Overview of MHD Computation for Magnetic Confinement Fusion

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MFE MHD computations include magnetic-island evolution and relaxation from magnetic tearing.

- Magnetic islands alter the confining topology, providing a conduit for enhanced energy transport.
- They also impede plasma flow.

Field-line traces and puncture plot of a 3D MHD tokamak computation.

- Transient 3D MHD activity leads to spheromak formation.

Spheromak simulation results on B-topology and temp. for SSPX [PRL 94].
Applications to edge-localized modes and disruptions show plasma-surface distortion and movement.

- ELMs concentrate heat flux temporally and alter the deposition location.

- Vertical displacement instability moves the plasma torus into the first wall.

Density (left) and temp. (right) from a JOREK simulation by Huysmans, et al., PPCF 51, 124012 (2009).

Poloidal flux contours from an M3D simulation by Strauss et al., PoP 17, 082505 (2010).
Simulations of MFE macroscopic dynamics are based on single- and two-fluid plasma models.

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot \left( D_n \nabla n - D_h \nabla \nabla^2 n \right)
\]

continuity with diffusive numerical fluxes

\[
m n \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla \sum_{\alpha} n T_{\alpha} - \nabla \cdot \mathbf{\Pi}
\]

flow evolution

\[
\frac{3}{2} n \left( \frac{\partial}{\partial t} + \mathbf{V}_{\alpha} \cdot \nabla \right) T_{\alpha} = -n T_{\alpha} \nabla \cdot \mathbf{V}_{\alpha} - \nabla \cdot \mathbf{q}_{\alpha} + Q_{\alpha}
\]

temperature evolution

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[ \eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{T_e}{ne} \nabla n + \frac{m_e}{ne^2} \frac{\partial}{\partial t} \mathbf{J} \right]
\]

Faraday's / Ohm's law

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

low-\(\omega\) Ampere's law

• Large resistivity keeps current density negligible outside the plasma part of the central region, and small mass density maintains accurate inertia.

• Two-fluid contributions appear in underlined terms.
The closure for stress ($\Pi$) can be a combination of Braginskii ion gyroviscosity and anisotropic viscous stress.

$$\Pi_{gv} = \frac{m_i p_i}{4eB} \left[ \hat{b} \times \mathbf{W} \cdot (\mathbf{I} + 3\hat{b}\hat{b}) - (\mathbf{I} + 3\hat{b}\hat{b}) \cdot \mathbf{W} \times \hat{b} \right], \quad \left( \mathbf{W} \equiv \nabla \mathbf{v} + \nabla \mathbf{v}^T - \frac{2}{3} \mathbf{I} \nabla \cdot \mathbf{v} \right)$$

$$\Pi_\perp \sim -\frac{3p_i m_i^2}{10e^2 B^2 \tau_i} \mathbf{W}$$

$$\Pi_\parallel = \frac{p_i \tau_i}{2} \left( \hat{b} \cdot \mathbf{W} \cdot \hat{b} \right) \left( \mathbf{I} - 3\hat{b}\hat{b} \right)$$

Similarly, the closure for conductive heat-flux density for each species ($q_\alpha$) can include different effects.

$$q_\alpha = -n \left[ \chi_{\parallel \alpha} \hat{b}\hat{b} + \chi_{\perp \alpha} \left( \mathbf{I} - \hat{b}\hat{b} \right) \right] \cdot \nabla T_\alpha + \frac{5}{2} \left( \frac{p_\alpha}{q_\alpha B} \right) \hat{b} \times \nabla T_\alpha$$

- The above relations are for large magnetization ($\Omega_\alpha \tau_\alpha >> 1$).
- MFE computations often use simplified closure relations.
Variation of magnetization in time and space has been used in at least one application.

- Non-inductive startup from localized current injection is studied experimentally in the Pegasus Toroidal Experiment [Eidietis et al., JoFE 26, 43 (2007), Battaglia, et al., NF 51, 073029 (2011)].

- Simulations with the NIMROD code model electrical current development and relaxation [O’Bryan and Sovinec, PoP 19, 080701 (2012)].

Isosurfaces of normalized parallel current density ($J_{||}/B$) for early (left) and late (center) in the driven phase, and after cessation of localized injection (right).

- Regions between filaments are unmagnetized early in time, and variable magnetized effects on heat-flux density are modeled.

- Variable magnetization for ion viscous stress has been implemented.
Modeling of plasma/neutral dynamics is important for small experiments and for edge conditions.

- A fluid model for interacting plasma and neutral species is developed in Meier and Shumlak, PoP 19, 0872508 (2012).
  - Collisional effects include scattering and reaction (ionization, recombination, charge exchange).
  - Neutral and plasma species are coupled through source/sink terms in the continuity, momentum density, and energy equations.
- The model has been implemented in the HiFi code [Glasser and Tang, CPC 164, 237 (2004); Lukin, PhD thesis, Princeton Univ. (2008)].

Simulation result for the Electrodeless Lorentz Force thruster using HiFi with dynamic neutral modeling. [Meier, PhD thesis, Univ. of Washington (2011).]

Kinetic effects from minority and majority species are included in some calculations.

- Energetic ions from beams, RF resonance, and fusion reactions have significant effects on macroscopic MHD modes.
  - Use of evolving-weight simulation particles for minority energetic particles, coupled to MHD equations through PIC-like deposition of a hot-particle pressure tensor is developed in Park, et al., PFB 4, 2033 (1992).
  - The method was implemented in M3D-K and later in NIMROD.

- Nonlocal majority-species kinetics are important at high temperature.
  - Formulation of closures based on simultaneous solution of drift-kinetic equations is presented in Ramos, PoP 17, 082502 (2010) and Ramos, PoP 18, 102506 (2011). Implementations for NIMROD (Held) and for M3D-C1 (Lyons) are being tested.

Comparison of non-resonant n=1 mode structure in NSTX without (left) and with (energetic) particle effects [Wang, et al., PoP 20, 102506 (2013)].
Numerical methods for MFE MHD computations address stiffness and anisotropy.

• Temporal scales vary widely in high-performance experiments.
  • Global Alfvén-wave propagation times ($\tau_A$) are of order 0.1-1 µs.
  • Global resistive diffusion times ($\tau_r$) are of order 1-10 s.
  • Magnetic island development can be very slow with growth times up to 1/10 of $\tau_r$.

• Effective time-advance methods are a focus of numerical development.
  • Implicit and semi-implicit methods are applied.
  • Some computations solve reduced models that eliminate the fastest MHD dynamics analytically.

• Extreme anisotropy with respect to the evolving direction of $\mathbf{B}(\mathbf{x})$ is another major consideration.
  • Several codes (NIMROD, M3D-C1, HiFi, JOREK, Psi-Tet) use high-order finite element methods, which helps resolve anisotropy.
  • Another approach tailors numerical heat flux densities for anisotropy [Günter et al., JCP 226, 2306 (2007)].

• MFE codes are not designed for shock capturing.
Methods for marching nonlinear calculations in time include a range of implicitness.

- Early developments [Jardin, JCP 29, 101 (1978); Aydemir and Barnes, JCP 59, 108 (1985)] treat the fast wave implicitly in analogy to computation for nearly incompressible fluids.

- The quasi-implicit method of the original M3D code applies implicit fast-wave computation with a potential representation [Park, et al., NF 30, 2413 (1990)].

- Adaptation of semi-implicit methods from weather modeling stabilizes all MHD waves without a full implicit treatment [Harned and Kerner JCP 60, 62 (1985); Schnack, et al. JCP 70, 330 (1987); Lerbinger and Luciani, JCP 97, 444 (1991)]. Currently used in DEBS, XTOR (MHD), NIMROD (MHD).

- M3D-C1 has a range of two-fluid options, where the implicit operator is based on linearization about each time-step [Jardin, et al., JCP 226, 2146 (2007); Ferraro and Jardin, JCP 228, 7742 (2009)].

- Two-fluid computations with NIMROD use an implicit leapfrog to avoid solving all fields simultaneously [Sovinec and King, JCP 229, 5803 (2010)].

- Several codes now use nonlinear implicit solves for *implicit balance* of all fields [Chacón, et al., JCP 178, 15 (2002); Glasser and Tang, CPC 164, 237 (2004); Reynolds, et al., JCP 215, 144 (2006); Chacón, PoP 15, 056103 (2008); Lütjens and Luciani, PoP 229, 8130 (2010)].
Many recently developed MFE MHD codes use high-order finite elements in their spatial representation.


- The M3D-C1 code uses reduced-quintic triangles, in combination with 1D Hermite cubics, to make values and derivatives of potential fields continuous across element borders [Jardin, JCP 200, 133 (2004)].

- The JOREK code now uses Bézier surfaces and elements with 1D finite Fourier series [Czarny and Huysmans, JCP 227, 7423 (2008)].

- The Psi-Tet code adapts Nedelec elements from electromagnetics for a high-order representation that separates longitudinal and solenoidal parts of vector fields in tetrahedra [Hansen, PhD thesis, Univ. of Washington (2014)].

Tetrahedral mesh used for Psi-Tet simulations of the HIT-SI experiment. [Hansen, PhD thesis]
Parallel computation is essential for modeling 3D evolution.

- Fusion MHD codes use 3D domain decomposition for distributed-memory parallelism.
- Parallel computation tends to be communication-intensive.
  - Physical information propagation is fast relative to the dynamics of interest.
  - Computations with time-steps larger than global wave propagation times are common.
- Most of the wall-clock time goes to solving the algebraic systems for implicit time advances in typical MFE applications.
  - Linear systems are usually solved with Krylov-space methods (GMRES, CG, etc.).
  - Block-diagonal preconditioning with sparse direct solves, such as SuperLU_DIST [Li and Demmel, ACM TMS 29, 110 (2003)], is applied in NIMROD and M3D-C1.
  - Nonlinearly implicit computations have been made possible with “matrix-free” Newton-Krylov solves using physics-based preconditioning [Chacón, PoP 15, 056103 (2008)].
Discussion Points

• Some of the features of MFE MHD simulation codes may be useful for MHD power generation:
  • 3D physics
  • Two-fluid and finite-Larmor-radius modeling
  • Implicit time-stepping
  • High-order spatial representation
• Plasma/neutral modeling is relatively new but developing.
• Effect that are not modeled may be needed:
  • Shock development
  • Plasma-surface interaction
  • Other effects?