

A MASSIVELY PARALLEL EULER-LAGRANGE STRATEGY FOR SIMULATING FLUIDIZED BED REACTORS

2012 MULTIPHASE CONFERENCE
MAY 22, 2012

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FUNDED IN PART BY:
*DOE OFFICE OF BIOMASS PROGRAM
NATIONAL RENEWABLE ENERGY LABORATORY*



Cornell University
Computational Thermo-Fluids
Laboratory

Motivation

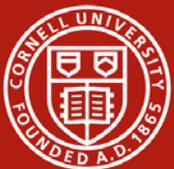
- Particle-laden flows are an important type of multiphase flow
- Common in many natural and industrial processes
- Fluidized bed reactors are ideal for gasification /pyrolysis
 - Easily scalable
 - Efficient mixing
 - Uniform temperature distribution
- Range of phenomenon exist in particle flows
 - Bubbling
 - Clustering



Spiegel (2009)

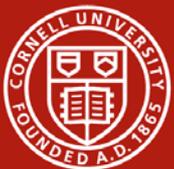
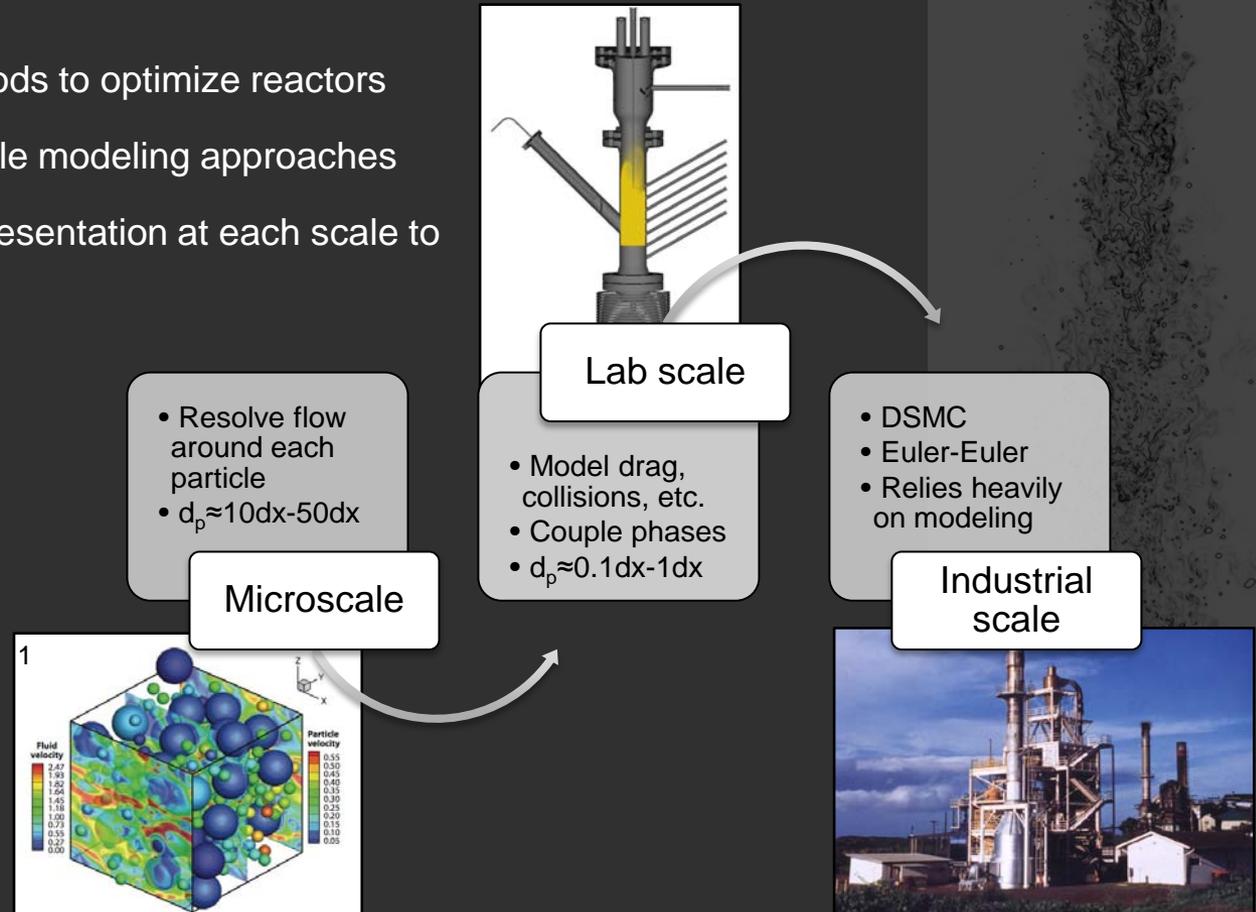


Horio & Kuroki (1994)



Objective

- Develop simulation strategy investigate complex multiphase flow dynamics
- Use first-principle based methods to optimize reactors
- Provide closures for larger scale modeling approaches
- Useful to have successful representation at each scale to elucidate the physics



Mathematical Formulation

POINTWISE DESCRIPTION

- *Gas phase:* Variable-density / low-Mach Navier-Stokes equations

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{u}_f) = 0$$

$$\frac{\partial}{\partial t} (\rho_f \mathbf{u}_f) + \nabla \cdot (\rho_f \mathbf{u}_f \otimes \mathbf{u}_f) = \nabla \cdot \boldsymbol{\tau} + \rho_f \mathbf{g}$$

$$\boldsymbol{\tau} = -p \mathbf{I} + \mu \left[(\nabla \mathbf{u}_f + \nabla \mathbf{u}_f^\top) - \frac{2}{3} (\nabla \cdot \mathbf{u}_f) \mathbf{I} \right]$$

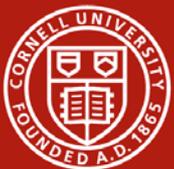
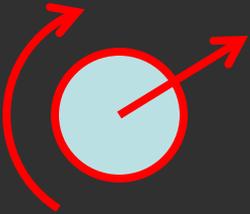
- *Particles:* Newton's second law of motion

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$$

$$m_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} \, d\mathbf{y} + F^{\text{col}} + m\mathbf{g}$$

$$I_p \frac{d\boldsymbol{\omega}_p}{dt} = \int_{S_p} \frac{d_p}{2} \mathbf{n} \times (\boldsymbol{\tau} \cdot \mathbf{n}) \, d\mathbf{y} + \sum_{j=1}^{n_p} \mathbf{f}_{t,j \rightarrow p}^{\text{col}}$$

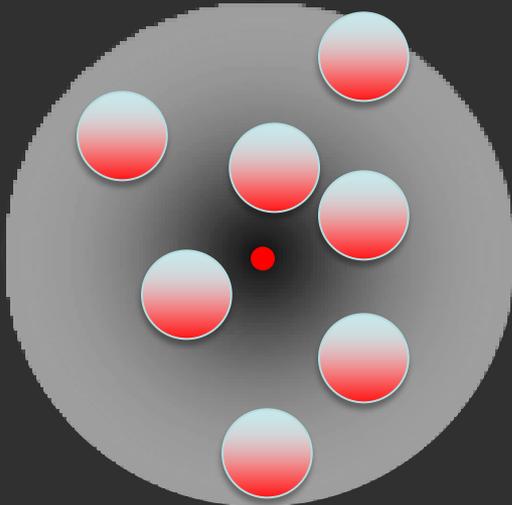
- *Boundary conditions:* no-slip and no-penetration at surface of particle
- *Collision force:* contact mechanics



Mathematical Formulation

VOLUME-FILTERED DESCRIPTION

- Following the work of Anderson & Jackson (1967)
- Objective: formulate equations for particle-laden flows that allow $\Delta x \gg d_p$
- Introduce **local volume filter** based on convolution product with kernel $g(r)$
 - Characteristic width of kernel needs to be $\gg d_p$
 - Flow features on the scale of the particles are filtered out, enabling $\Delta x \gg d_p$
 - Leads to **local mean voidage**:

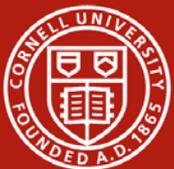


$$\varepsilon_f(\mathbf{x}, t) = \int_{V_{f\infty}(t)} g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

- Allows to define **filtered variable** \bar{a} from point variable a :

$$\varepsilon_f \bar{a}(\mathbf{x}, t) = \int_{V_{f\infty}(t)} a(\mathbf{y}, t) g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

- No commutation between filtering and differentiation due to particle surface contributions



Mathematical Formulation

VOLUME-FILTERED DESCRIPTION

- For variable density flows $\tilde{a} = \frac{\varepsilon_f \overline{\rho_f a}}{\varepsilon_f \overline{\rho_f}}$
- In general, decompose point variable a as $a = \bar{a} + a'$ or $a = \tilde{a} + a''$

$$\frac{\partial \varepsilon_f \overline{\rho_f}}{\partial t} + \nabla \cdot (\varepsilon_f \overline{\rho_f} \tilde{\mathbf{u}}_f) = \mathbf{S}_\rho$$

$$\frac{\partial}{\partial t} (\varepsilon_f \overline{\rho_f} \tilde{\mathbf{u}}_f) + \nabla \cdot (\varepsilon_f \overline{\rho_f} \tilde{\mathbf{u}}_f \otimes \tilde{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \overline{\rho_f} \mathbf{g} + \mathbf{S}_{\rho u} - \mathbf{F}^{\text{inter}}$$

$$\mathbf{S}_\rho = \sum_{i=1}^{n_p} \int_{S_p} \rho_f \mathbf{n} \cdot \frac{d\mathbf{r}_i}{dt} g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

$$\mathbf{F}^{\text{inter}} = \sum_{i=1}^{n_p} g(|\mathbf{x} - \mathbf{x}_p|) \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} d\mathbf{y}$$

$$\mathbf{S}_{\rho u} = \sum_{i=1}^{n_p} \int_{S_p} \rho_f \mathbf{n} \cdot \frac{d\mathbf{r}_i}{dt} \otimes \mathbf{u}_f g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

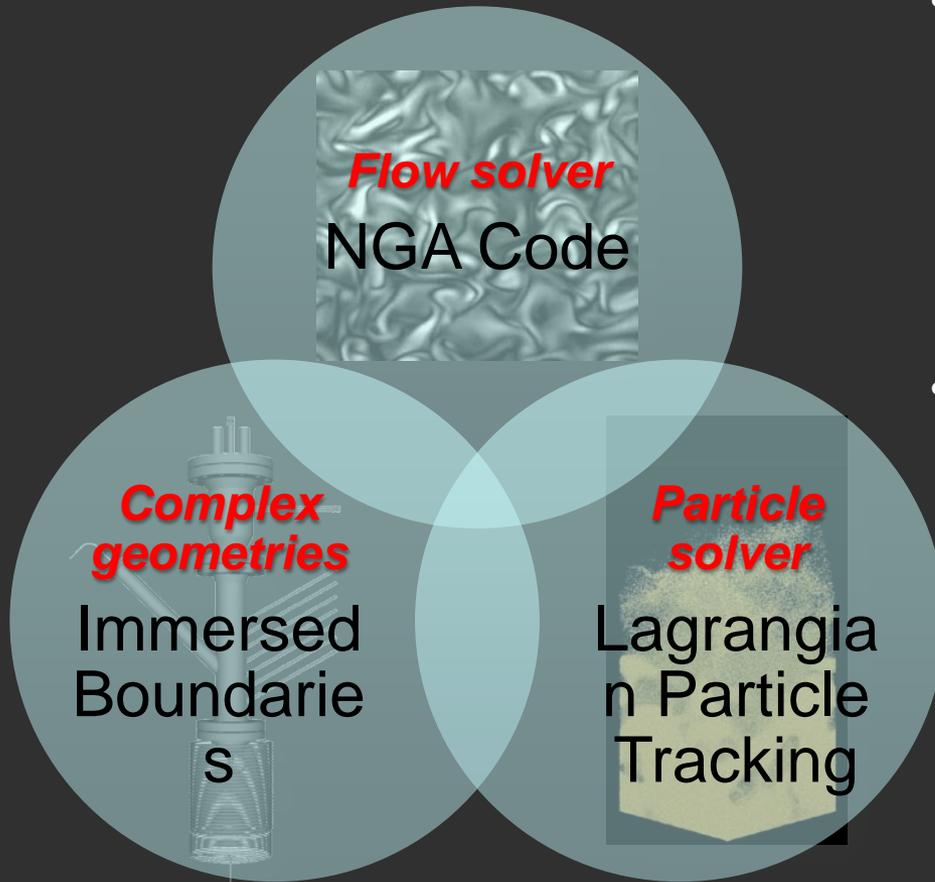
$$\int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} d\mathbf{y} \approx V_p \nabla \cdot \bar{\boldsymbol{\tau}} + \mathbf{f}_p^{\text{drag}}$$

$$\bar{\boldsymbol{\tau}} = -p\mathcal{I} + \mu \left[(\nabla \bar{\mathbf{u}}_f + \nabla \bar{\mathbf{u}}_f^T) - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}}_f) \mathcal{I} \right] + \mathbf{R}_\mu$$

$$\mathbf{R}_u = \varepsilon_f \overline{\rho_f} \widetilde{\mathbf{u}_f'' \otimes \mathbf{u}_f''}$$



Computational Approach



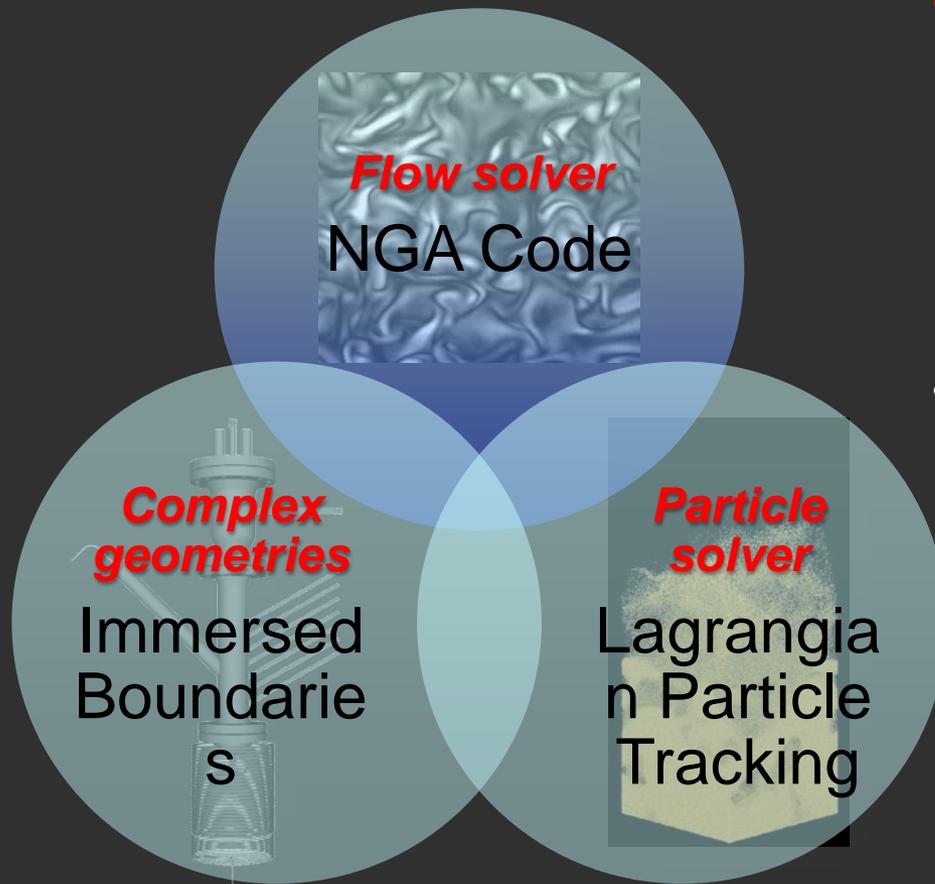
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 - Arbitrarily high-order DNS/LES code
 - Massively parallel
 - Conservation of mass, momentum, and kinetic energy
- Immersed Boundary³
 - Based on cut-cell approach
 - Discrete conservation of mass and momentum
 - Fully implicit implementation to handle small cut-cells

2. O. Desjardins, G. Blanquart, G. Balarac, H. Pitsch, High order conservative finite difference scheme for variable density low Mach number turbulent flows, *Journal of Computational Physics* 227 (2008) 7125– 7159.

3. P. Pepiot, O. Desjardins, Direct numerical simulation of dense particle-laden flows using a conservative immersed-boundary technique, Center of Turbulence Research, Summer program 2010.



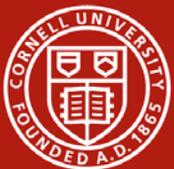
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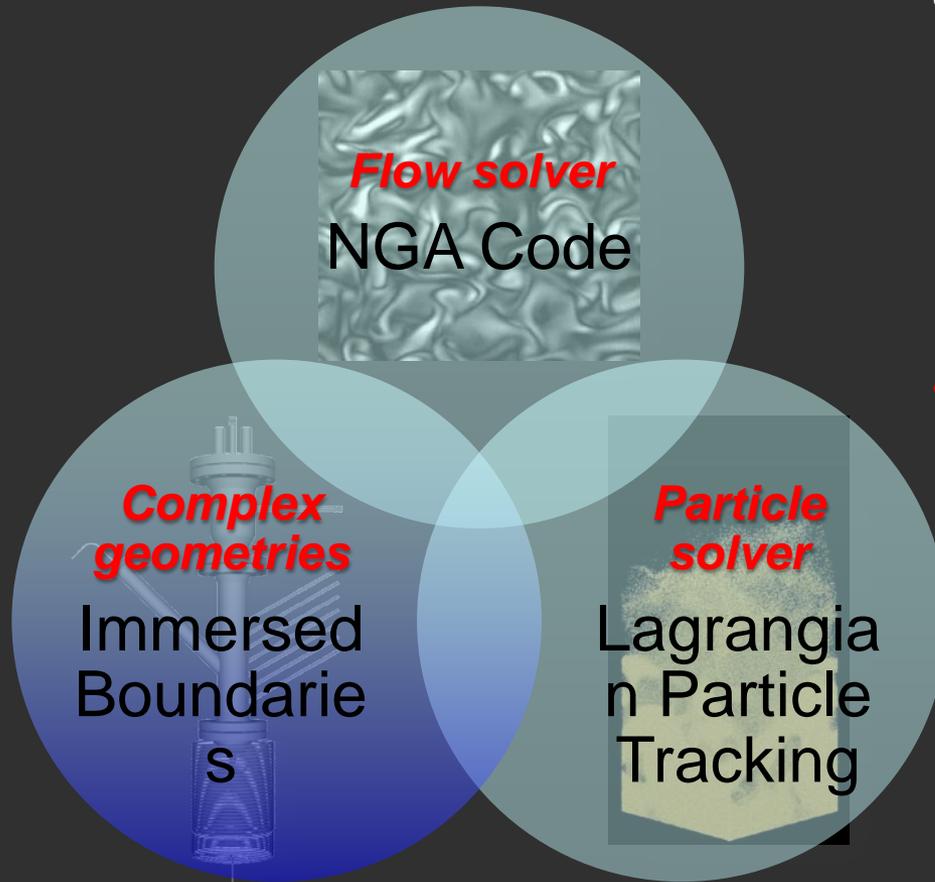
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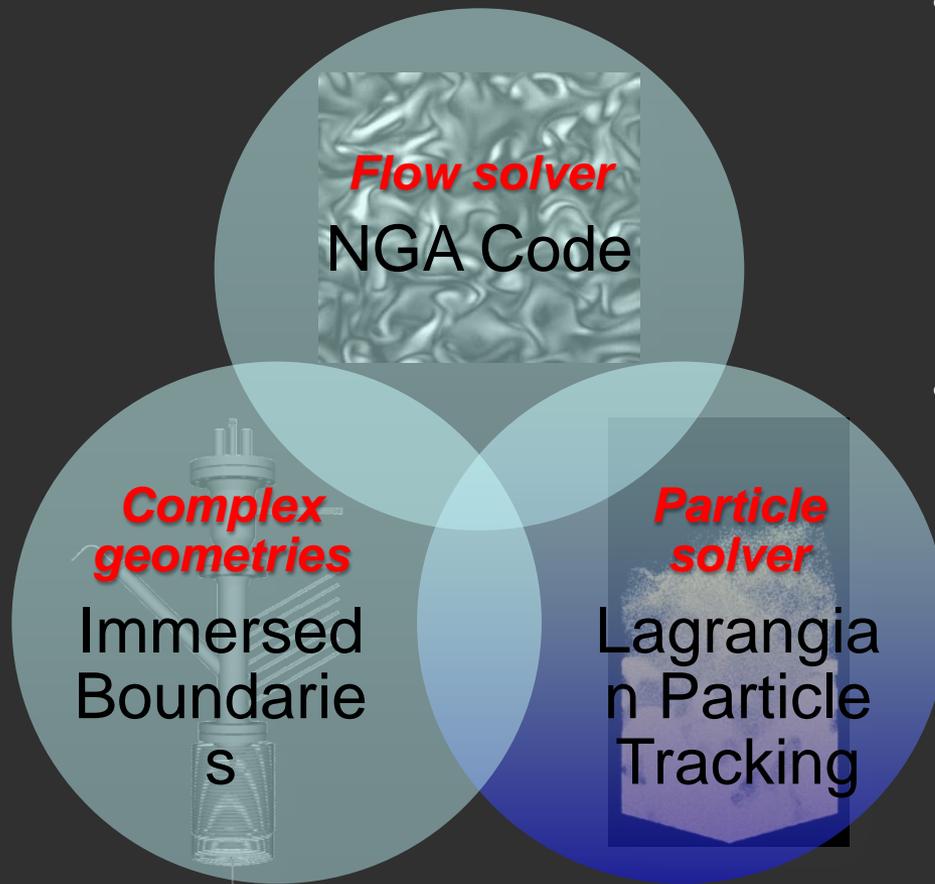
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NGA Computational Platform

- NGA solves the volume-filtered equations with the following assumptions & models

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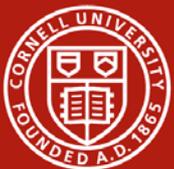


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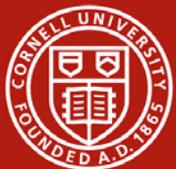
$$\mathbf{R}_u = -\nu_T \left(\nabla \tilde{\mathbf{u}}_f + \nabla \tilde{\mathbf{u}}_f^T \right)$$

Drag model of Tenneti & Subramaniam (2011)

$$\mathbf{f}_i^{\text{drag}} = \frac{1}{\tau_p} (\tilde{\mathbf{u}}_f - \mathbf{u}_p) F(\varepsilon_f, \text{Re}_p)$$

$$\tau_p = \frac{\rho_p d_p^2}{18\mu\varepsilon_f} \quad \text{Re}_p = \frac{\varepsilon_f \rho_f |\tilde{\mathbf{u}}_f - \mathbf{u}_p| d_p}{\mu}$$

$$F(\varepsilon_f, \text{Re}_p) = \frac{1 + 0.15 \text{Re}_p^{0.687}}{\varepsilon_f^2} + \frac{5.81(1 - \varepsilon_f)}{\varepsilon_f^2} + \frac{0.48(1 - \varepsilon_f)^{1/3}}{\varepsilon_f^3} + \varepsilon_f(1 - \varepsilon_f)^3 \text{Re}_p \left(0.95 + \frac{0.61(1 - \varepsilon_f)^3}{\varepsilon_f^2} \right)$$



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$$\bar{\boldsymbol{\tau}} = -p \mathbf{I} + \mu^* \left[(\nabla \bar{\mathbf{u}}_f + \nabla \bar{\mathbf{u}}_f^T) - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}}_f) \mathbf{I} \right] + \cancel{R_{\mu}} \quad \text{Drag model of Tenneti \& Subramaniam (2011)}$$

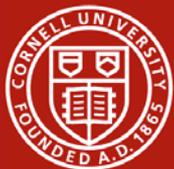
Effective viscosity model of Gibilaro et al.

~~(2007)~~
 $\mu^* = \mu \varepsilon_f^{-2.8}$

$$\mathbf{f}_i^{\text{drag}} = \frac{1}{\tau_p} (\tilde{\mathbf{u}}_f - \mathbf{u}_p) F(\varepsilon_f, \text{Re}_p)$$

$$\tau_p = \frac{\rho_p d_p^2}{18 \mu \varepsilon_f} \quad \text{Re}_p = \frac{\varepsilon_f \rho_f |\tilde{\mathbf{u}}_f - \mathbf{u}_p| d_p}{\mu}$$

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Lagrangian Particle Tracking

- A set of 9 coupled ODEs are solved for each individual particle
- Time advancement based on
 - 2nd order Runge-Kutta for particle ODEs
 - 2nd order coupling between gas and particles phase

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$$

$$m_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} \, d\mathbf{y} + F^{\text{col}} + m\mathbf{g}$$

$$I_p \frac{d\boldsymbol{\omega}_p}{dt} = \int_{S_p} \frac{d_p}{2} \mathbf{n} \times (\boldsymbol{\tau} \cdot \mathbf{n}) \, d\mathbf{y} + \sum_{j=1}^{n_p} \mathbf{f}_{t,j \rightarrow p}^{\text{col}}$$

- Collisions based on soft sphere approach⁶ modified for parallel efficiency
- Exchange between phases
 - Gas phase data is interpolated to the particle location using trilinear interpolation
 - Particle data is filtered onto the Eulerian mesh with an implicit/conservative smoothing operation



Lagrangian Particle Tracking

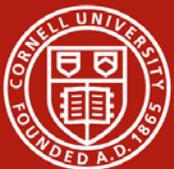
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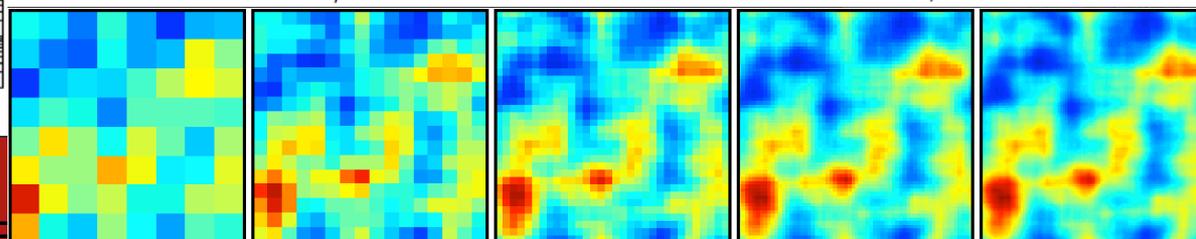
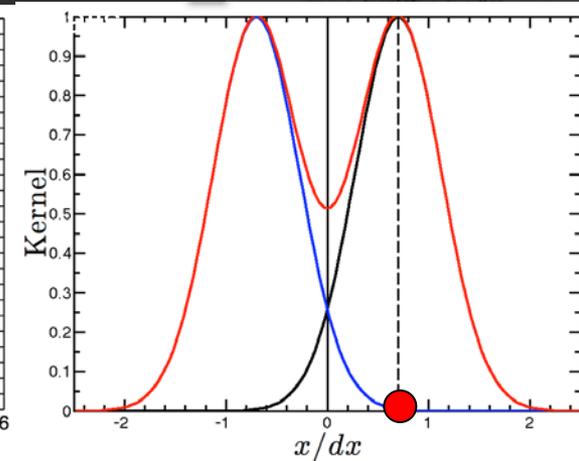
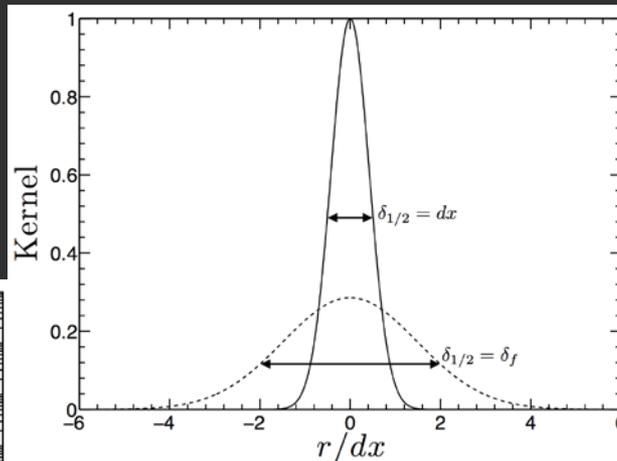
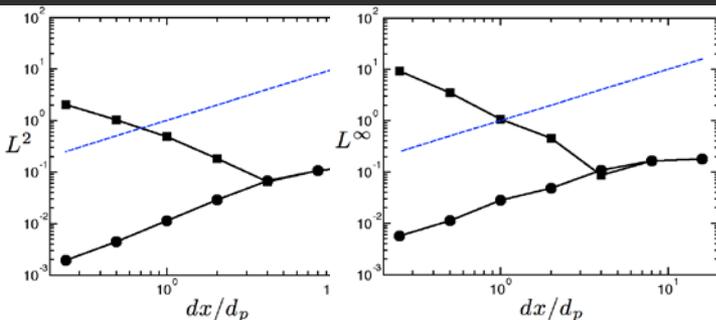


Particle Data to Eulerian Mesh

- Need to transfer ε_f and F^{inter} to underlying mesh consistent with mathematical formulation
- Filtering needs to be based on particle size not mesh
- Filter based on the convolution of mollification and Laplacian smoothing
 - Mollification: extrapolate particle data to neighboring cells
 - Diffusion: smooth data with specified width
- Filter width δ_f is independent of the mesh Gaussian diffusion

Introduce image particle to apply Neumann condition near

L-norm error



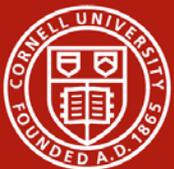
$dx = 4d_p$

$dx = 2d_p$

$dx = d_p$

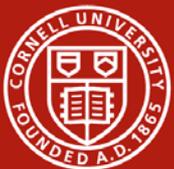
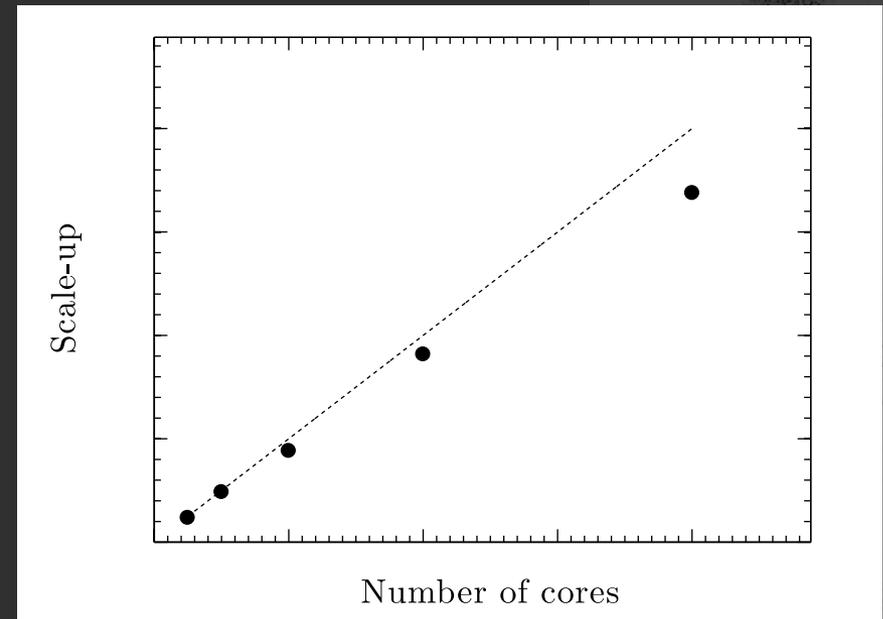
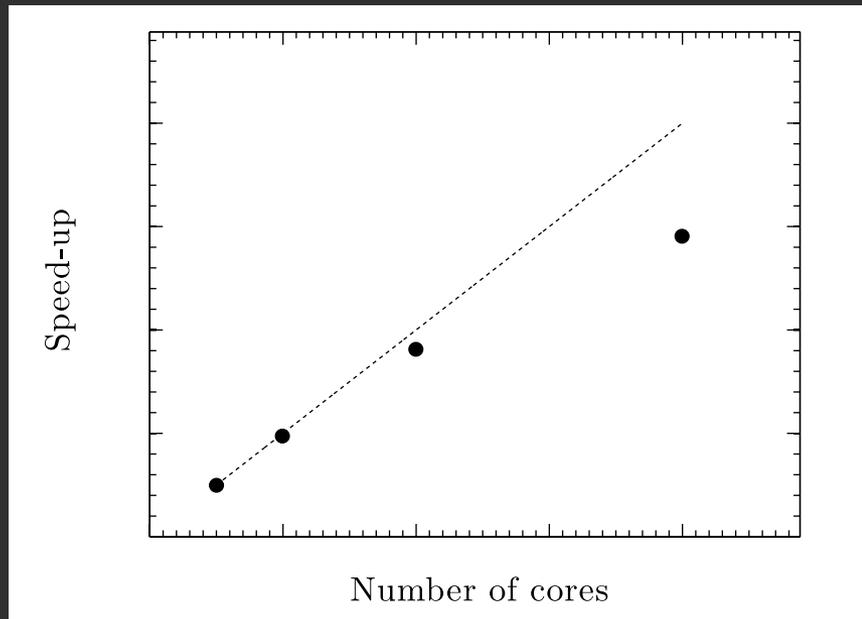
$dx = d_p/2$

$dx = d_p/4$



Parallel Performance

- Parallelization: MPI (domain decomposition)
- Scaling performed on Red Mesa (Sandia National Labs)
- 134 million cells, 383 million particles



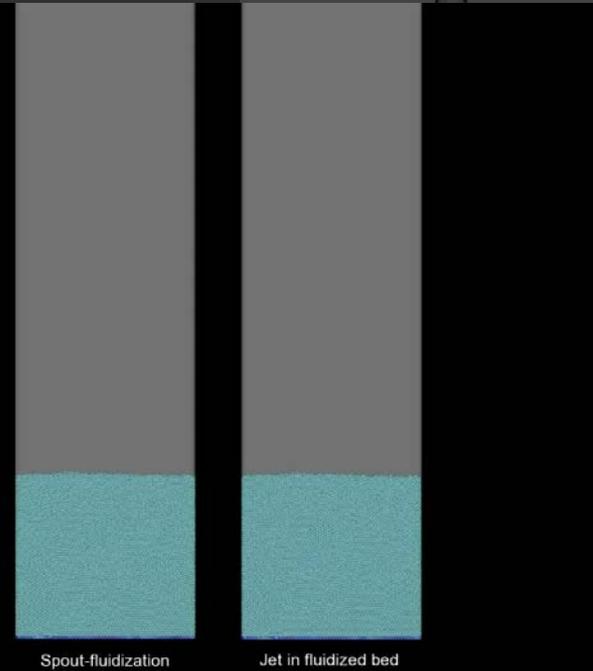
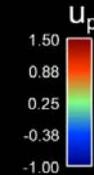
Initial validation

Spout fluidization (*Link et al., 2005*)

- Case A: $u_{bg} = 1.5$ m/s, $u_{sp} = 30$ m/s
- Case B: $u_{bg} = 3.0$ m/s, $u_{sp} = 20$ m/s

Bed dimensions [m]	0.75 x 0.15 x 0.015
Spout width [m]	0.01
Particle diameter [m]	0.0025
Particle density [kg/m ³]	2526
Number of particles [-]	245,000
Grid [-]	300 x 60 x 6

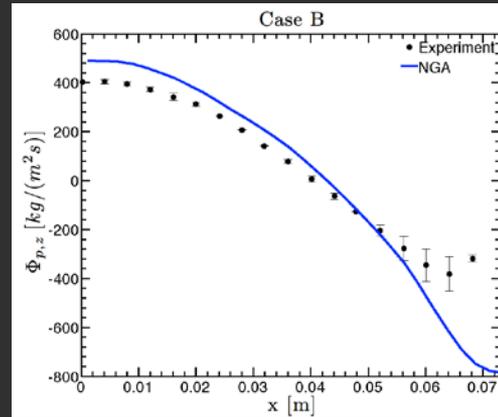
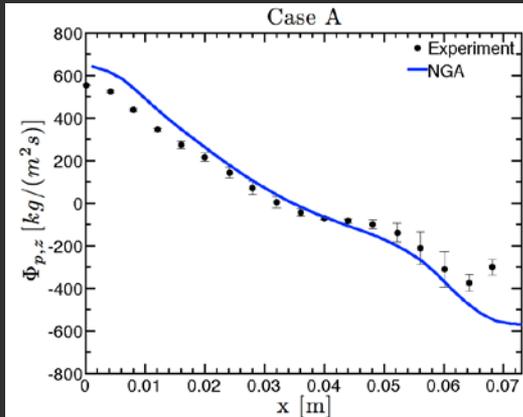
Time = 0.00000



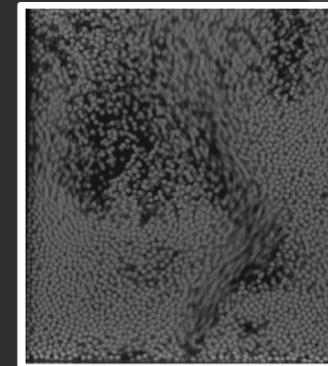
Spout-fluidization

Jet in fluidized bed

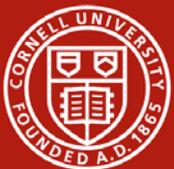
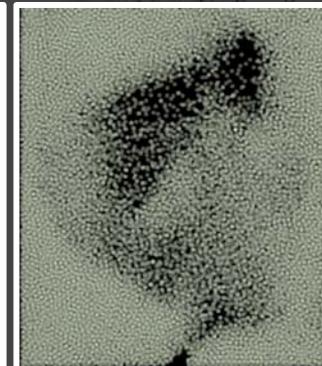
Particle mass flux



Case B (exp)



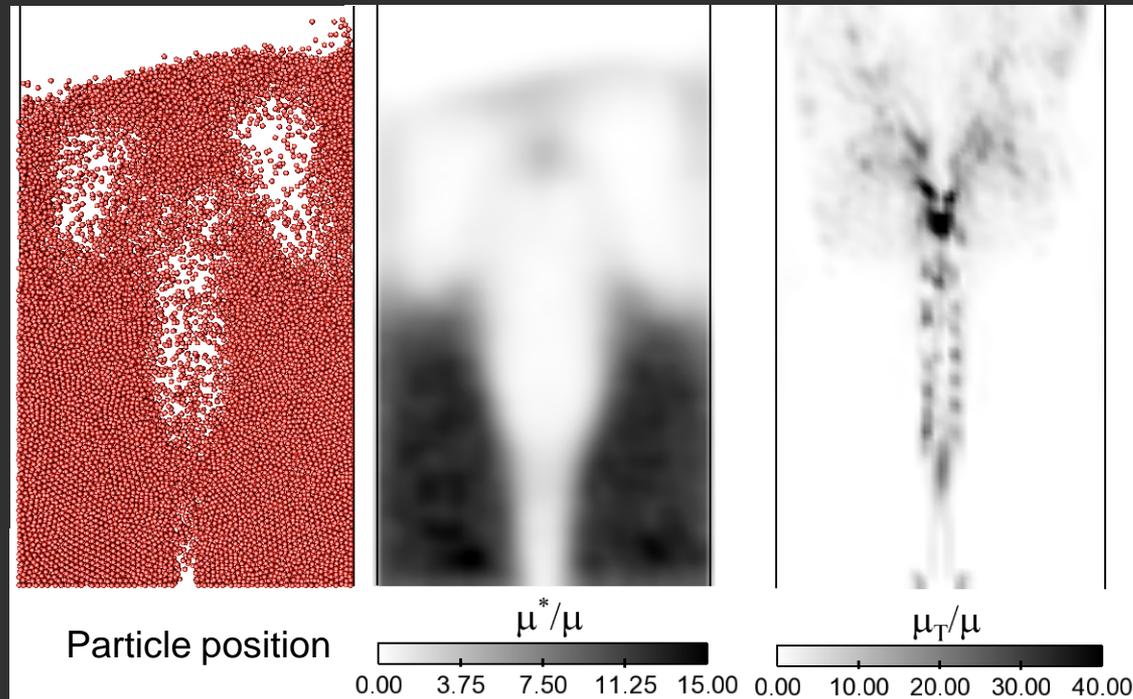
Case B (sim)



Initial validation

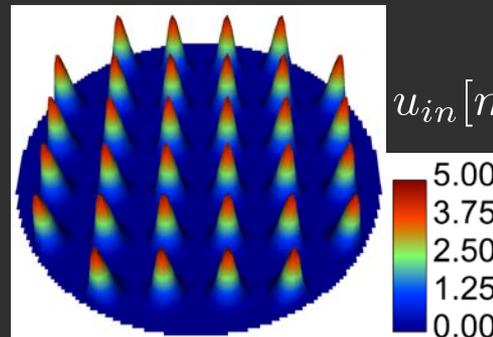
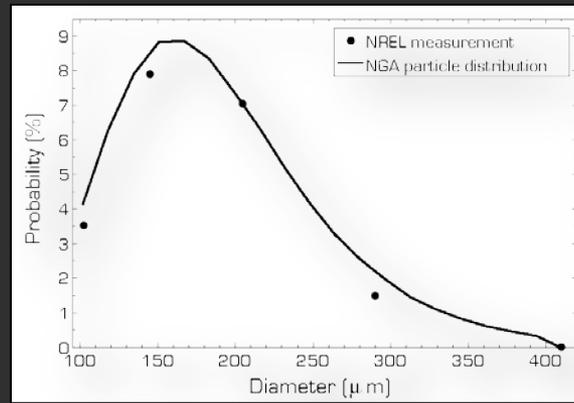
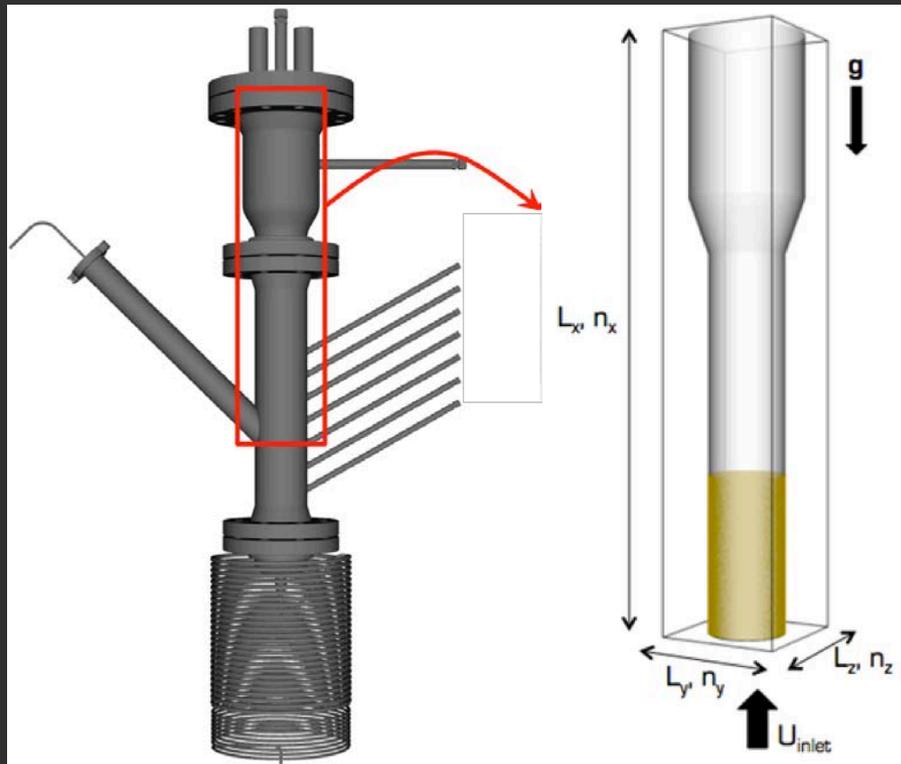
Spout fluidization, case A

- Dynamic Smagorinsky eddy viscosity model⁸ to close \mathcal{R}_{u}
- Based on Lagrangian averaging⁹
- SGS model does not account for turbulence modulation by particles
- Spatial segregation between models



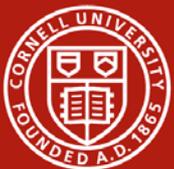
Half-scale simulation of NREL's 4-in fluidized bed reactor

- National Renewable Energy Laboratory operates a 4-inch fluidized bed reactor for biomass gasification

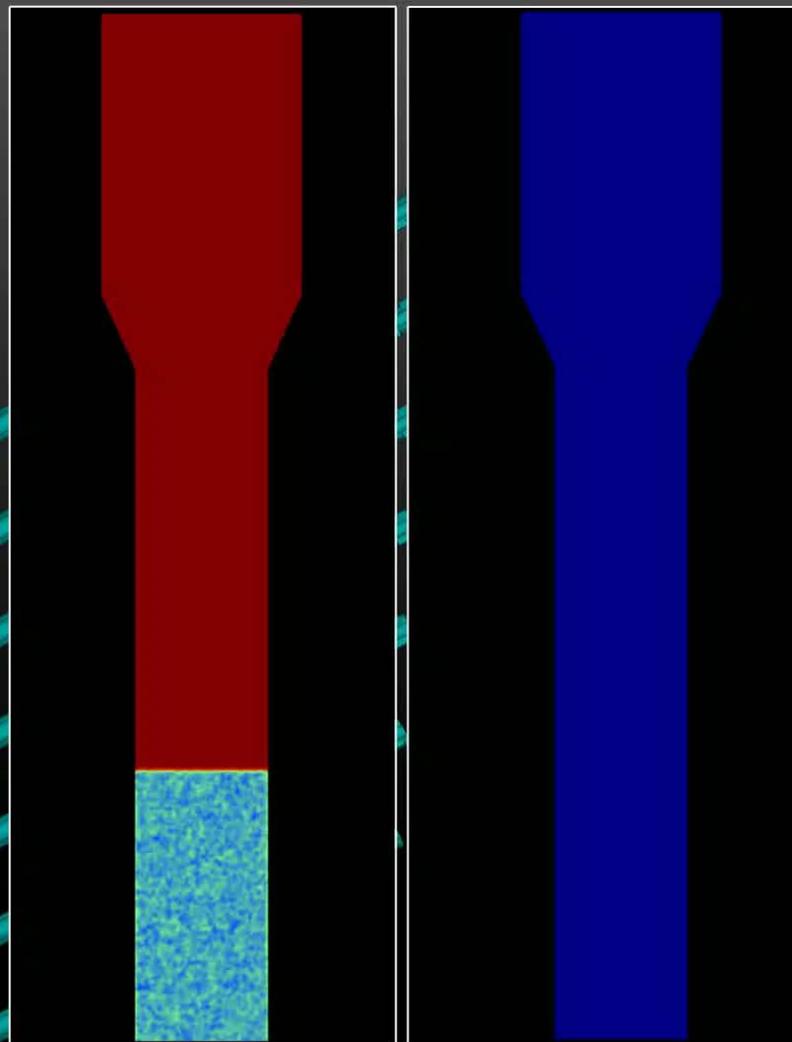
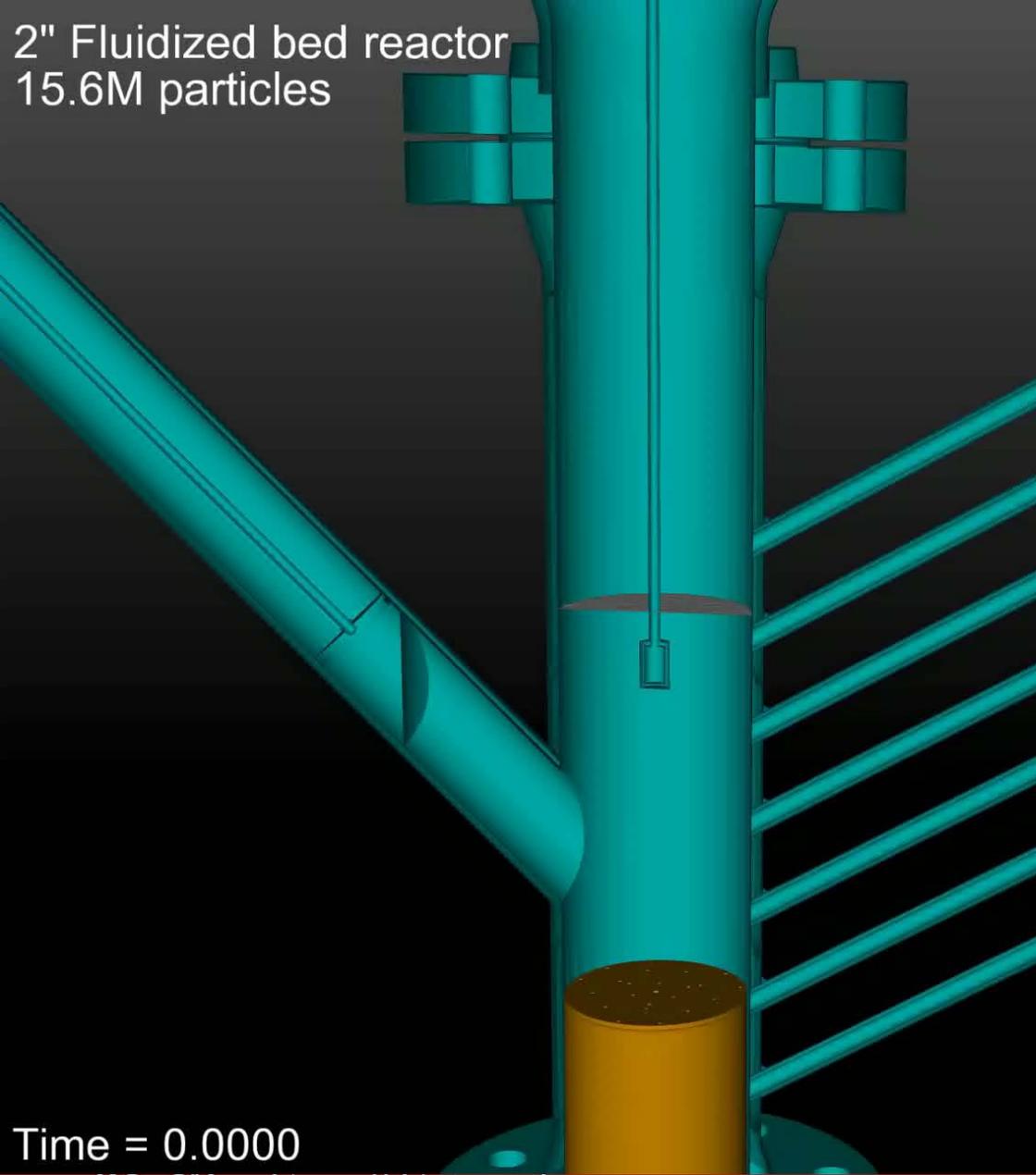


D_{bed}

- $u_{in} = 6 \rightarrow 12U_{mf}$
- 15.6 M particles
- 20 M grid cells
- 576 cores on Marvin (Cornell cluster)

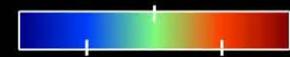


2" Fluidized bed reactor
15.6M particles



Volume fraction

0.34 0.67 1.00



0.50 0.83

Gas velocity

0.00 1.50 3.00



0.75 2.25

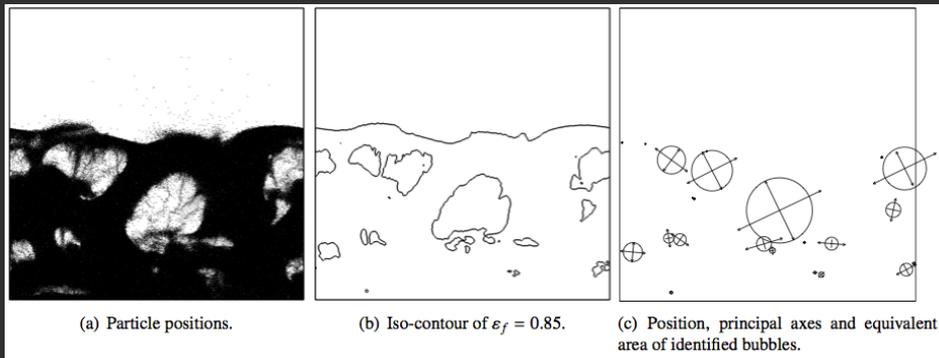
Time = 0.0000



Cornell University
Computational Thermo-Fluids
Laboratory

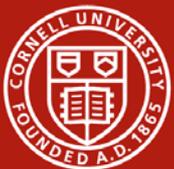
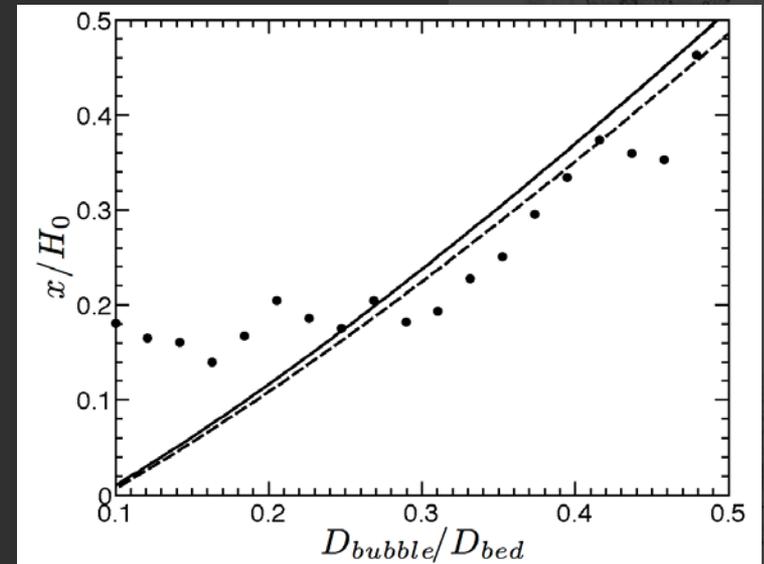
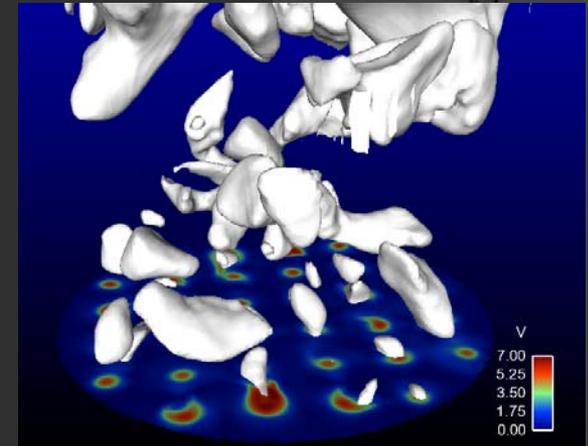
Bubble characterization

- **Structure identification** algorithm¹⁰
 - Band-growth algorithm to identify bubbles



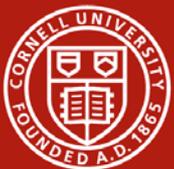
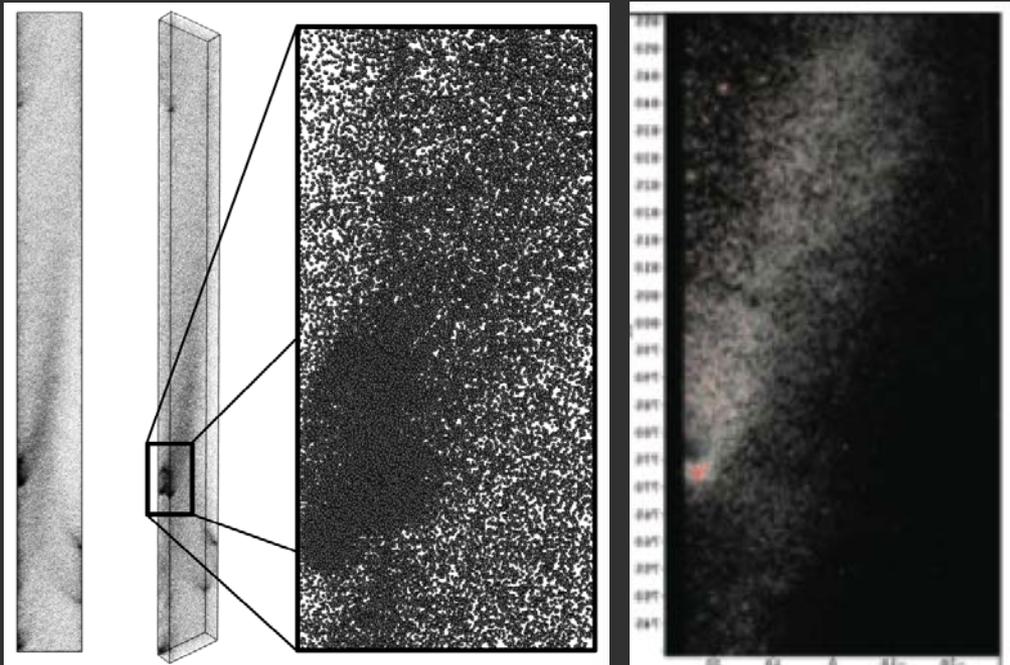
- Mean bubble diameter: $0.18 \cdot D_{bed}$
- On average 8 bubbles in bed at once
- Compared results with Darton¹¹ correlation
 - $U_{in} = 6 U_{mf}$ (solid line)
 - $U_{in} = 12 U_{mf}$ (Dashed line)

$$D_b = 0.54 (U_{in} - U_{mf})^{0.4} \left(h + 4\sqrt{A_0} \right)^{0.8} g^{-0.2}$$



Simulation of a Turbulent Riser

- Periodic in stream-wise directions
- 266,760 particles initially uniform distribution
- 800 x 82 x 26 mesh for 0.5 m domain
- Formation of clusters along walls is observed – excellent qualitative agreement with experiments¹²



Simulation of a Turbulent Riser

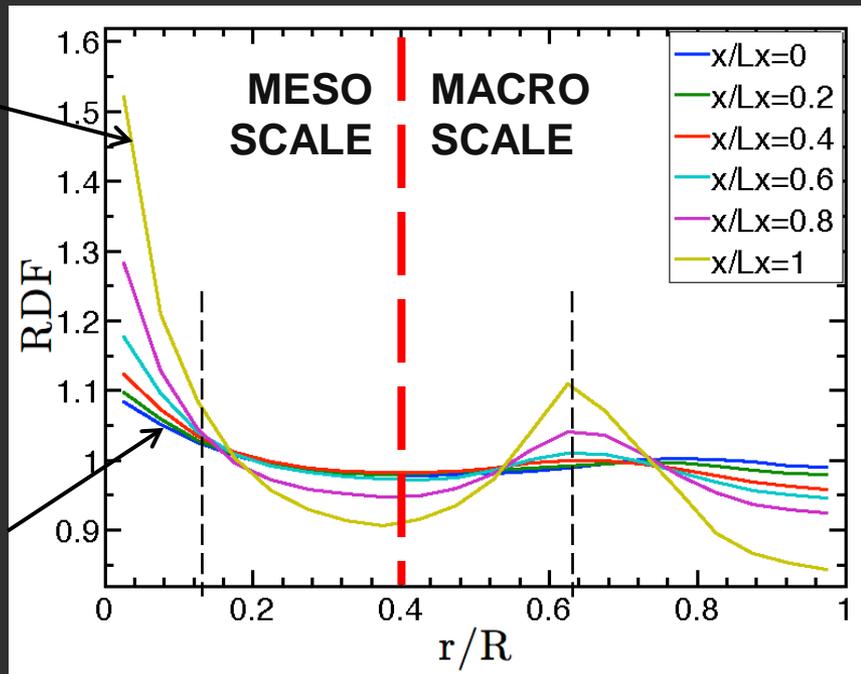
- Radial Distribution Function (RDF) to characterize clustering

$$RDF(x, r) = \frac{N_p L_z}{2drN(N-1)}$$

RDF=1: Uniform distribution

RDF>1: Clustering

- N: Number of particles in column
- N_p : Number of particle pairs
- Total possible number of pairs: $N(N-1)$

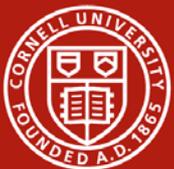
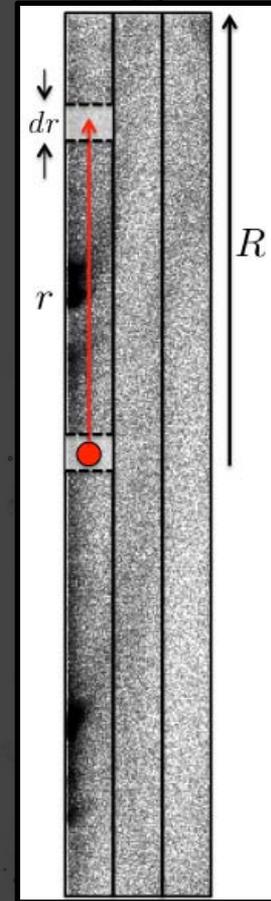


- Meso-scale

- Maximum clustering at the walls
- Characteristic cluster size $\sim 90 d_p$

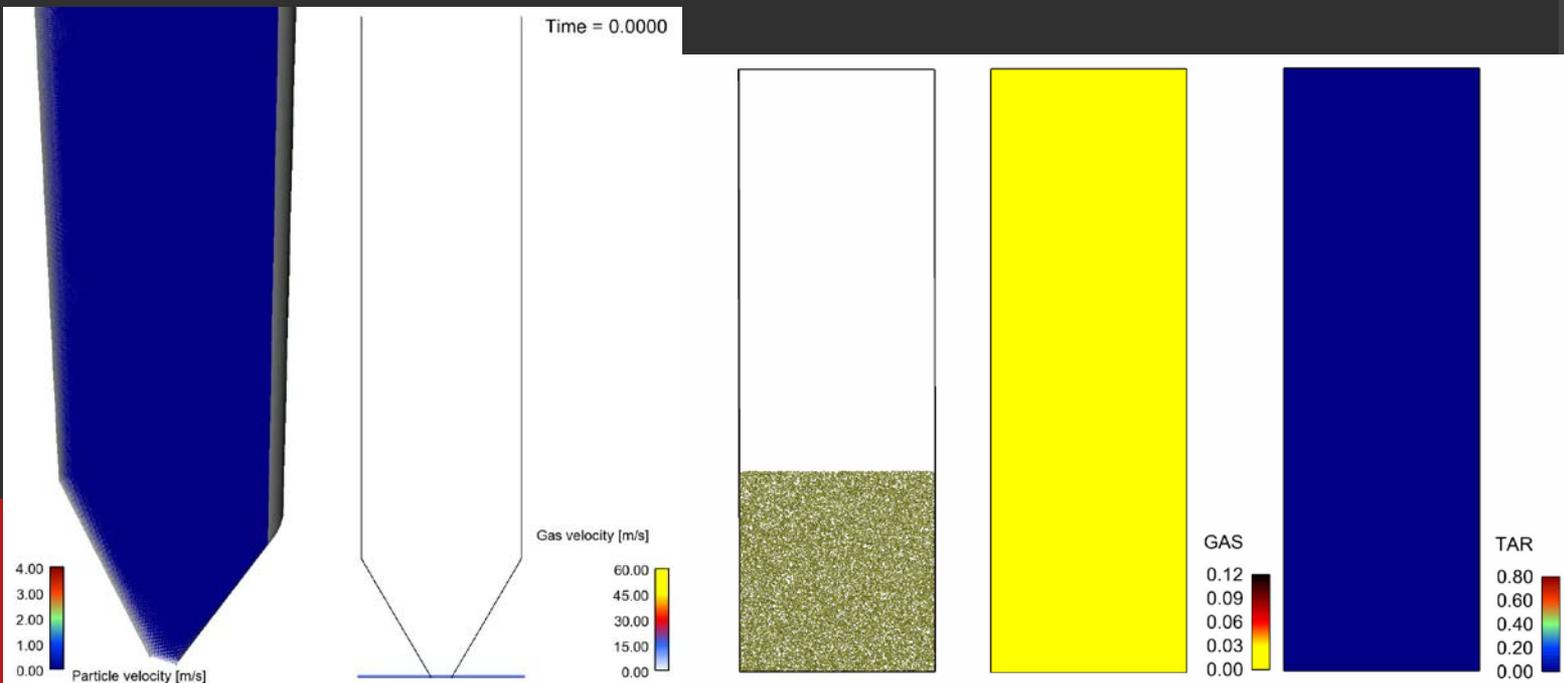
- Macro-scale

- Second peak in RDF
- Characteristic length scale $\sim L_z / 3$ (or $460 d_p$)



Conclusions

- Presented a simulation strategy for turbulent particle-laden flows in complex geometries
- Simulations of fluidized bed reactors show good agreement with experimental data
- Looking forward:
 - Further validation
 - Chemistry is currently being incorporated (Pepiot research group)
 - Investigate clustering in turbulent risers
 - Use resolved particle simulations for exploring the closure of the filtered equations
 - Transition to industrial scale approaches (QMOM, parcels...)



Time = 0.00000

Questions?

