Discretization Details

We approach the magnetostatic problem using a method described in [3].

- Mesh nodes $\mathcal{N}$, mesh edges $\mathcal{E}$, mesh faces $\mathcal{F}$ must be mapped
- Magnetic potential, current density, and electric field is discretized on $\mathcal{E}$

$$\mathbf{a}_h = \left(\frac{1}{|e_j|}\int_{e_j}\mathbf{a} : e_j\in\mathcal{E}\right)$$

$\mathbf{e}_h, \mathbf{j}_h$ is defined similarly

- Magnetic pressure and electrical potential is discretized on $\mathcal{N}$

$$\lambda_h = \left(\lambda(\mathbf{x}_j) : \mathbf{x}_j\in\mathcal{N}\right)$$

- We construct operators $\mathcal{GRAD}_h : \mathcal{N}\to\mathcal{E}$ and $\mathcal{CURL}_h : \mathcal{E}\to\mathcal{F}$ defined in terms of stokes theorem.

$$\mathcal{GRAD}_h\, f_h(e_j) = \frac{f(\mathbf{x}_i) - f(\mathbf{x}_{i+1})}{|e_j|}, \mathbf{x}_i, \mathbf{x}_{i+1}\in e_j$$

$$\mathcal{CURL}_h\,\mathbf{u}_h(f_j) = \frac{1}{|f_j|}\sum_{e_i\in f_j}\sigma_{ij}|e_j|\mathbf{u}_h(e_j), \quad \sigma = \pm 1$$

- We construct quadrature matrices $\mathbb{Q}_\mathcal{N}$, $\mathbb{Q}_\mathcal{E}$ and $\mathbb{Q}_\mathcal{F}$ which produce an inner product using nodes, edges, and faces respectively. These matrices discretize the L^2 inner product.
- Discrete static Maxwell system is defined as follows

$$\begin{bmatrix} \mathbb{Q}_\mathcal{E} & -\mathbb{Q}_\mathcal{E}\mathcal{GRAD}_h & & \\ -\mathcal{GRAD}_h\mathbb{Q}_\mathcal{E} & & \mathcal{GRAD}_h^T\mathbb{Q}_\mathcal{E}\sigma_h[\mathbf{u}_\times]\mathcal{CURL}_h & \\ -\sigma_h\mathcal{Q}_\mathcal{E} & & \mathcal{CURL}_h^T\mathbb{Q}_\mathcal{N}\mathcal{CURL} - \sigma_h[\mathbf{u}_\times]\mathbb{Q}_\mathcal{N}\mathcal{CURL}_h & \mathbb{Q}_\mathcal{E}\mathcal{GRAD}_h \\ & & \mathcal{GRAD}_h^T\mathbb{Q}_\mathcal{E} & \end{bmatrix}\begin{bmatrix}\mathbf{e}_h\\ \psi_h\\ \mathbf{a}_h\\ \lambda_h\end{bmatrix} = \begin{bmatrix}0\\0\\ \mathbf{j}_{\mathrm{Hall}}\\0\end{bmatrix}$$

Sensitivity Experiments

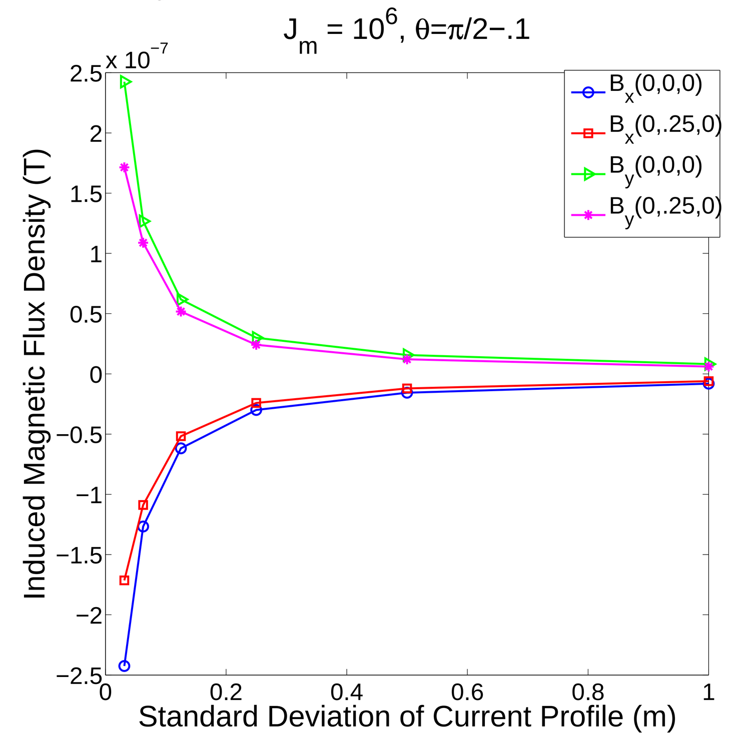
We explore the response of a magnetic field to features we expect to occur in **MHDG** by computing sensitivities to

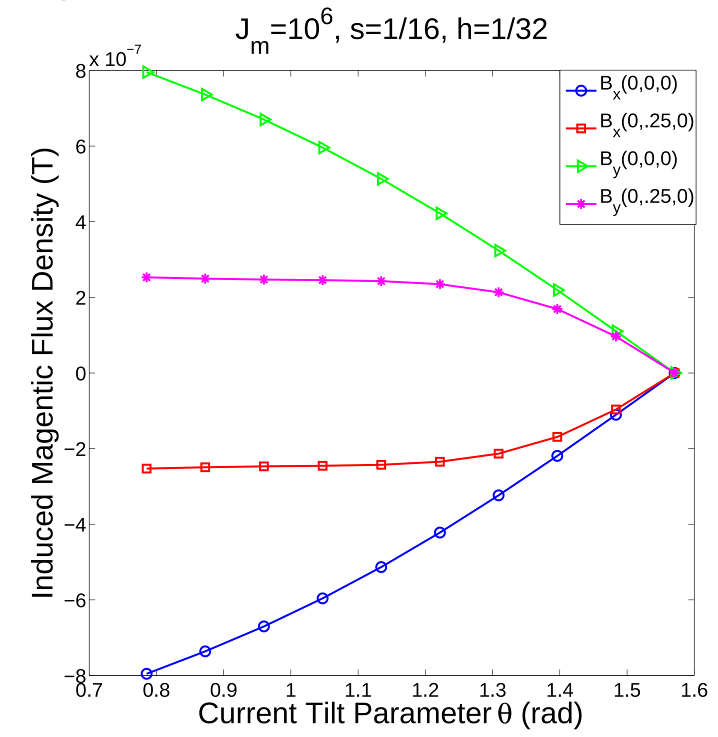
- Variation in current density from diffuse to concentrated.
- Variation in the tilt (angle) of the current.

We will exclude electrostatic effects and instead consider a current density profile which mimics **MHD** features.

$$\mathbf{j}(x,y,z) = \frac{j_m}{\sqrt{2\pi s^2}}\mathbf{v}\exp\left(\frac{1}{2s^2}\left|\left(\mathbb{I} - \mathbf{v}\mathbf{v}^T\right)\begin{pmatrix}x\\y\\z\end{pmatrix}\right|^2\right), \ \mathbf{v} = \begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}$$

The parameter space is determined as followed: j_m which determines the total current, s which determines current density, θ determines tilt mimicking the Hall effect.





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Acknowledgments

This work was supported by the DOE NETL Grant RES1100426 / 015.

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