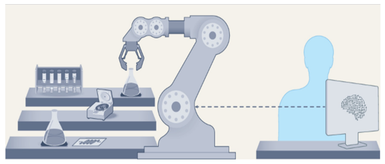
Optimizing Batch Crystallization with Model-based Design of Experiments

Hailey Lynch[a], Aaron Bjarnason[b], Daniel Laky[a], Cameron Brown[b], Alexander W. Dowling[a]

[a]*Department of Chemical and Biomolecular Engineering, University of Notre Dame, Notre Dame, IN 46556, USA*
[b]*EPSRC Future Manufacturing Research Hub for Continuous Manufacturing and Advanced Crystallisation, University of Strathclyde, Glasgow G1 1RD, UK*

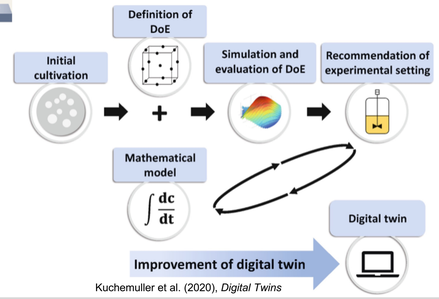
Advances in Design of Experiments[1,2]



Abolhasani & Kumacheva (2023), *Nature Syn*

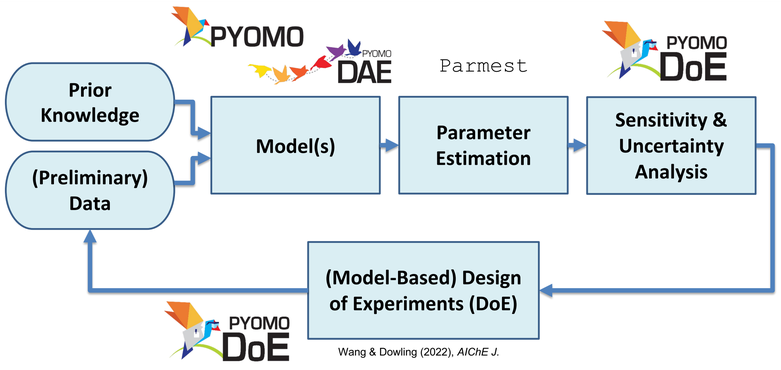
Adaptive sequential experiment optimization makes more accurate digital twins.

Self-driving laboratories select the next best experimental conditions for maximizing material performance.



Kuchemuller et al. (2020), *Digital Twins*

Science-based Modeling Workflow[3,4]

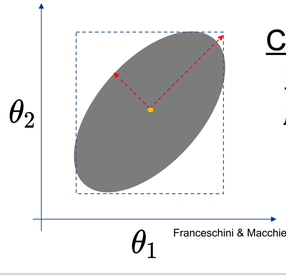


Wang & Dowling (2022), *AIChE J.*

What are the most informative data to reduce uncertainty (θ) and derisk technology optimization and scale-up?

Science-based Design of Experiments[3,4]

How do we sequentially choose the experimental conditions (φ) that will maximize information gain?



Franceschini & Macchietto (2008), *Chem. Eng. Sci.*

Confidence Ellipsoid

$$\boldsymbol{M} \approx \boldsymbol{V}^{-1}$$

Optimization Problem

$$\underset{\varphi}{\text{argmax}}\ \boldsymbol{M}(\hat{\theta}, \varphi)$$
$$\text{s.t.}\ \ \boldsymbol{M} = \boldsymbol{Q}^T \Sigma_\epsilon \boldsymbol{Q}$$

Sensitivity Matrix

$$\boldsymbol{Q}(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial f(x_1, \boldsymbol{\theta})}{\partial \theta_1} & \cdots & \frac{\partial f(x_1, \boldsymbol{\theta})}{\partial \theta_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f(x_n, \boldsymbol{\theta})}{\partial \theta_1} & \cdots & \frac{\partial f(x_n, \boldsymbol{\theta})}{\partial \theta_m} \end{bmatrix}$$

Batch Crystallization Example

A batch cooling crystallization system is defined using population balance equations[5] to determine the kinetic parameters for the driving mechanisms of crystallization: nucleation and growth.

Method of Moments

$$\frac{d\mu_0}{dt} = B_p + B_s$$
$$\frac{d\mu_1}{dt} = G\mu_0$$
$$\frac{d\mu_2}{dt} = 2G\mu_1$$
$$\frac{d\mu_3}{dt} = 3G\mu_2$$
$$\frac{dT}{dt} = -\beta$$
$$\frac{dC_{\text{crys}}}{dt} = -\rho_{\text{crys}} k_v \frac{d\mu_3}{dt}$$
$$\frac{dM_T}{dt} = -\frac{dC_{\text{crys}}}{dt}$$

Birth and Growth Rate

$$B_p = k_{b_p} S^P$$
$$B_s = \epsilon M_T k_{b_s} S^{s_1}$$
$$G = k_g S^{g_1}$$

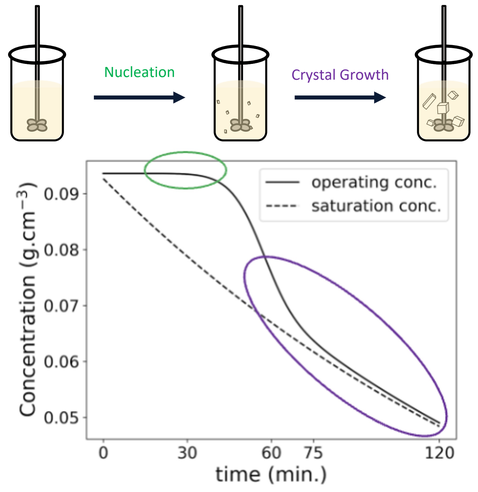
Saturation Concentration

$$S = \frac{c - c_{\text{sat}}}{c_{\text{sat}}}$$
$$C_{\text{sat}} = A + BT + CT^2$$

Supersaturation occurs as solubility is reduced when the temperature lowers (i.e., *cooling* crystallization).

Initial Conditions

$$\mu_i(0) = \mu_{i_{\text{seed}}}$$
$$T(0) = T_0$$
$$C_{\text{crys}}(0) = C_0$$



How can we systematically explore information of different cooling rates (i.e., β=0, 0.1, 0.2, and 0.3 °C min^{-1})?

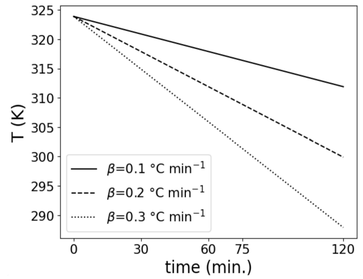
Notation

μ_i: i-th moment of the crystal size distribution
B_p: Primary nucleation rate
B_s: Secondary nucleation rate
G: Crystal growth rate
T: Operating temperature of the system
β: Cooling rate (°C min^{-1})
C_{crys}: Solute concentration in the liquid
M_T: Solids concentration in the slurry
S: Relative supersaturation
C_{sat}: Saturation concentration at operating temperature T
A, B, C: Solubility quadratic fit parameters
k_v: Crystal shape factor
ϵ: Power density (W/kg)
k_{b_p}: Primary nucleation rate constant
p: Primary nucleation exponent
k_{b_s}: Secondary nucleation rate constant
s_1: Secondary nucleation exponent
k_g: Crystal growth rate constant
g: Crystal growth rate exponent

$$\theta = \{k_g, k_{b_p}, k_{b_s}, g, p, s_1\}$$
$$\varphi = \{\beta\}$$

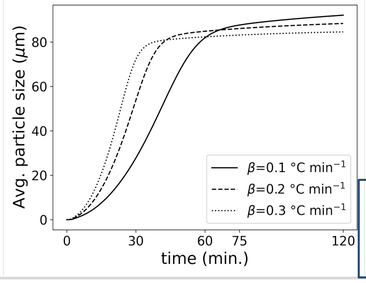
Goal: Utilize science-based design of experiments to understand which cooling rates are the most informative and compare to current experimental best practices.

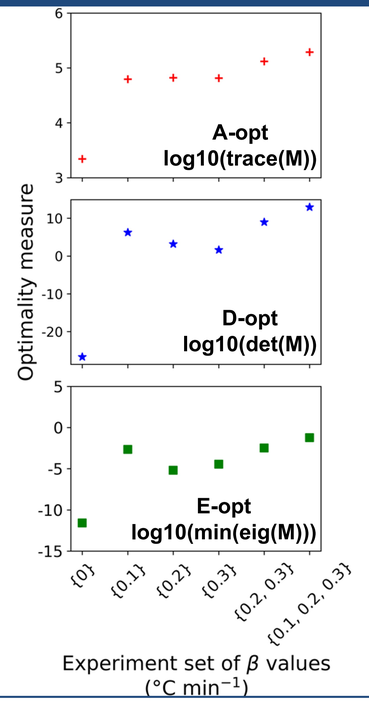
Conclusions



Low cooling rate (i.e., β=0.1) provides the most information rich experiment.

Aligns with heuristic that lower cooling rates give more information on growth kinetics.





Multiple experiments (i.e., β=0.2+β=0.3 and β=0.1+β=0.2+β=0.3) lead to higher A-, D-, and E-optimality.

Future Work

- Incorporate uncertainty quantification for batch crystallization studies.
- Perform enumerative exploratory analysis to evaluate experimental design space.
- Automate DoE workflow for cooling crystallization.

Acknowledgements

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Contact: Alexander W. Dowling, adowling@nd.edu

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[3] Wang, J., and Dowling, A.W. (2022) Pyomo.DoE: An open-source package for model-based design of experiments in Python, *AIChE Journal*, 68(12): e17813.

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[5] Hulburt, H.M and Katz, S. Some problems in particle technology: A statistical mechanical formulation. Chemical Engineering Science 19 (8)