



University of Colorado at Boulder
Chemical and Biological Engineering



QUANTIFYING THE UNCERTAINTY OF KINETIC-THEORY PREDICTIONS OF CLUSTERING

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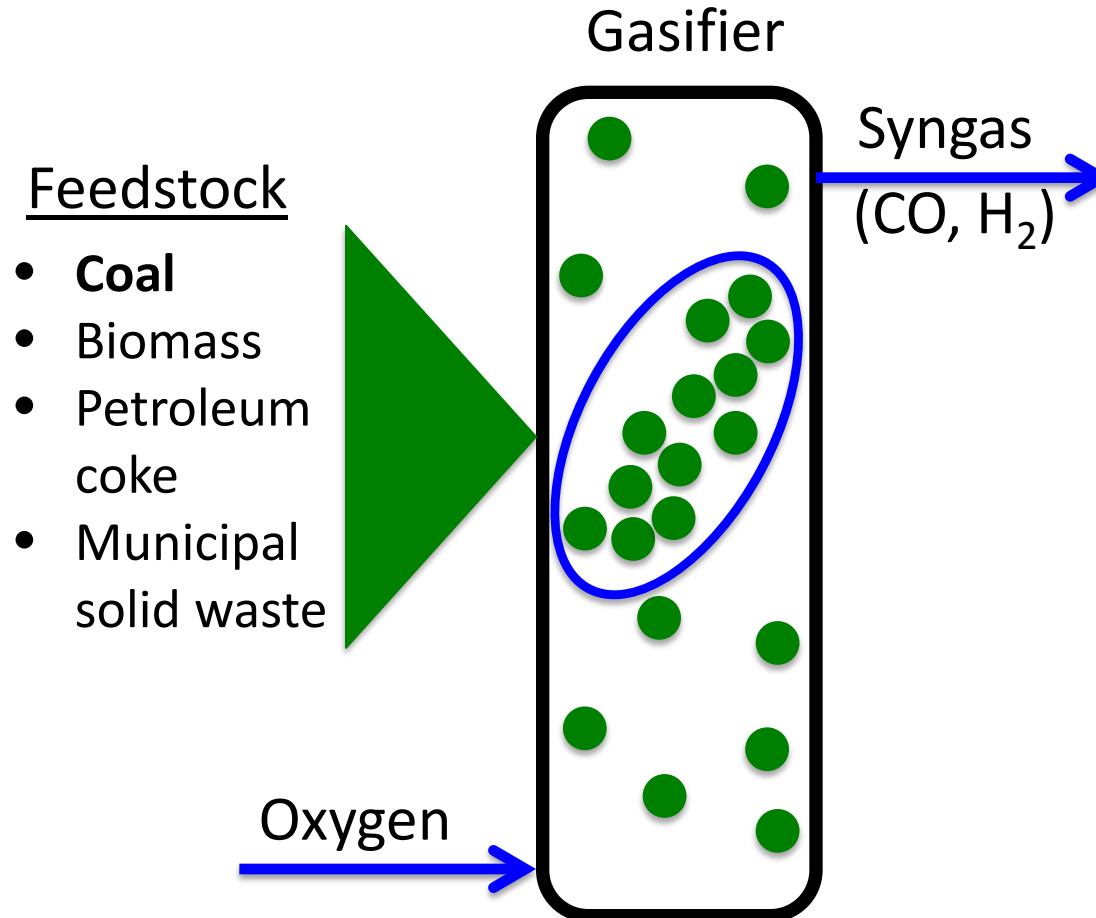


May 31st, 2012

University Coal Research Conference

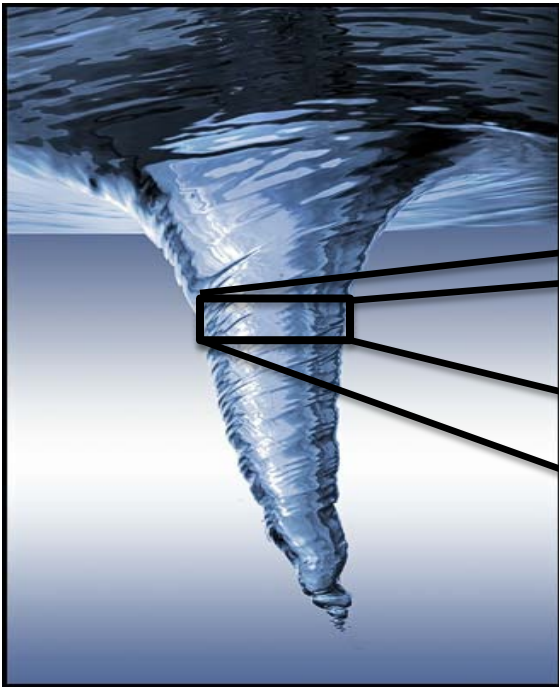
Pittsburgh, PA

Motivation: Granular instabilities



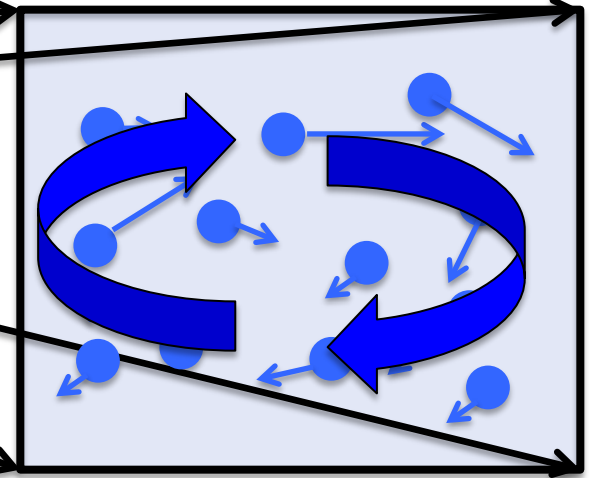
Fluid Analogy: Continuous vs. Discrete

Continuum perspective

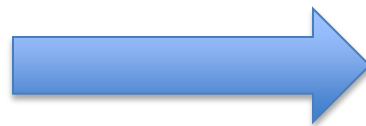


Navier Stokes eqns

Molecular perspective

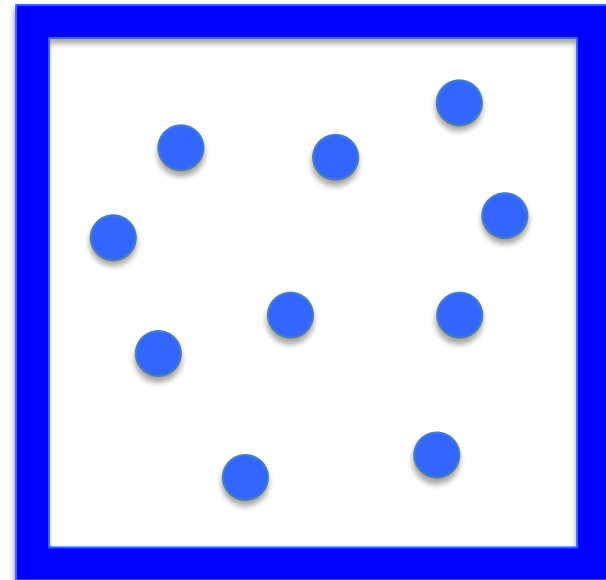


Newton's laws



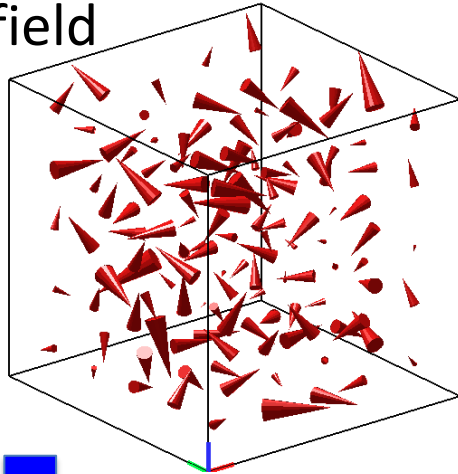
System of Interest: Granular Flow

- The Homogeneous Cooling System (HCS)
 - No external forces
 - Periodic boundaries
 - No gradients in the hydrodynamic variables
- Particle properties
 - Constant coefficient of restitution (e)
 - Monodisperse particles
 - No enduring contacts



Background

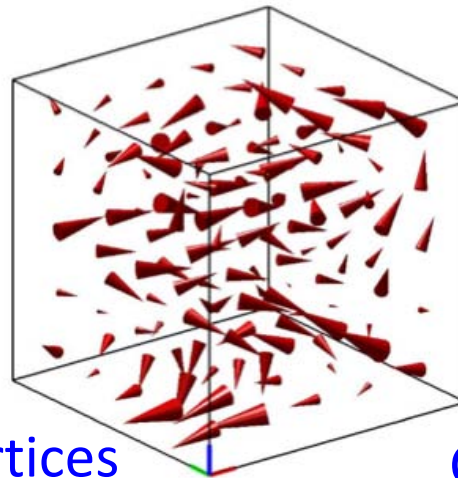
Velocity field



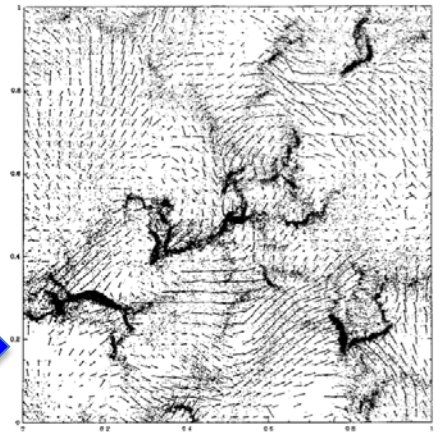
Molecular dynamics (MD)
simulations of the HCS

- Dissipative collisions
- Sufficiently large system domain

Velocity field



Particle locations

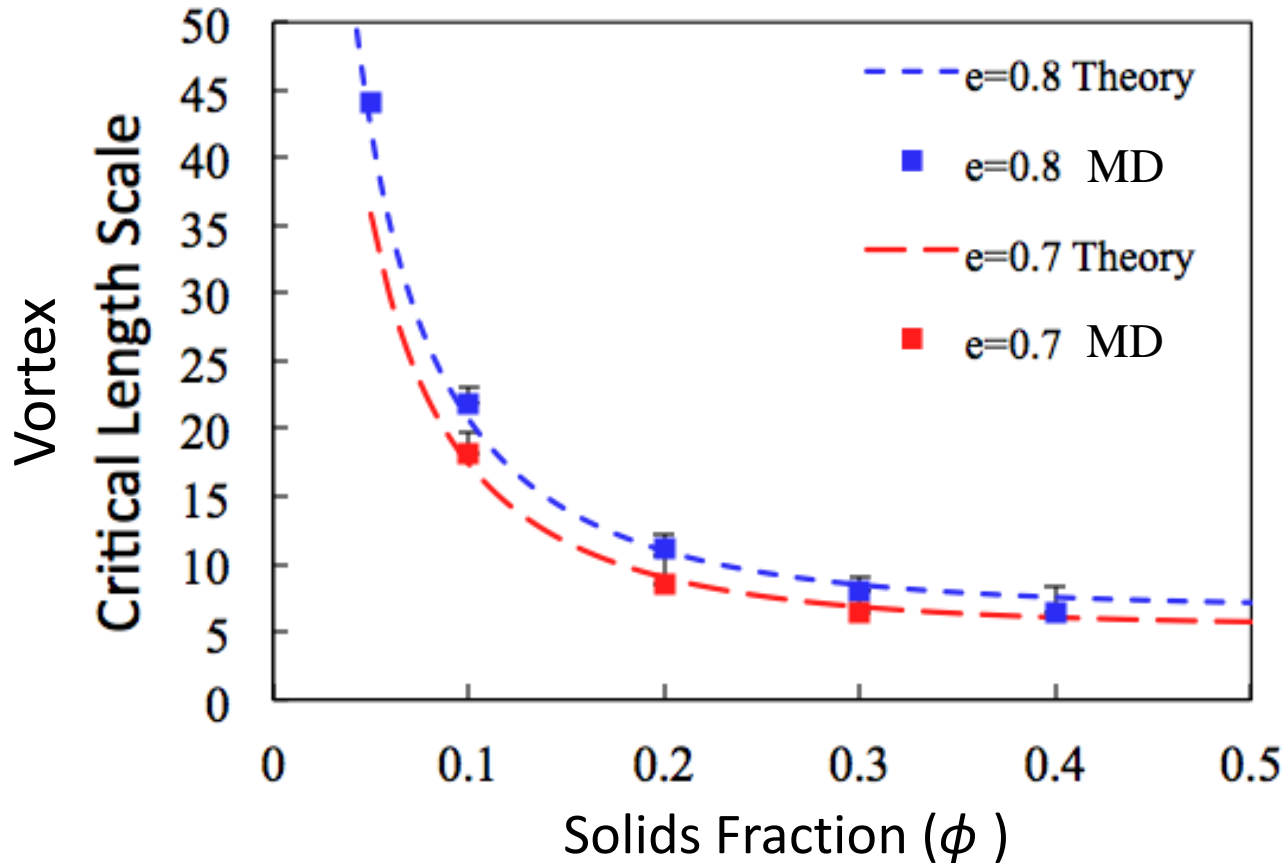


Vortices

Clusters

Goldhirsch, Tan, Zanetti, J. Sci. Comput. (1993)

Background



Mitrano et al., Phys. Fluids (2011)

Kinetic-Theory-based stability analysis: Garzó, 2005

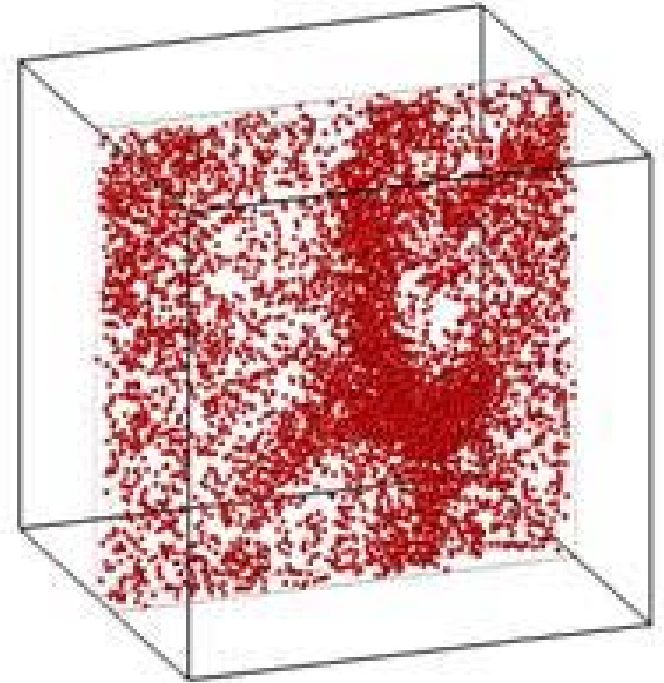
Objectives

Quantitatively assess Kinetic-theory-based predictions of instabilities via MD simulations

- Clustering instabilities
 - MD vs. CFD theory solution
- Effect of friction on instabilities
 - MD vs. *linear* stability analysis (LSA) of theory

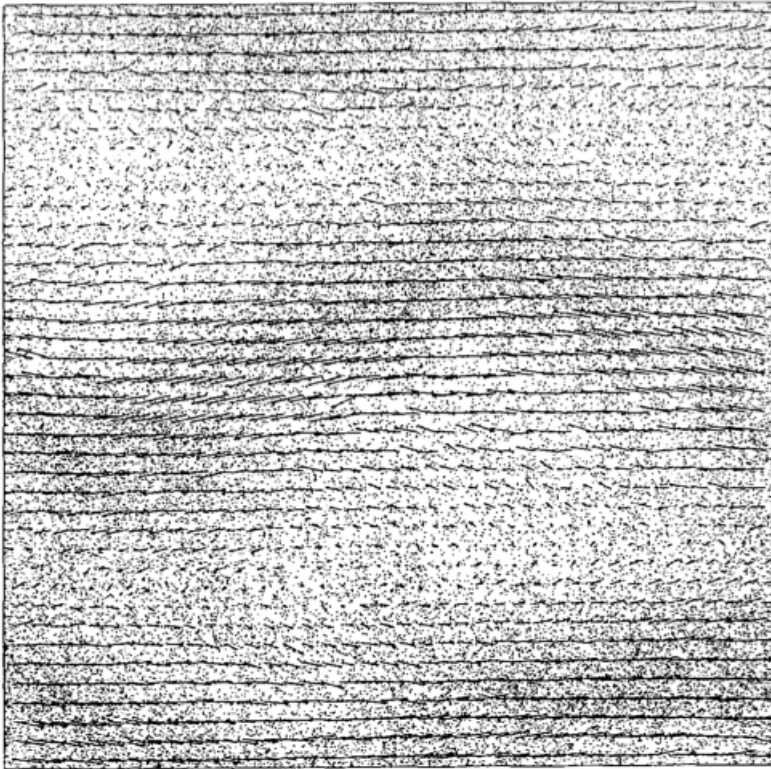
Molecular Dynamics

- Input
 - System length scale (L/d)
 - Restitution coefficient (e)
 - Volume fraction (ϕ)
- 3-dimensional domain
- Hard sphere collision model
 - Binary, instantaneous collisions
- Relevant Output
 - Particle positions & velocities



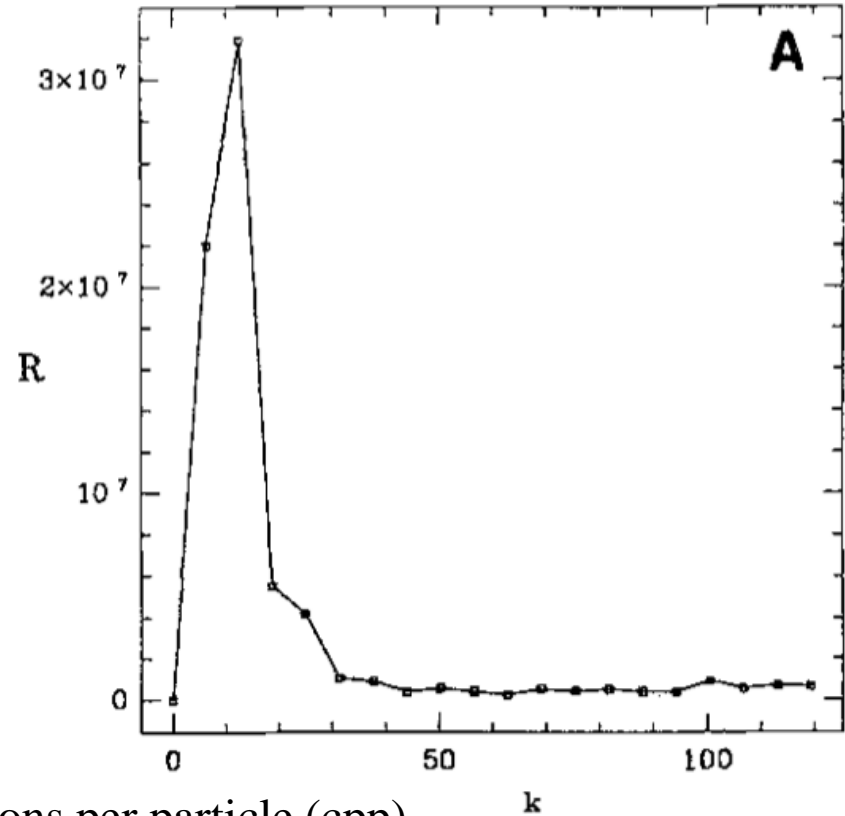
MD: Fourier Analysis

Particle positions (2D MD simulation)

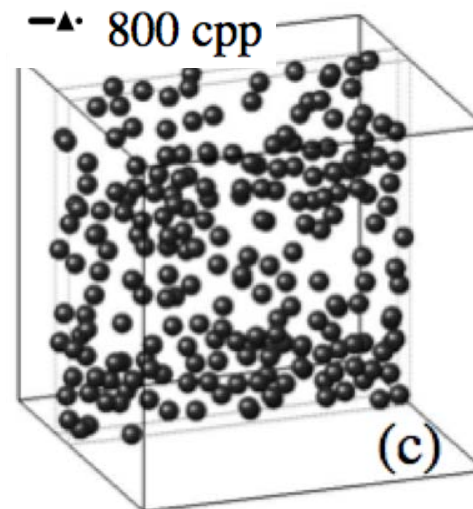
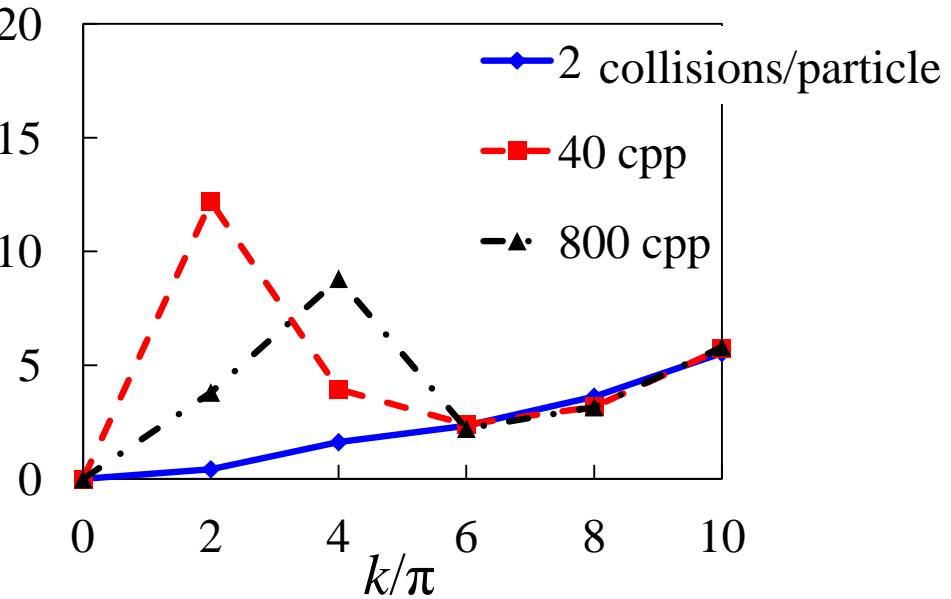
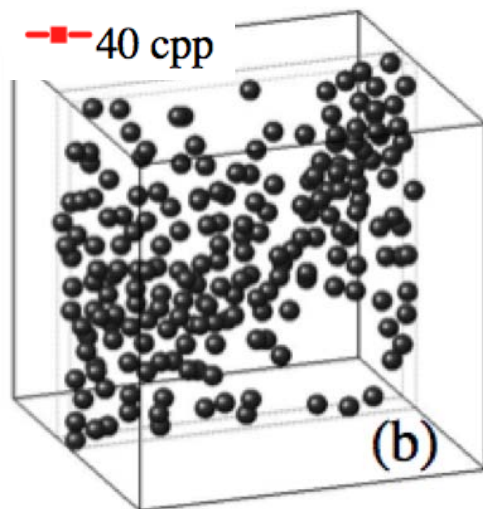
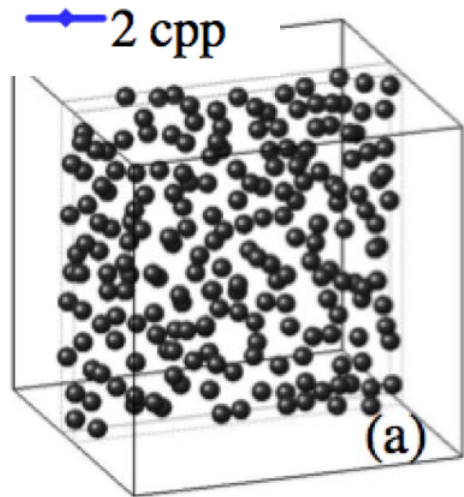


At 400 collisions per particle (cpp)

“Mass Mode” vs. wavenumber



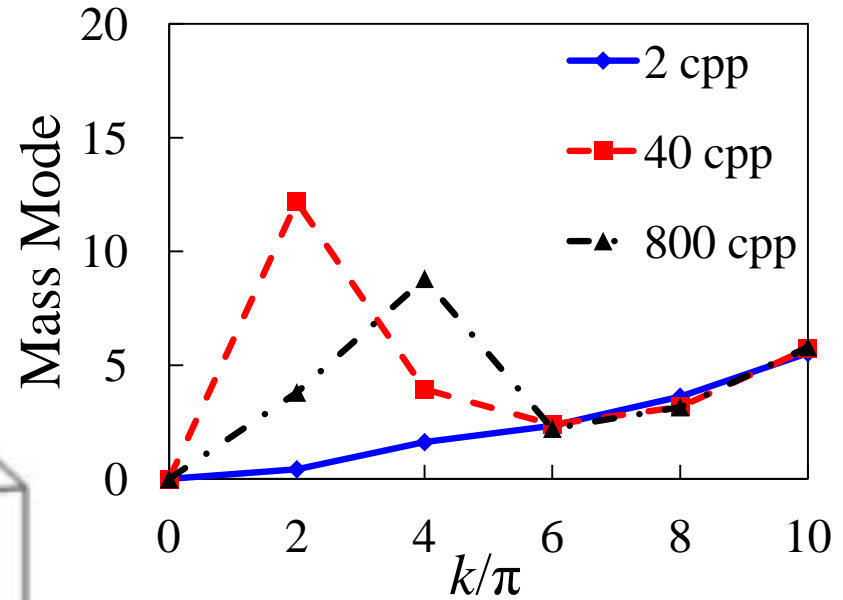
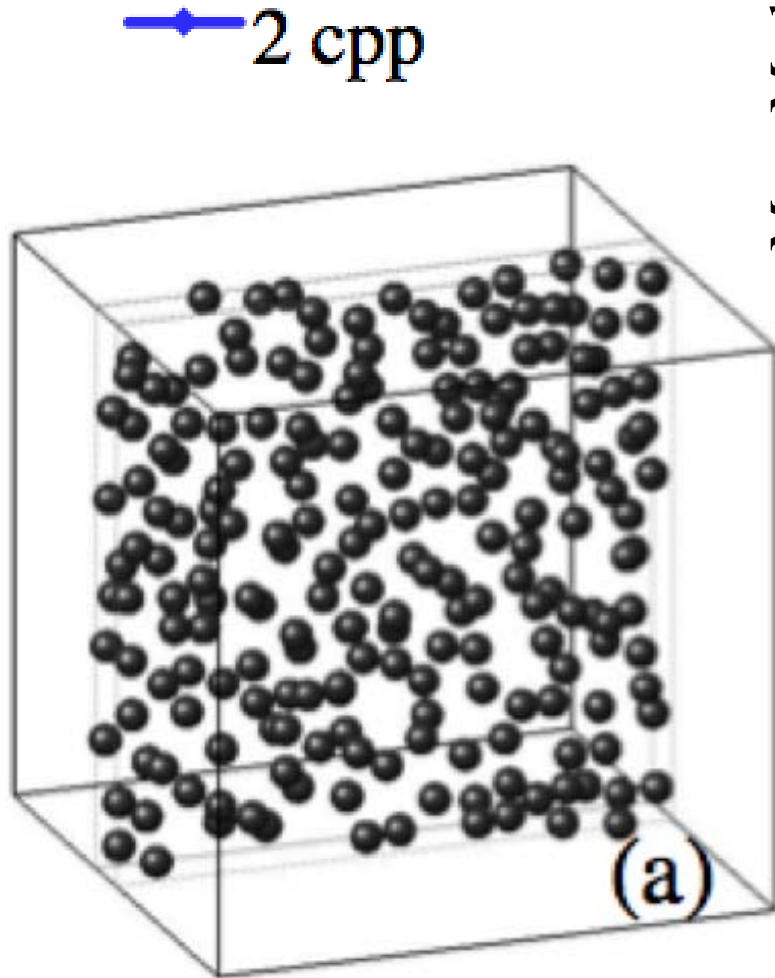
MD: Fourier Analysis



$e=0.6, \phi=0.2$
 $N=2000$

Mitrano et al.,
PRE (2012)

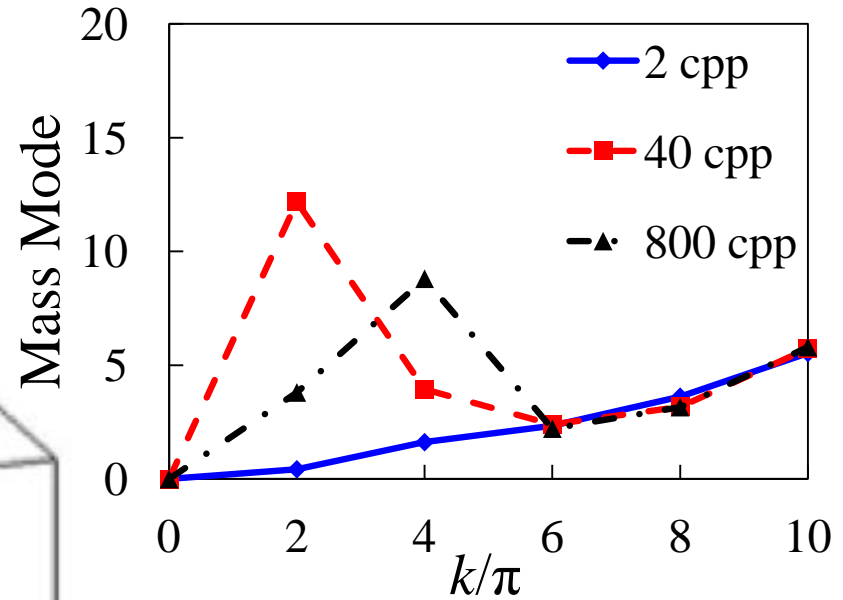
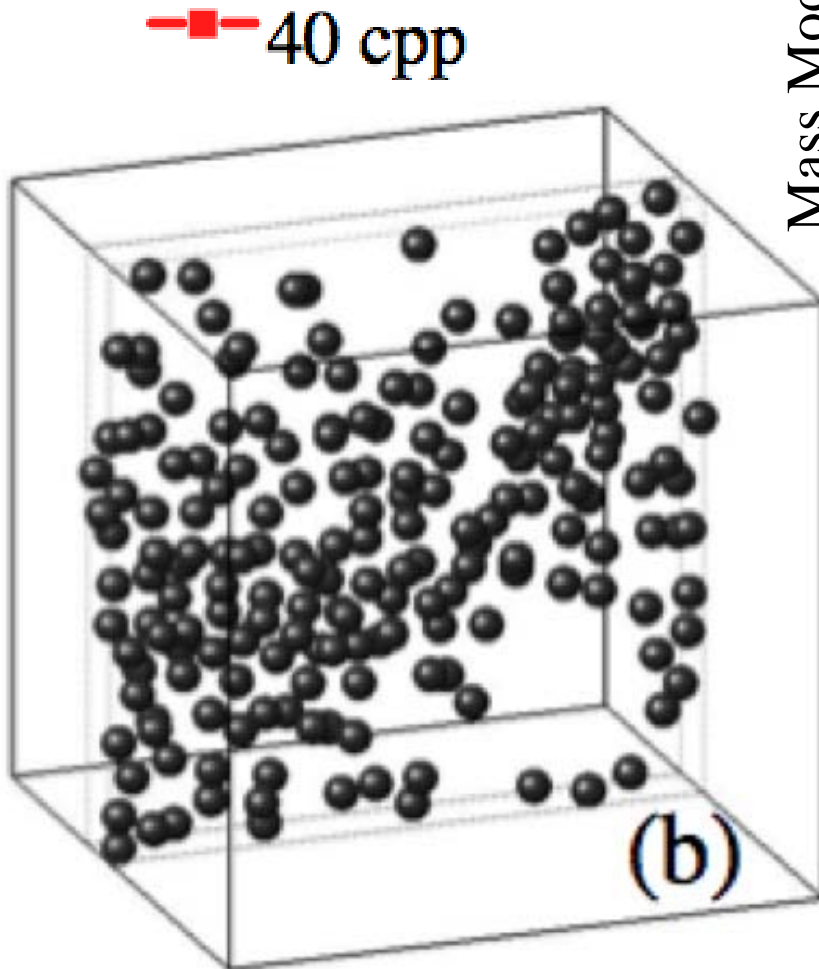
MD: Fourier Analysis



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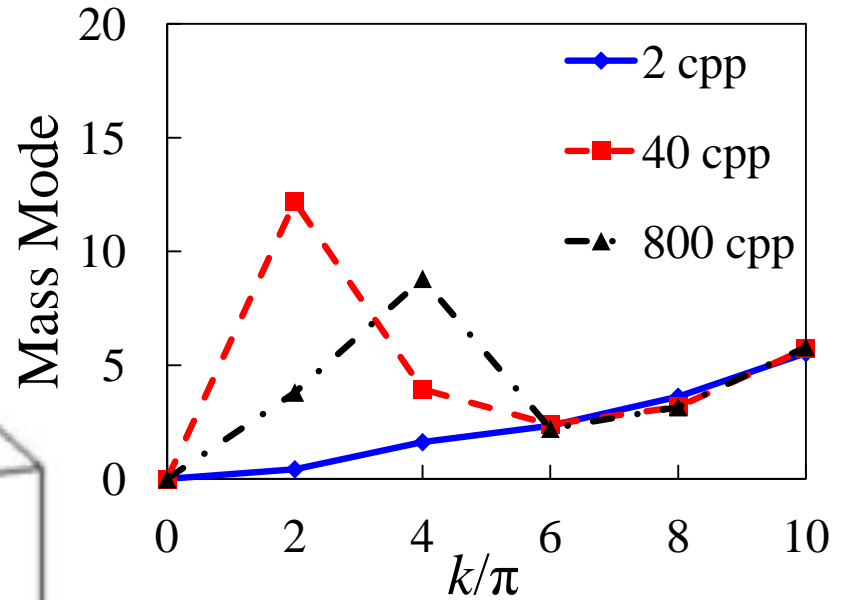
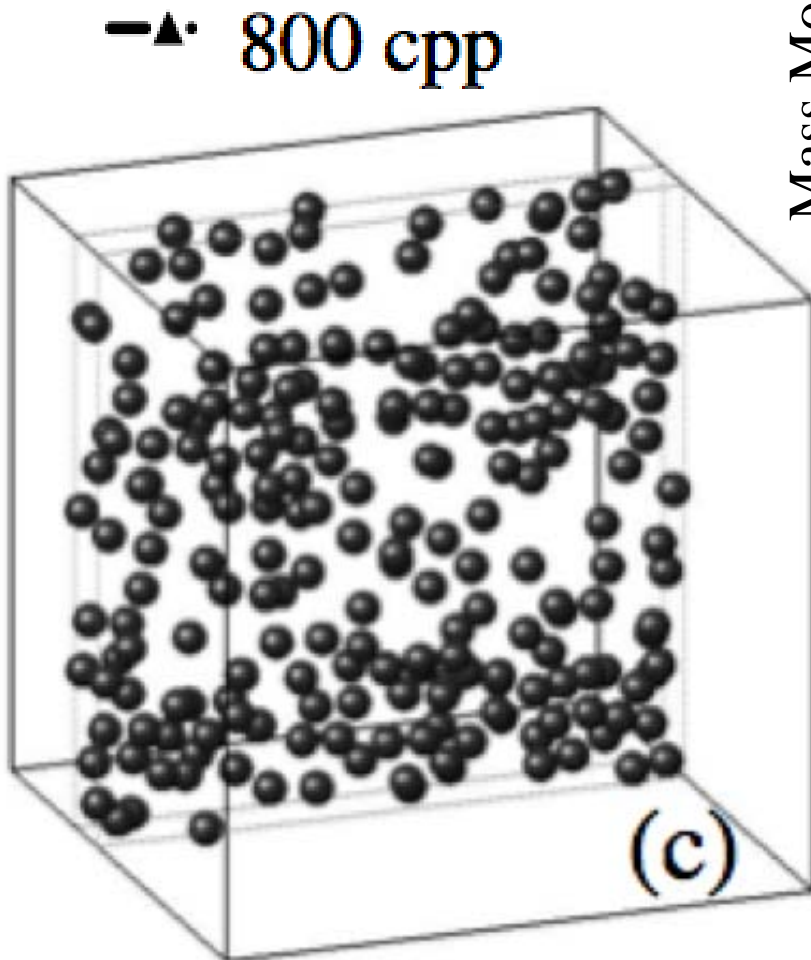
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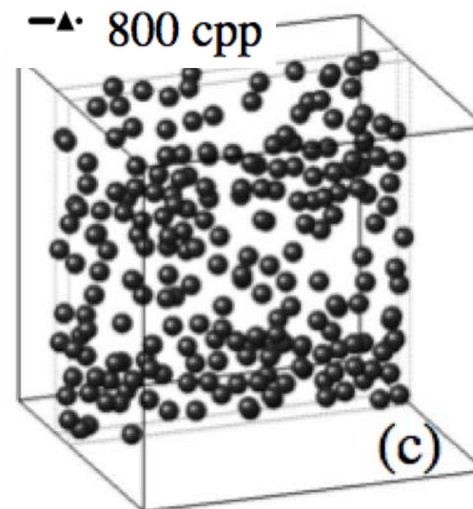
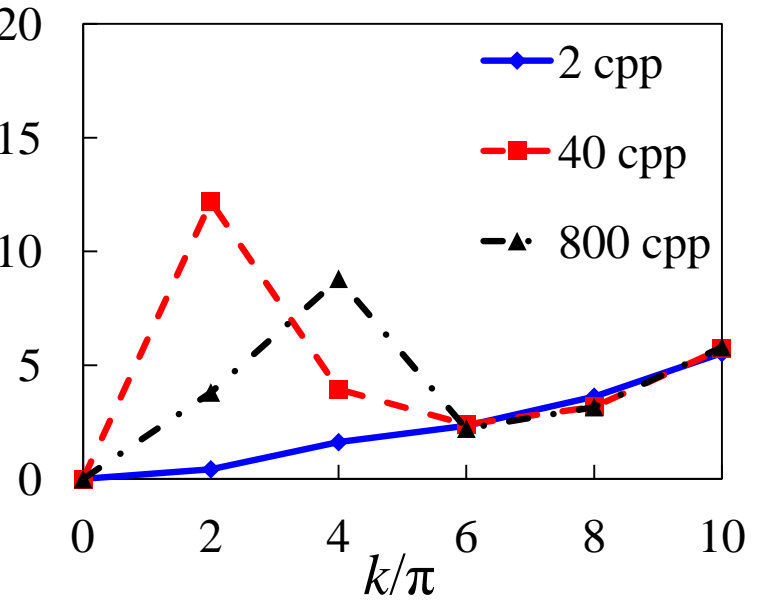
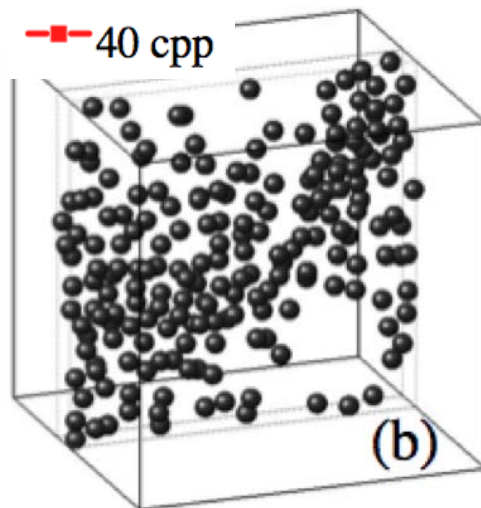
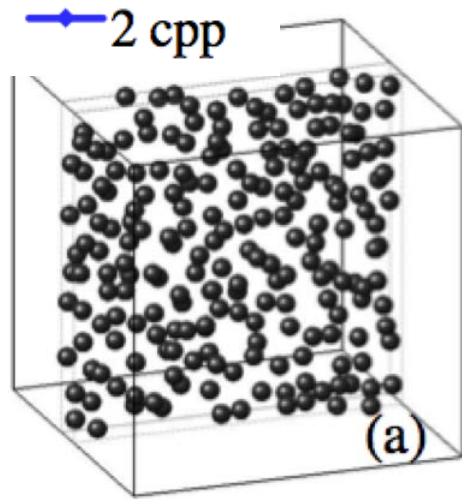
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Mitrano et al.,
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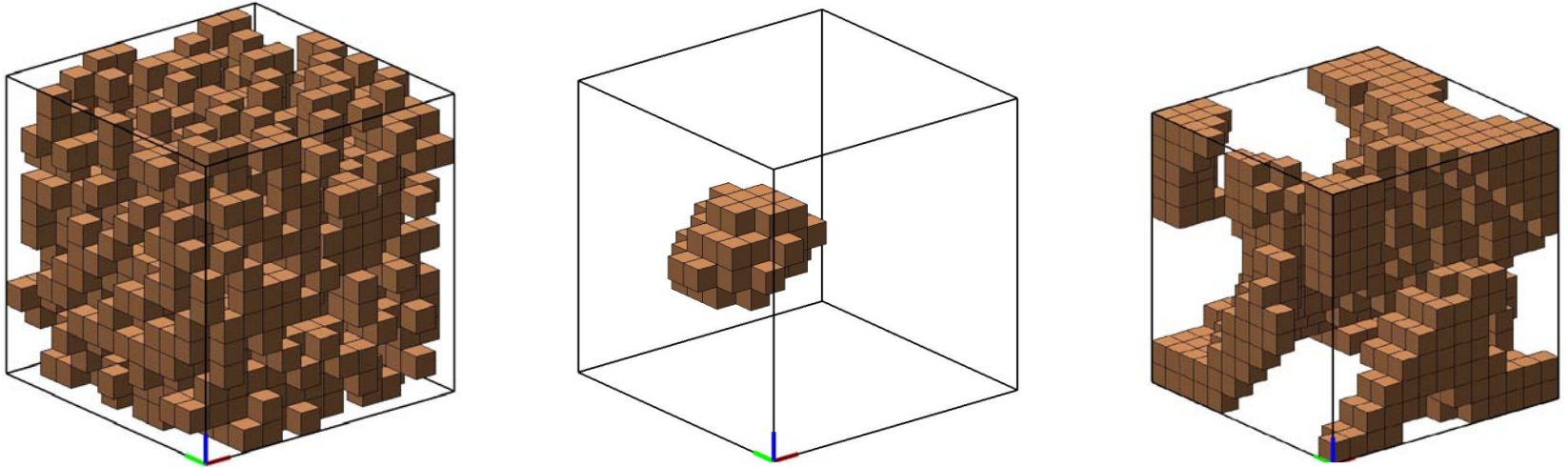
MD: Fourier Analysis



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Mitrano et al.,
PRE (2012)

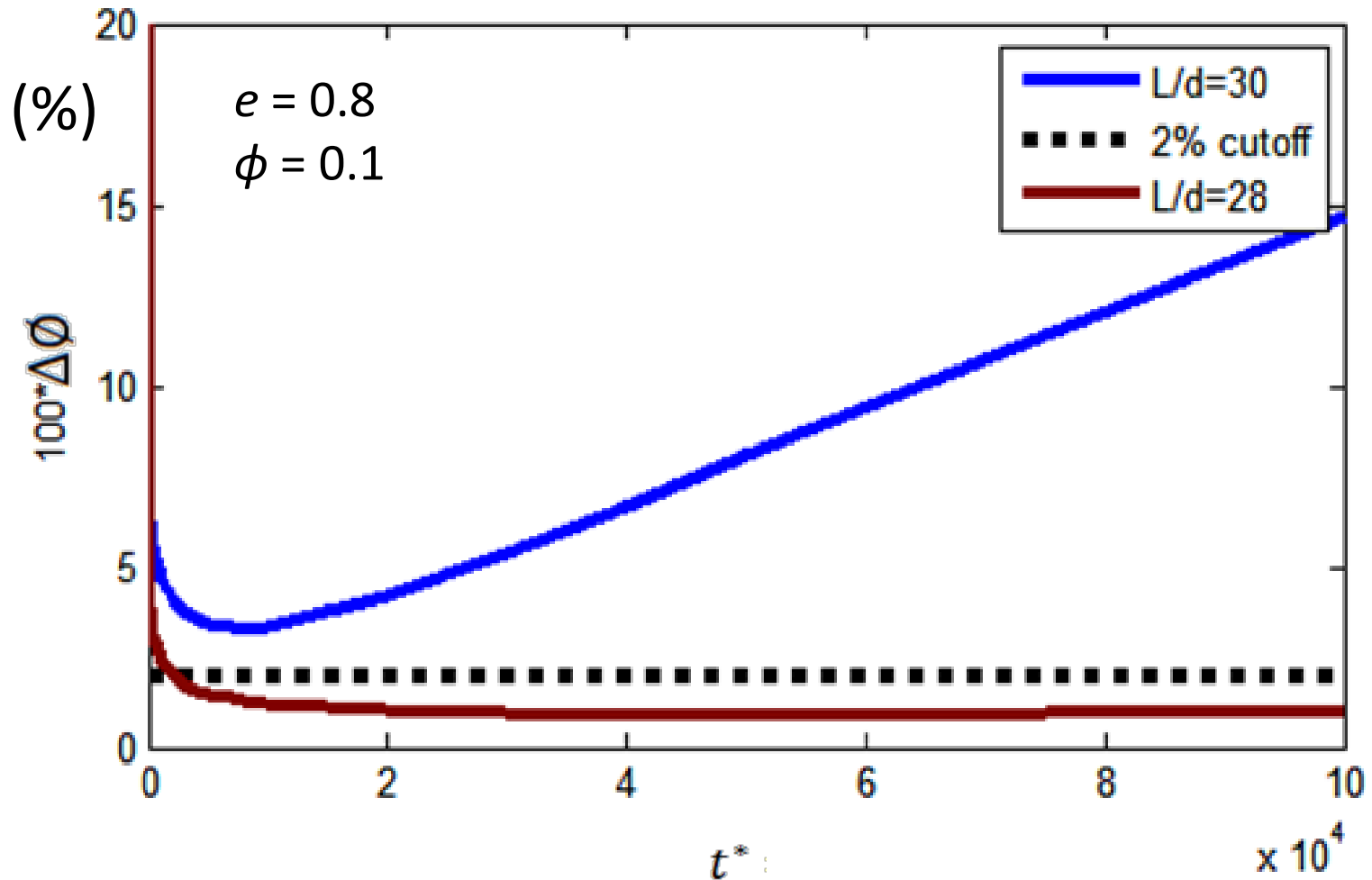
CFD: Cluster Detection



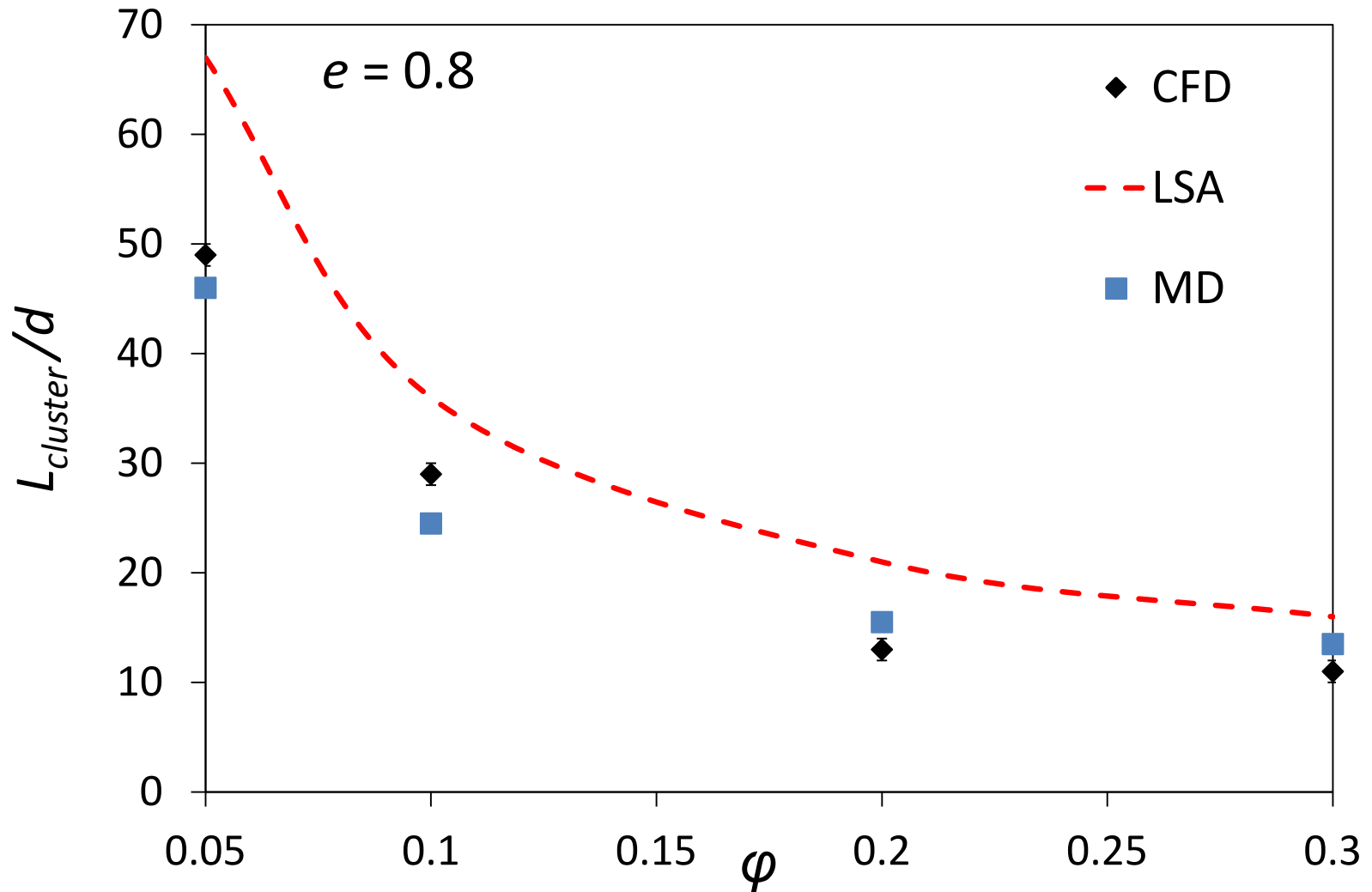
time

$$\Delta\phi = \frac{\phi_{cell,max} - \phi_{cell,min}}{\phi}$$

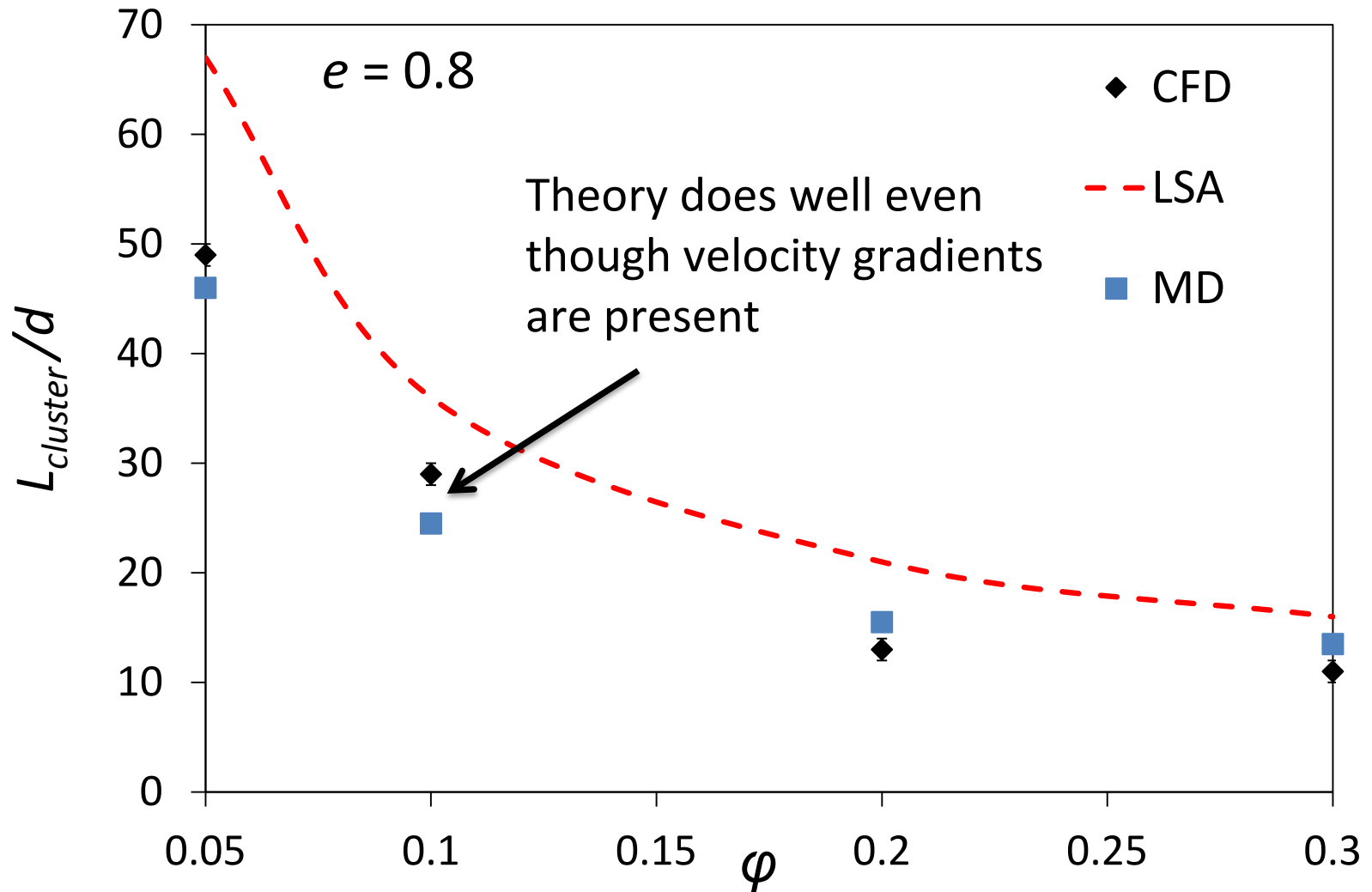
CFD: Cluster Detection



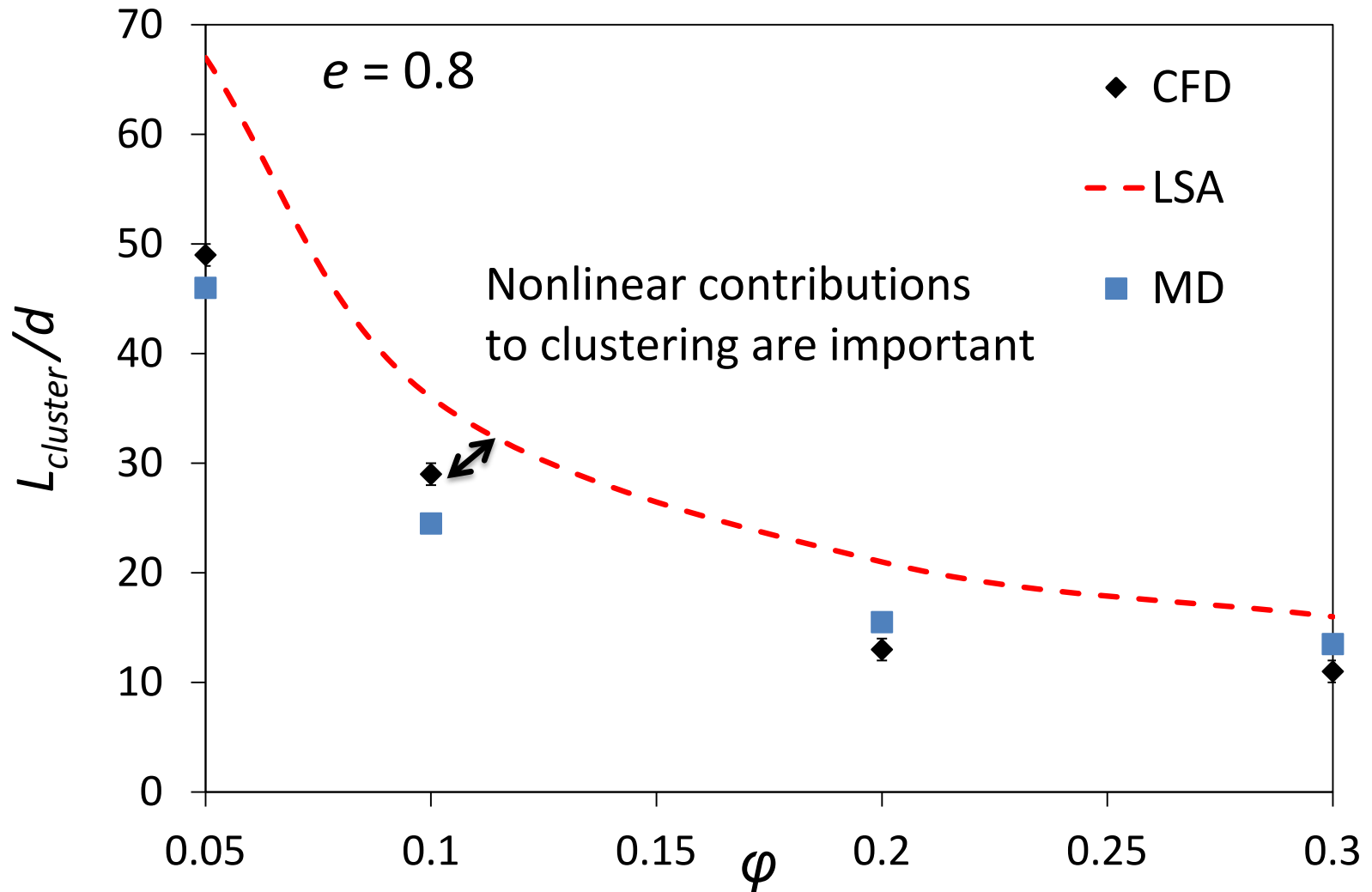
Clustering Onset: CFD-MD-LSA



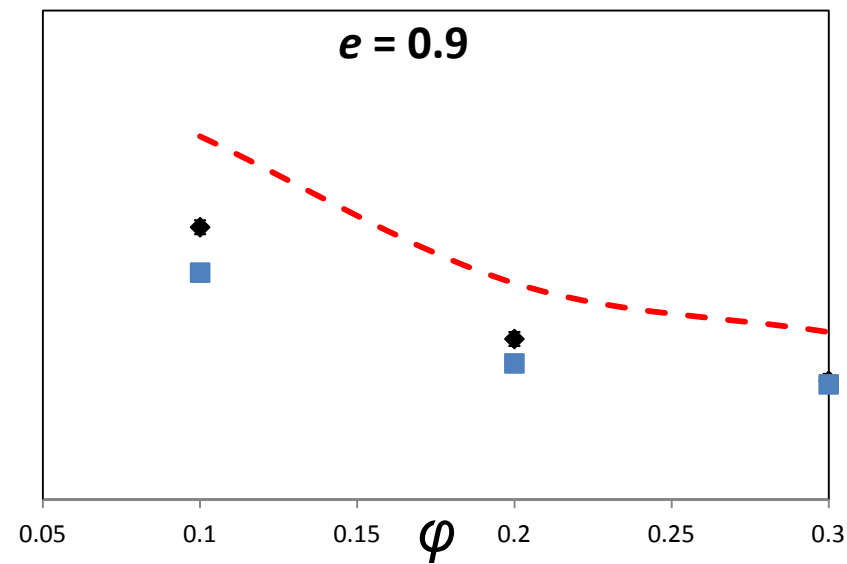
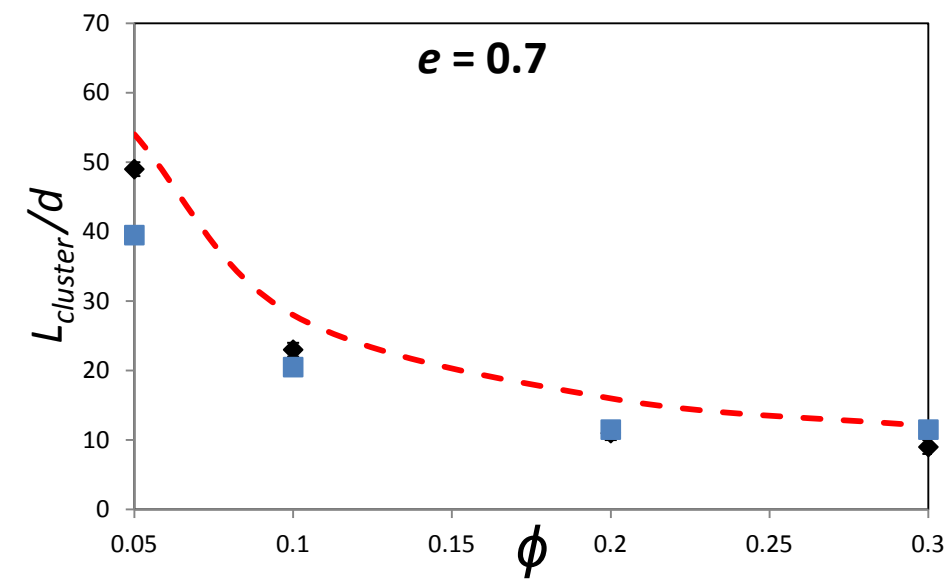
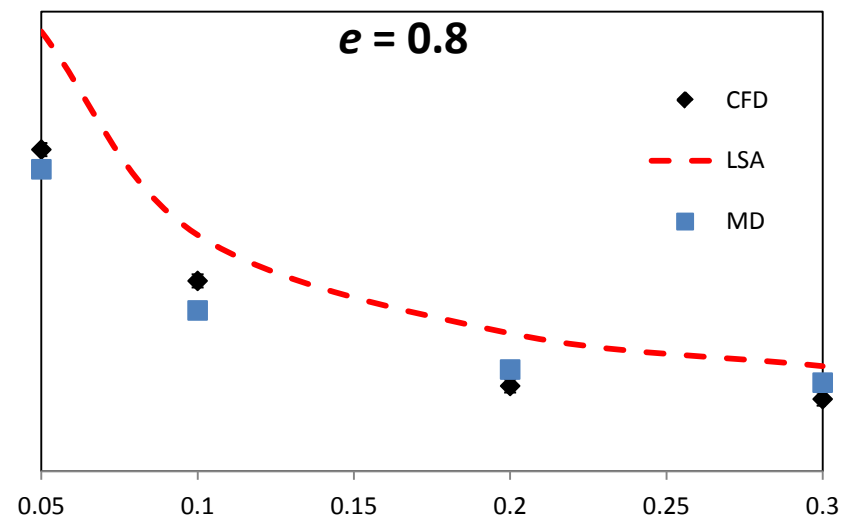
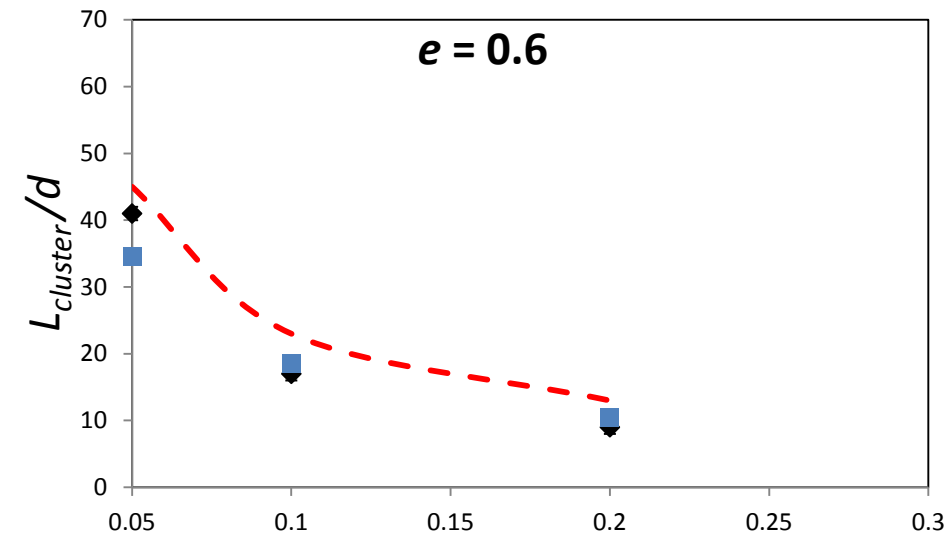
Clustering Onset: CFD-MD-LSA



Clustering Onset: CFD-MD-LSA

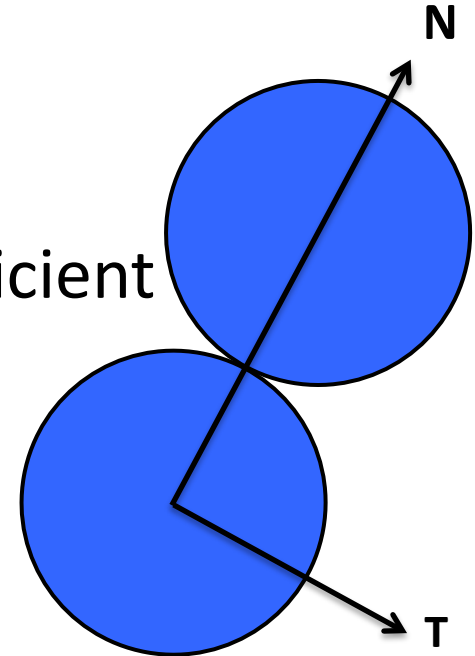


Clustering Onset: CFD-MD-LSA



Types of Dissipation

- Normal dissipation
 - Constant normal restitution coefficient
 - $0 \leq e \leq 1$
- Tangential dissipation
 - Constant tangential restitution coefficient
 - $-1 \leq \beta \leq 1$



Types of Dissipation

- Normal dissipation

- Constant normal restitution coefficient

$$0 \leq e \leq 1$$

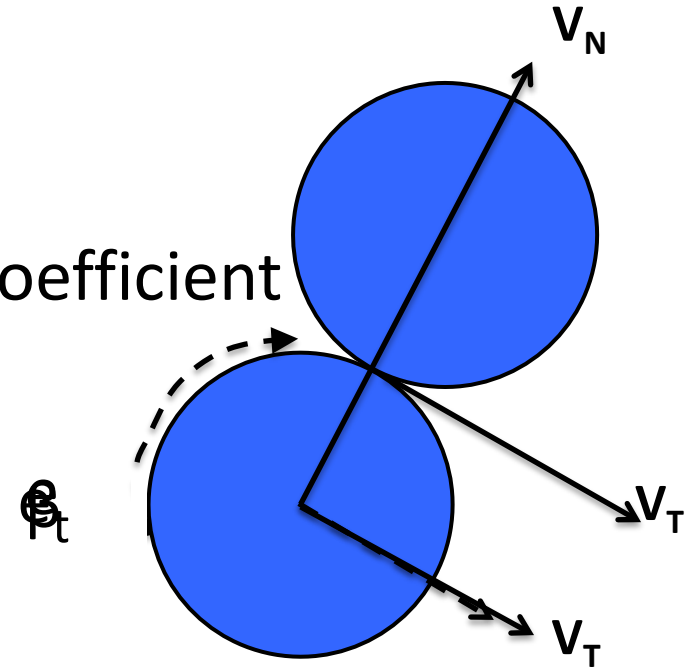
- Tangential dissipation

- Constant tangential restitution coefficient

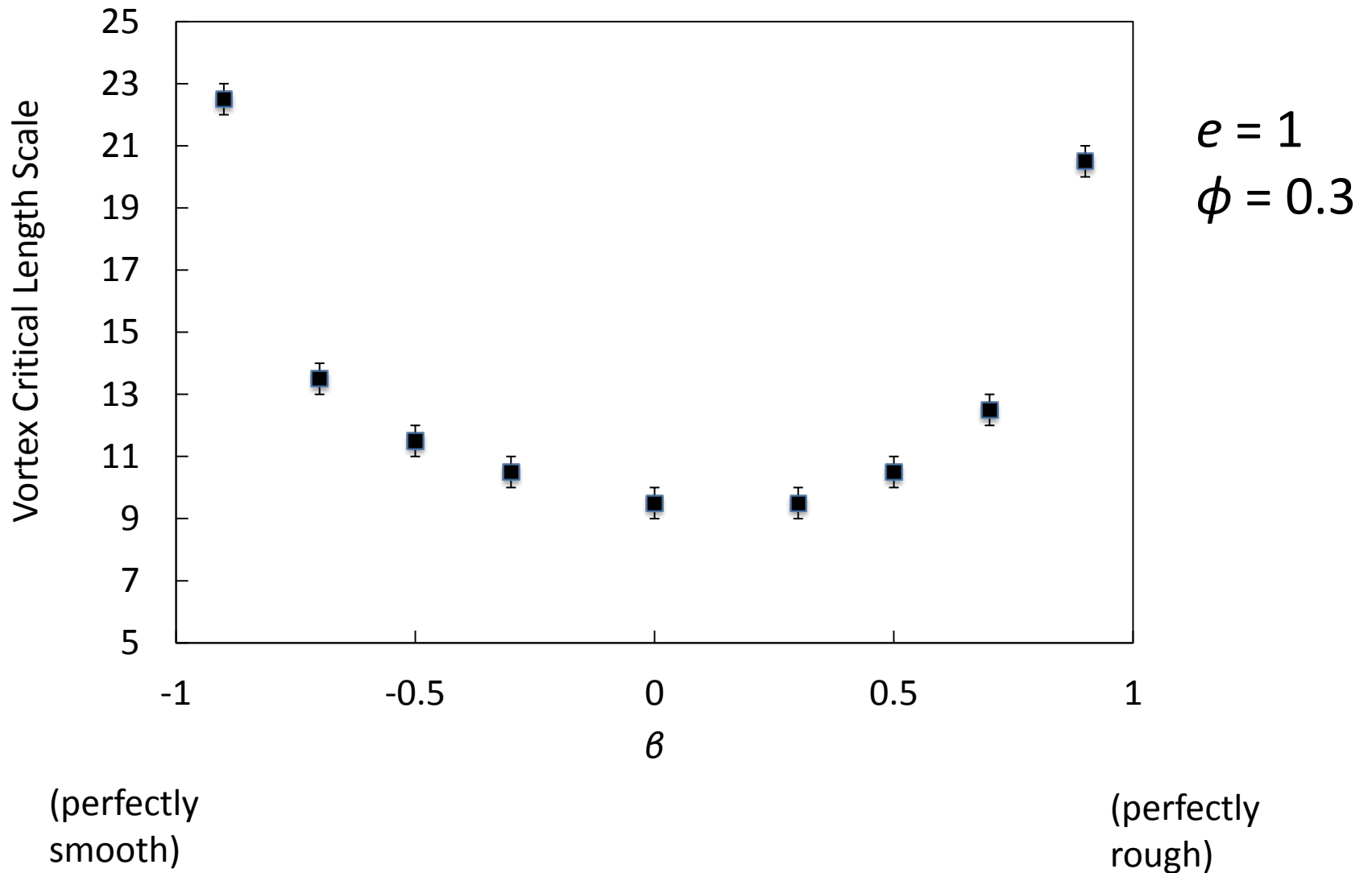
$$-1 \leq \beta \leq 1$$

No tangential impulse:
“perfectly smooth”

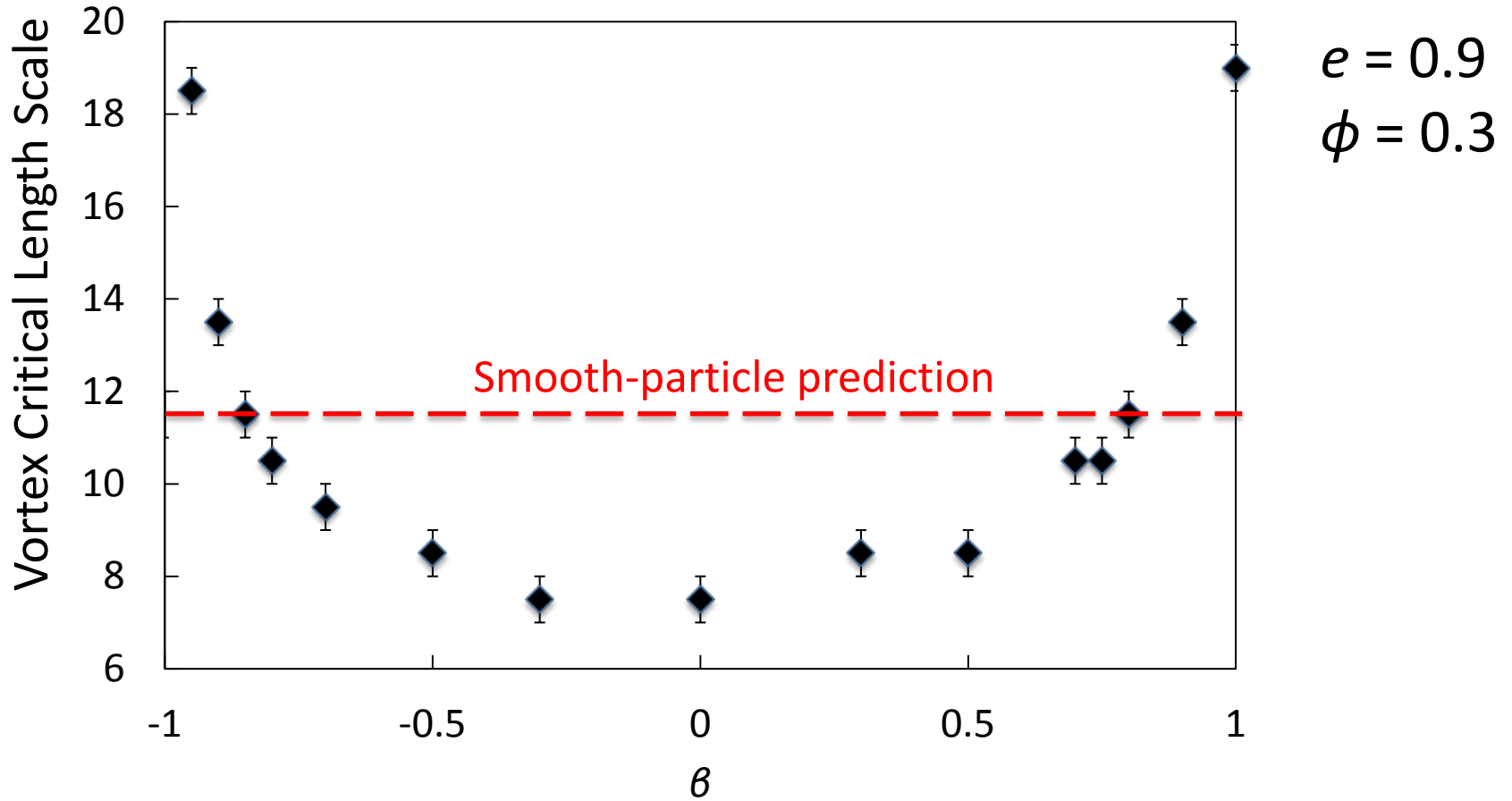
Elastic tang. Impulse:
“perfectly rough”



Elastic Results



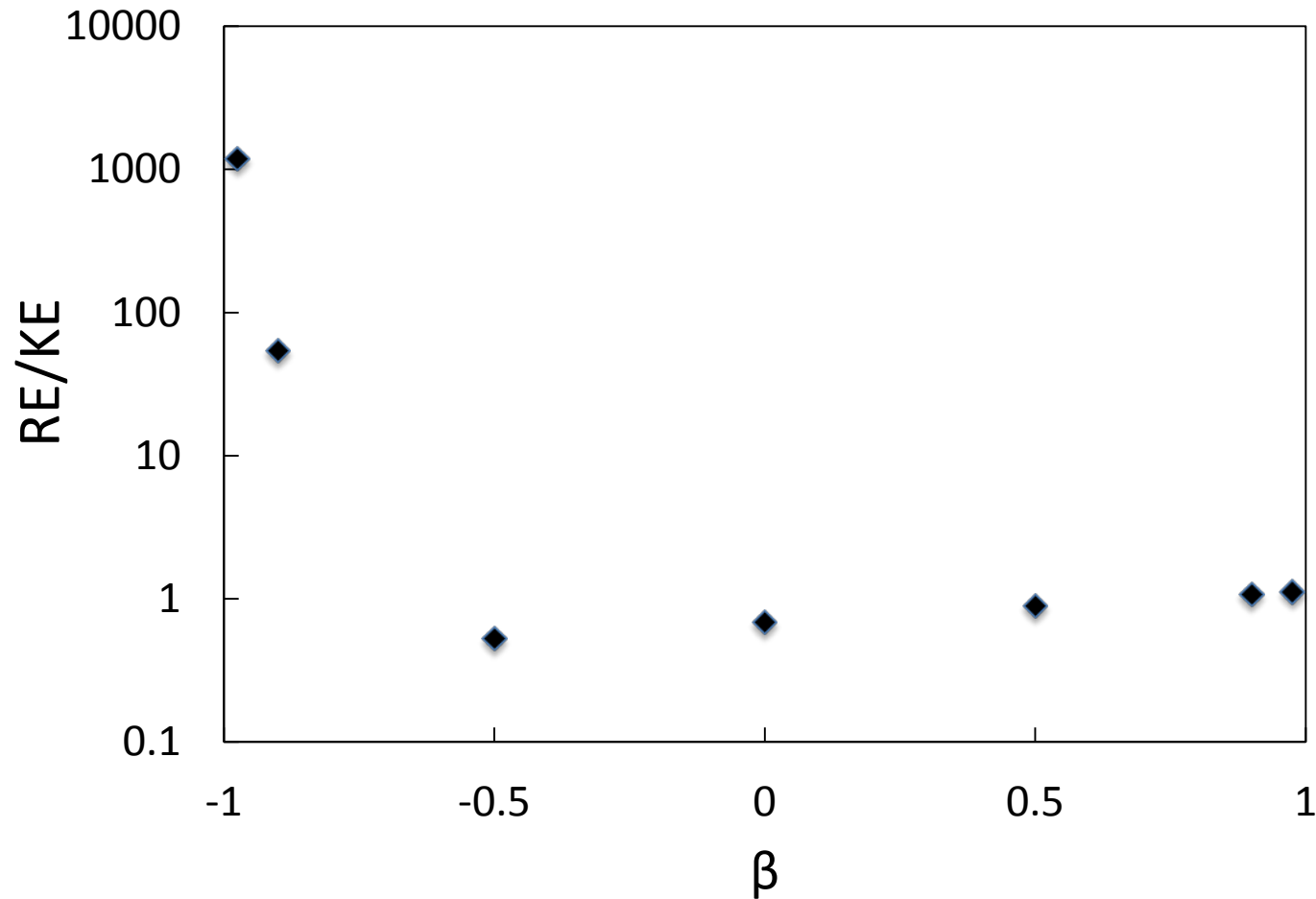
Frictional Results



Extra note (not in original presentation)

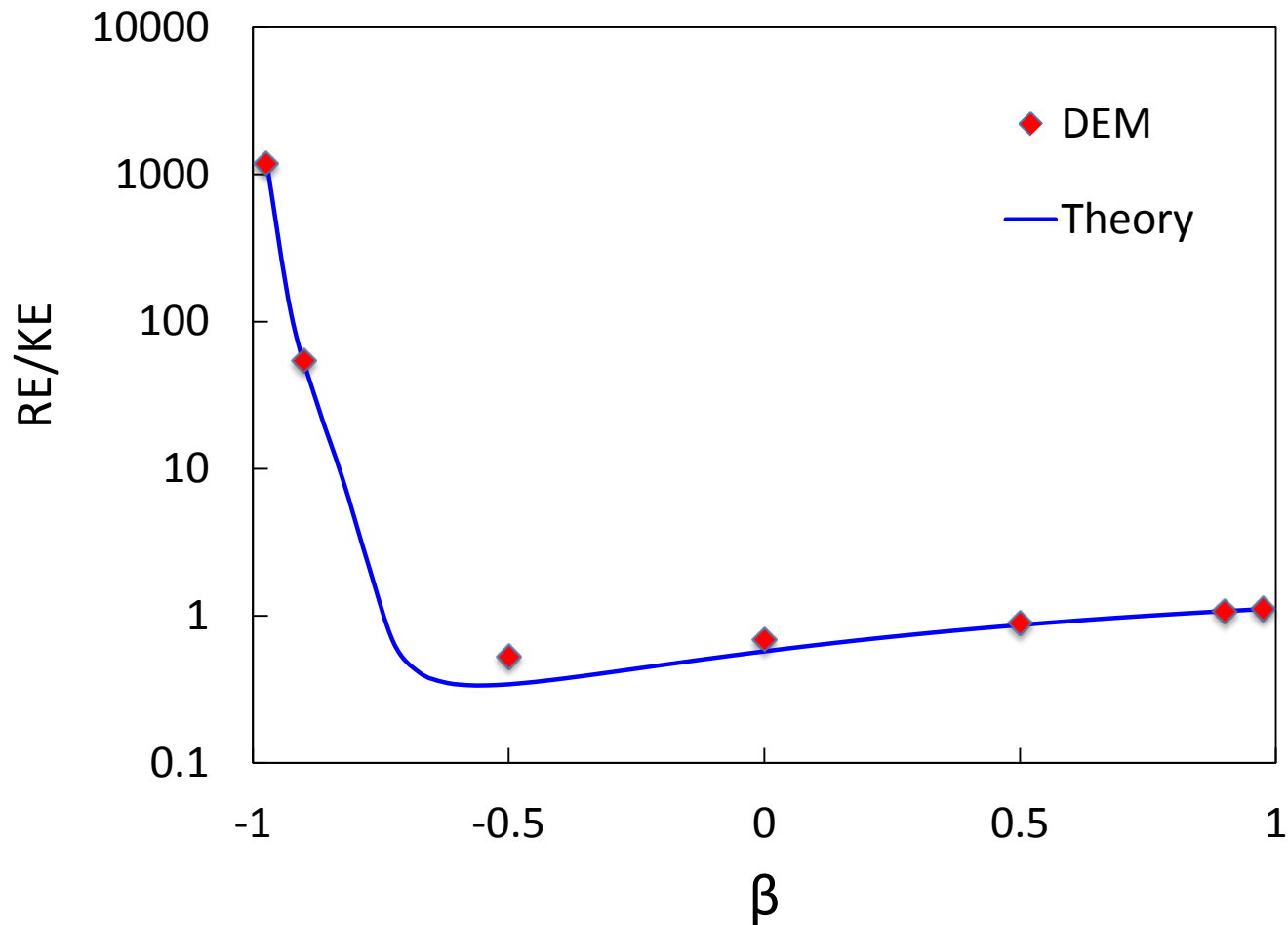
- Very strange behavior for nearly smooth and nearly perfectly (elastically) rough particles can be traced to the energy ratio and more directly the fact that we only allow for “sticking” collisions that depend on the relative tangential overall velocity. Highly rotating particles are caused to separate since the tangential component is so large giving to a large tangential impulse. (vortex motion is dependent on the tangential translation alignment). E_t is a tangential translational restitution coefficient that is well correlated to vortex motion- high e_t values hinder vortex formation. Next slide shows that the particle rotation is very high on the left side. As particles become more and more rough the tangential impulse is inherently larger. We briefly examine a friction model that allows for either sticking or coulomb-governed sliding collisions a few slides later.

Temperature Ratio (Rotation/Translation)



$e = 0.9$
 $\phi = 0.3$

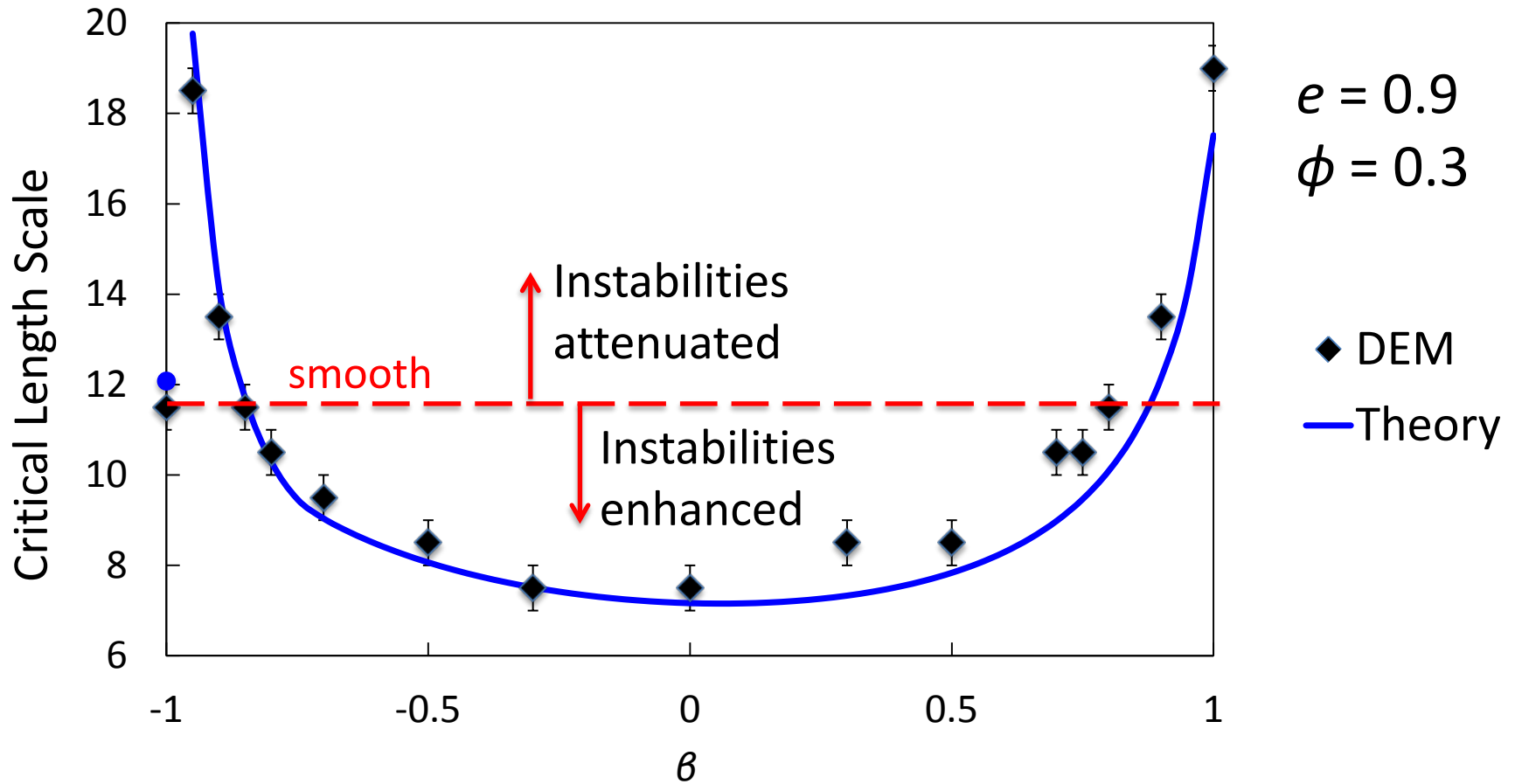
Temperature Ratio (DEM-theory comparison)



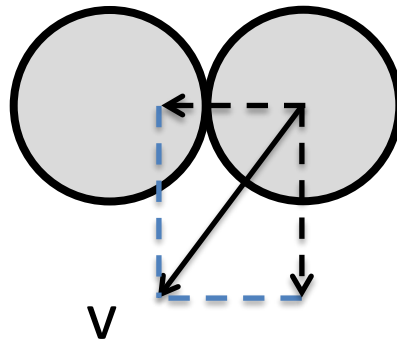
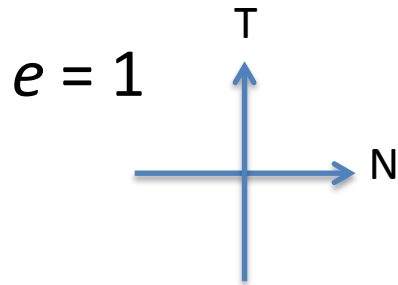
$e = 0.9$
 $\phi = 0.3$




Theory: Santos, Kremer, Garzó, *Prog Theor Phys, Suppl* (2010)

Frictional Results

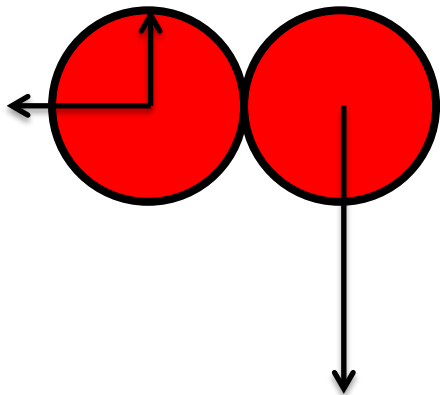


Tangential *Translational* Restitution Coefficient (e_t)

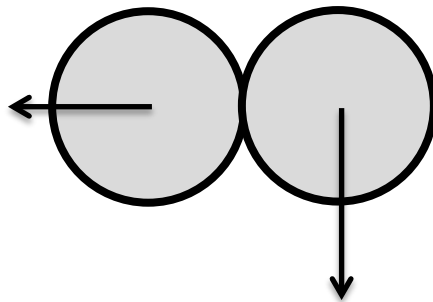


-  Increased rel. tang. velocity:
Vortices Suppressed
-  No change
-  Decreased rel. tang. velocity:
Vortices Enhanced

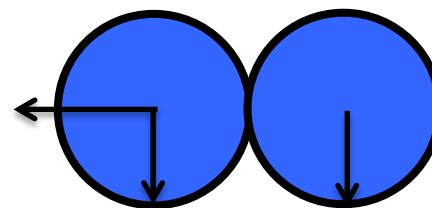
$e_t = -2$



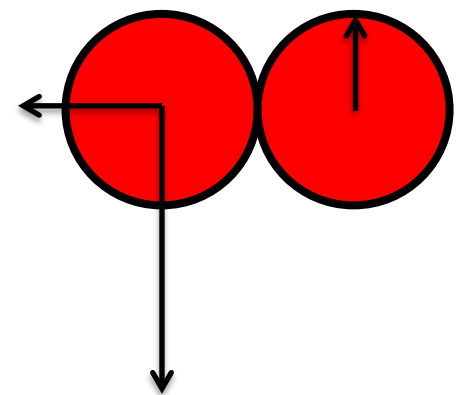
$e_t = -1$



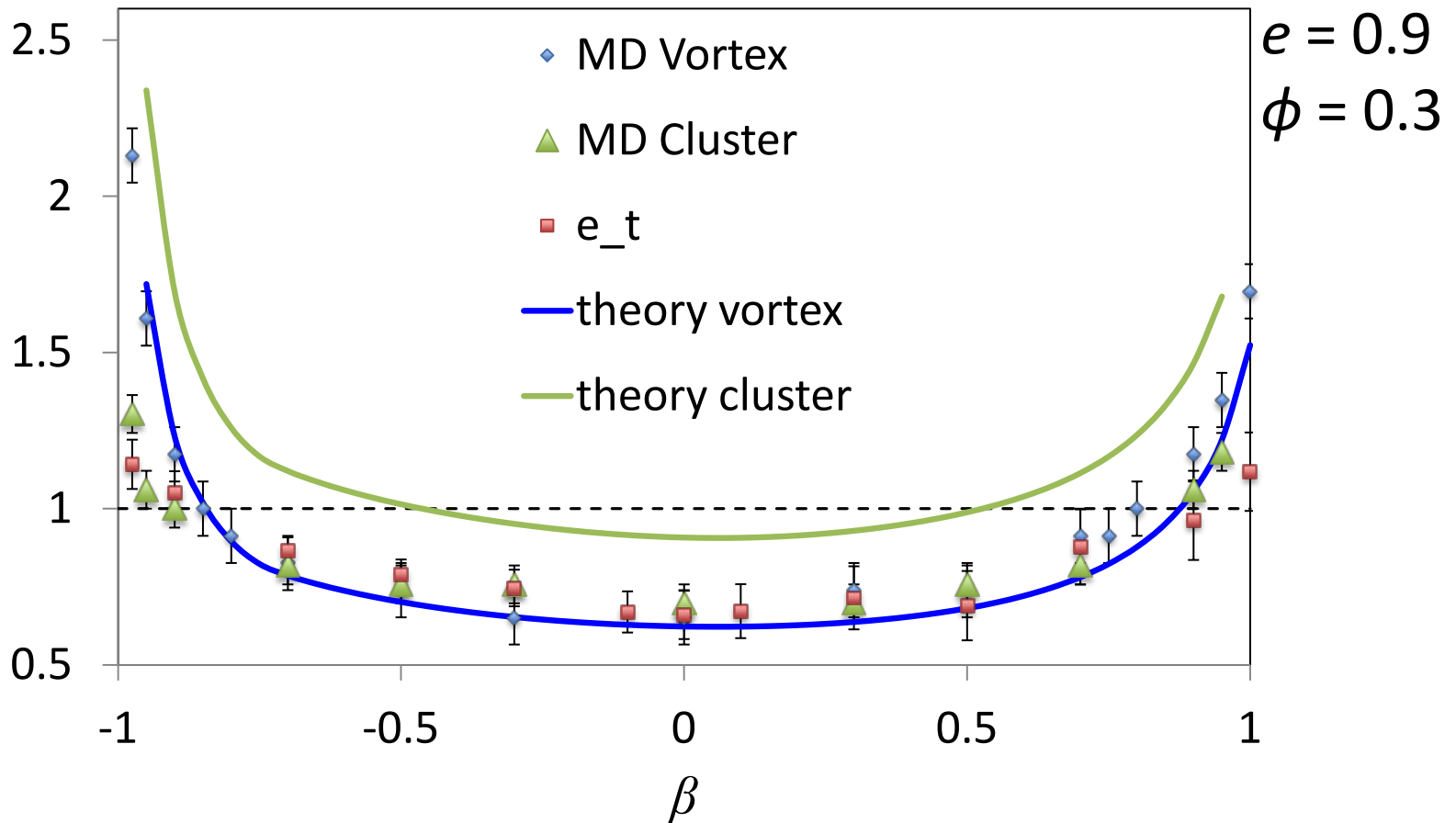
$e_t = 0$



$e_t = 2$



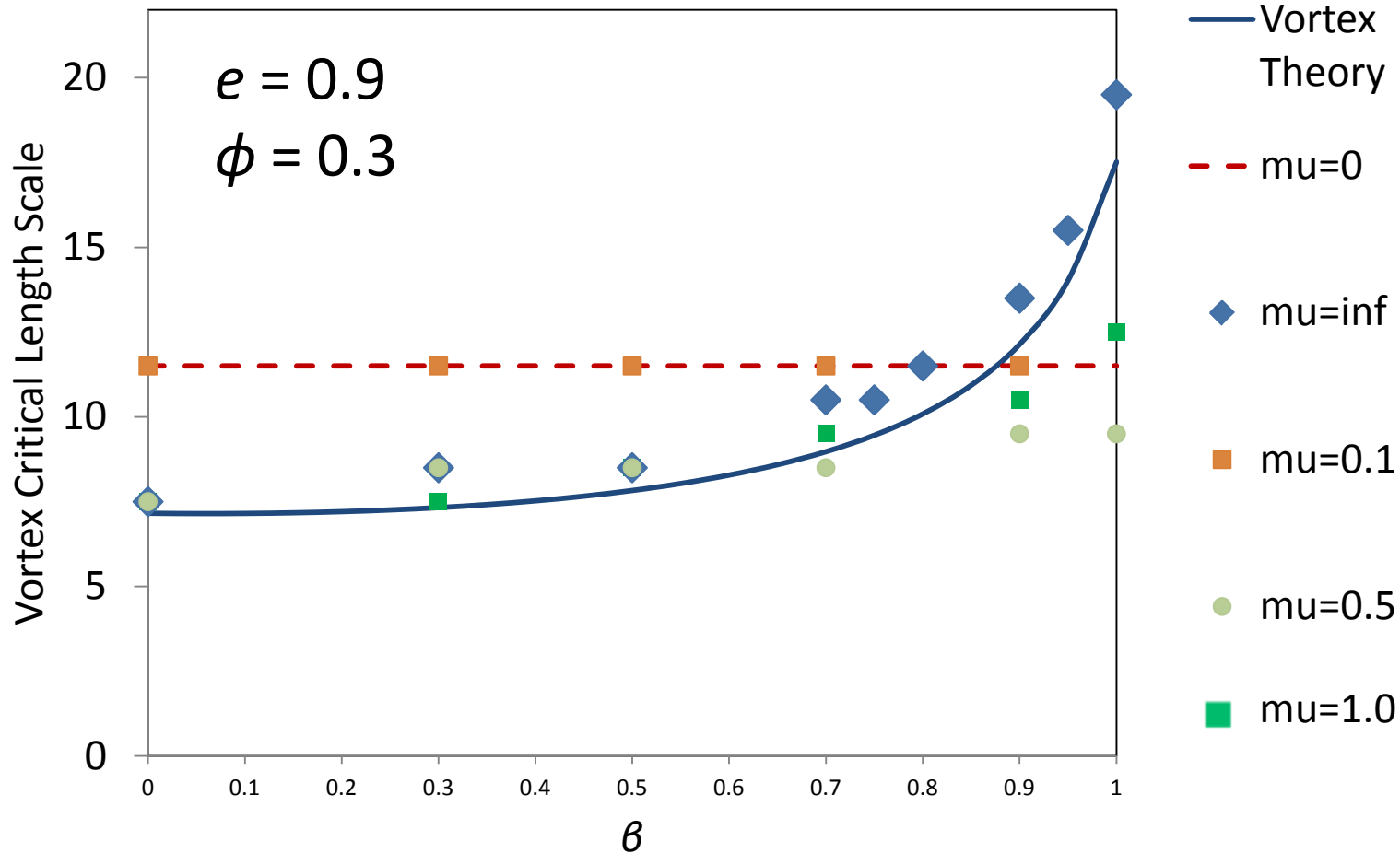
Onsets normalized to smooth-particle value



Extra note 2

- The ϵ_t shown is not just averaged
- First take absolute value of ϵ_t
- Take \log_{10}
- Average
- Raise 10 to the average
- This is because we want $\epsilon_t=0.1$ and 10 to average to 1 not close to 5

A Coulomb-friction model: Onset of vortices

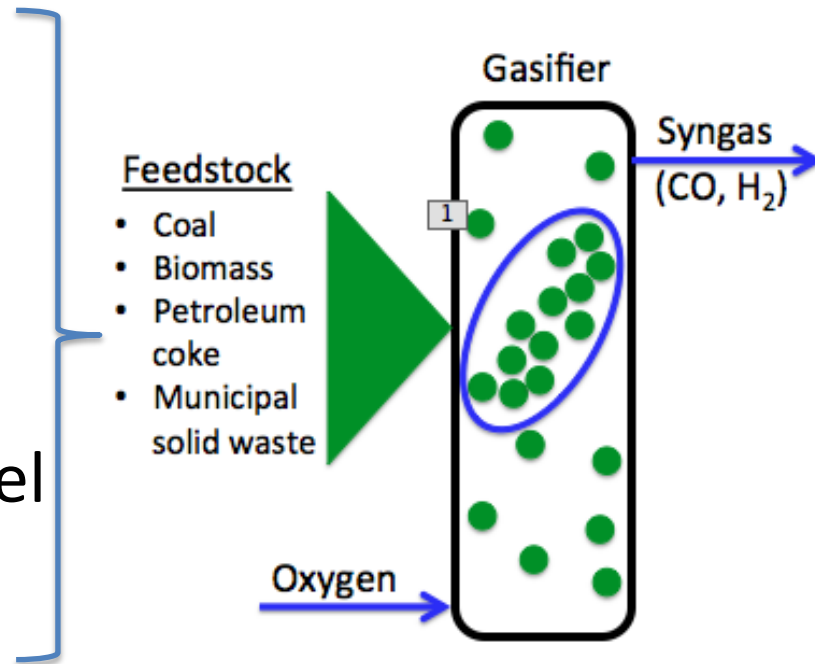


Concluding Remarks

- MD vs CFD vs LSA
 - Excellent agreement between kinetic theory and MD simulations
 - Small-gradient, molecular chaos assumptions of theory are not so restrictive
 - Nonlinear mechanisms are important for clusters
- Frictional dissipation
 - All dissipation is not created equal
 - A frictional cooling rate alone does well
(other transport coef.'s neglect friction)

Future Work

- Increased system complexity
 - Polydisperse particles
 - Non-spherical particles
 - Fluid phase
 - Bulk flow
 - Improved dissipation model
 - Wall boundaries





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