SIMPLE ANALYTICAL SOLUTIONS FOR THE DEVONIAN SHALE GAS RESERVOIR BEHAVIOR

PROGRESS REPORT

BY

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CONTENTS

ABSTRACT

Pressure transient methods to analyze wells and the reservoir are not available for Devonian Shale gas wells because the motion of gas in the Devonian Shale gas reservoir is different than gas flow in conventional gas reservoirs. This is due to the desorption characteristic of the Devonian Shale. Gas transport through the shale exhibits two types of regimes: (1) flow through natural fractures or highly permeably regions, and (2) diffusion through very tight regions which are the matrices of the shale. Transport through the matrices of the shale is a diffusional type of flow, i.e. a significant portion of gas flows into fractures and permeable pores of the shale from the matrices of the shale through the process of physically adsorbed gas. Diffusion of the gas through the matrix of the shale is a much slower process than flow in the fractures.

Mathematical models are presented for the purpose of investigating the transient behavior of Devonian Shale gas wells and the reservoir producing at a constant rate or at a constant pressure.

The results show that the presence of the diffusion-desorption process in the matrix of the shale will considerably affect the pressure and the production history of such systems. Further, the diffusion constant, size of the shale matrix elements, and the desorption isotherm of the system will have a marked affect on the production and pressure history.

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1. PHYSICAL AND MATHEMATICAL BASIS FOR DEVONIAN SHALE RESERVOIR MODELING

From a practical viewpoint, justification of the techniques used for Devonian Shale gas reservoir modeling requires a convincing demonstration of reliable predictive capability. Unfortunately, a review of existing Devonian Shale models reveals that such a demonstration of predictive capability does not yet exist. From a more theoretical viewpoint, the basis for modeling gas flow in Devonian Shale reservoirs must be based on what appears to be a a very complex transport phenomena. It is believed that physical desorption is a contributing factor to gas production. Thus, in attempting to assess the present state of Devonian Shale gas reservoir modeling capability, the many assumptions and approximations that are involved in developing the flow equations for the shale must be understood. These relationships must be reduced to a form which is sufficiently manageable for Devonian Shale modeling applications.

The flow equations for Devonian Shale gas are complex and not amenable to exact solution except under special conditions. Analytic solutions typically require very simple and idealized reservoir models and flow conditions (initial and boundary conditions and source/sink terms). Such results cannot be expected to model accurately all of the important features of real producing geophysical systems. Nonetheless, analytic solutions have received a considerable amount of attention, and they may contribute in several ways to the goal of predictive reservoir modeling.

At the most fundamental level, analytic solutions to idealized gas flow problems help to provide a better understanding of the basic flow phenomena. They may also be used to explore, in a very general and qualitative manner, the sensitivity of modeling predictions to a variety of different effects.

A much more immediate connection between analytic solution techniques and the modeling of real systems is the fact that many existing gas reservoir models actually involve a combination of both analytical and numerical methods. Typically, the detailed conceptual model of a real geologic system will contain elements which are thought to have a secondary, but significant effect on reservoir behavior. In order to keep the numerical analysis within manageable bounds, these elements might be approximated in a manner which allows an analytic solution. This solution is then combined interactively with numerical treatments of the other system elements within the overall numerical approach. Unfortunately, this type of approach may sometimes be motivated more by the need for a tractable model than by the reliability of the assumptions involved.

Because they do offer exact solutions to well-defined problems, analytic methods also provide very useful benchmarks for model testing and refining and for revealing potential problem areas with the more cumbersome (but hopefully much more powerful) numerical solution techniques. Certainly, the accuracy or reliability of any large-scale computer code should never be assumed without extensively testing the ability of that code to reproduce the results of a variety of exact analytic solutions. It must also be recognized that such comparisions still do not assure that the numerical routine will continue to be accurate and reliable when applied to more realistic and complex problems.

Another interesting and sometimes useful class of analytic solutions to flow equations arises in connection with the interpretation of well test flow data which are taken in order to learn about the local in situ reservoir properties. Such well tests are extremely important as sources of the data needed to define reservoir model parameters. The analytic solutions used in well test analysis usually apply only to rather specialized (often very inconvenient) test conditions such as a well shut down after flowing at a constant rate (pressure buildup), or the opening of a shut-in well to a constant flow rate (pressure drawdown). The available analytic solutions also require highly idealized models of the reservoir properties near the well of interest. In practice, the required assumptions are never entirely fulfilled, and one is forced to choose between several qualitatively different "type curves," one of which must then be used to perform semi-quantitative interpretation of the data to obtain an estimate of local reservoir parameters. Despite the many difficulties, analytic solutions to well test problems have apparently been of significant value over the years in the petroleum industry. With the development of improved and more readily available numerical flow models, the use of analytic solutions is now decreasing in favor of the more general-purpose computer methods. This is currently an active research and development area in the petroleum industry. Similar trends might be expected in connection with well test analyses.

In summary, much of our general, qualitative understanding of gas flow equations is based upon exact analytic solutions to special cases. In addition, these exact solutions are directly relevant to Devonian Shale gas reservoir modeling as a basis for testing and refining more powerful numerical solution methods. The importance of this role should not be underestimated, as the possibility of otherwise undetectable deficiencies in the numerical calculations is very real. Although analytic solution methods have also been extensively employed (and still are to some extent) for special problems such as well-test analysis, current emphasis is being placed on the development of more general-purpose numerical schemes. It should not be expected, however, that analytic solution methods alone will be capable of predictively modeling the detailed behavior of real Devonian Shale gas reservoirs.

2. MATHEMATICAL MODEL

The gas transport through the shale appears to exhibit two types of regimes: (1) flow through natural fractures or highly permeably regions, and (2) diffusion through very tight regions which are the matrices of the shale (i.e., a significant portion of gas flows into fractures and highly permeable pores of the shale from the matrices of the shale due to desorption of physically adsorbed gas). Diffusion of the gas through the matrices of the shale is a much slower process than flow in the fractures. Depending on the size of the matrix and its diffusion constant, diffusion may not be the controlling factor in production. This type of flow model was originally developed by Price and Ancell[1] for the investigation of the feasibility of obtaining methane from coal and was subsequently adapted for use in the Devonian Shale.[2]

The new approach by Price and Ancell[1] leads to a diffusivity equation with a source term:

$$\nabla \{\rho \frac{k}{\mu} \nabla p\} + q_v = \frac{\partial}{\partial t} (\phi\rho) \qquad (2.1)$$

Where: ρ = density, g/cc

k = permeability, darcy

μ = viscosity, cp

ϕ = porostiy

p = pressure, atm

t = time, sec

q_v = rate of desorption, g/cc/sec

This equation is a general form of the gas motion equation relating density, pressure, time and space in which gravitational forces are neglected.

The source term, q_v, in this equation includes the effect of mass transfer from the matrix of the shale into the fracture voids due to desorption of gas from the matrix of the shale.

This euqation is solved analytically with a few simplifying assumptions for an infinite shale reservoir producing at a constant rate or a constant pressure. Nevertheless, the analytical solutions are important for investigating the behavior of gas motion through Devonian Shales and demonstrating the effect of the desorption on the production performance.

In this report the vigorous mathematical treatment of Equation 2.1 is omitted, and only the results from the solution will be presented.

3. RESULTS AND DISCUSSIONS

For the purpose of computing reservoir histories, the following values of reservoir parameters were chosen for a typical Devonian Shale gas reservoir.

They are:

Initial Reservoir pressure = 500 psia

Flowing wellbore pressure = 16 psia

Formation thickness = 622 feet

Permeability = 0.1 md

Porosity = 2%

Reservoir Temperature = 110^{o}F

Wellbore radius = 0.292 ft.

Specific gas gravity = 0.604

The above reservoir parameters, except permeability, can be measured with a satisfactory level of confidence for the Devonian Shale. Since the Devonian Shale may be fractured and is a very tight reservoir, a special measurement technique is needed for permeability. Devonian Shale reservoirs require other measurements. Size of the matrix, radius of the sphere which represents the matrix, the diffusion coefficient, and the desorption isotherm are very important parameters for the calculation of Devonian Shale performance. The measurement of some of these parameters is expensive and difficult. The effect of these parameters on Devonian Shale reservoir behavior will be discussed in the following paragraphs.

Since, in calculations a numerical Laplace transform technique has been used, a comparison will be made between the constant pressure solution of a conventional gas reservoir and our solution for the Devonian Shale reservoir. Slider[11] presented an equation for cumulative production in an infinite gas reservoir producing at a constant pressure as:

$$Q_{MSCF} = \frac{0.111\ \phi h\ r_w\ c\ (p_i^2 - p_{wf}^2)\ Q_{tD}}{z_{avg}\ T_f} \qquad (3.1)$$

Q_{tD} can be read from the van Everdingen - Hurst table for a given t_D.

Where:

ϕ = porosity

h = formation thickness, ft.

r_w = wellbore radius, ft.

c = compressibility of gas, psia^{-1}

p_i = initial reservoir pressure, psia

p_{wf} = flowing wellbore pressure psia

Q_{tD} = dimensionless cumulative production

z_{avg} = average compressibility factor, calculated at $(p_i + p_{wf})/2$ and T_f

T_f = formation temperature, R

$$t_D = \frac{6.33\ 10^{-3}\ k\ t}{\phi\mu\ c\ r_w^2}$$

t = time, day

k = permeability, md

μ = viscosity, cp

The cumulative production values calculated from the solution of Equation 2.1 for a small value of the diffusion coefficient, $D = 10^{-22}$, should be the same as cumulative production calculated from Equation 3.1. It is assumed that no gas flows from the shale matrix because of the very small diffusion coefficient. Table 1 presents the cumulative production values calculated from the solution of Equation 2.1 and the difference of these two cumulative production values. As seen from this table, the difference is less than 1.2%, even though Equation 3.1 is not a exact solution of the unsteady-state gas flow. According to Slider,[11] Equation 3.1 is an excellent approximation of unsteady-state gas flow in a porous medium.

In addition to the above properties of the reservoir, the following parameters were also chosen for a typical Devonian gas reservoir.[2,4,6]

Initial gas concentration in the matrix, ci = 4 cc STP/g shale

Diffusion coefficient, $D = 1 \times 10^{-10}$, cm^2/sec

Isotherm slope, $K_s = 1.000 \times 10^{-9}$, moles/cc shale/atm^2 cp

Radius of the sphere, a = 10 cm

Figure 3 shown the cumulative gas production calculated from the solution of Equations 2.1 and 3.1 as a function of time for the above properties of the shale gas reservoir. The upper curve in Figure 3 represents the cumulative production values for a dual porosity (the diffusion-desorption process occurs in the shale matrix) gas reservoir while the lower curve represents the cumulative production for a single porosity (conventional) gas reservoir. During the early life of the reservoir, the effect of desorption on the production is small. But after the initial flow period, the effects of gas flow from the matrix of the shale due to the diffusion-desorption process are pronounced.

The upper curve in Figure 3 represents the base case for the sensitivity analysis of each Devonian Shale parameter. In addition to conventional gas reservoir parameters, the diffusion coefficient, isotherm values, initial concentration of gas in the shale matrix, and shape and size of the shale matrix should be known for a complete reservoir analysis. The effect of these parameters on the cumulative production for a Devonian Shale gas reservoir which produces at a constant pressure will be discussed. Figure 4 is a graph of the cumulative production as a function of time. For comparison, the middle curve represents the base case ($D = 0.1 \times 10^{-9}$ cm^2/sec). All of the reservoir parameters except the diffusion coefficient are assumed to be the same in all three curves. The most noticeable characteristic of these curves is that they depend strongly on the diffusion coefficient of the system.

Figure 5 shows the effect of the varying K_s (the slope of the isotherm curve) while Figure 6 shown the effect of varying the initial gas concentration, c_i, in the shale matrix. These curves show that K_s and c_i strongly affect the production history.

Again, in Figure 7 as in Figures 4, 5, and 6, the cumulative production will depend strongly on the size of the sphere.

Figure 8 shown the effect of the shape of the shale matrix on production. It is assumed that the radius of the cylindrical membrane is the same as the radius of the sphere (10 cm) for both curves. The cumulative production would be 14% less at the end of thirty years if we assume a cylindrical source rather than a spherical source.

4. CONCLUSIONS

Devonian Shale Models presented here were simplified and idealized compared to real reservoirs. However, these analytical solutions provide a better understanding of basic flow phenomena. They can also be used for the calculations of recoverable reserves and the future prediction of the reservoir behavior. The results will not be exact, but for the first exploration of Devonian Shale reservoirs, they will be sufficient. For a complete analysis, more quantitative, sophiscated reservoir flow models will be required. These solutions can also provide useful benchmarks for model testing and refining.

This study yielded the following conclusions about Devonian Shale gas reservoirs.

1. The dual porosity model indicates a higher cumulative production than the single porosity (conventional gas reservoir) model.
2. The diffusion coefficient, adsorption isotherm, initial concentration of gas in the matrix, and size of the matrix strongly affect production history.
3. The shape of the matrix has somewhat less effect on production history than other variables (c_i, K_s, a, D).

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NOMENCLATURE

a	Radius of shale particle, cm
c	Concentration, g mole/cm^3
D	Diffusivity, cm^2/sec
k	Permeability, Darcy
M	Molecular weight, gr-mole
N_v	Rate of gas desorption per unit matrix volume, g/cm^2-sec
p	Pressure, atm
pwf	Flowing well pressure, psia
q_{sc}	Production rate, cm^3/sec
q_v	Rate of desorption, gr/cm^3/sec
r	Radius
r_w	Well radius
R	Universal gas constant, atm-cm^3/gr mole K
T	Temperature, ^oK
t	Time, sec
ϕ	Porosity
ρ	Density of gas, gr/cm^3
Ψ	Pseudo-pressure, atm^2/cp
μ	Viscosity of gas, cp
z	Compressibility factor

TABLE I

COMPARISON BETWEEN ANALYTICAL SOLUTIONS

TIME DAYS	CUMULATIVE PRODUCTION MMSCF. From Eq. 3.1	CUMULATIVE PRODUCTION MMSCF, From Sol. of Eq. 2.1	DIFFERENCE %
0.1	0.04535	0.045798	0.99
1.0	0.34572	0.34966	1.14
10.0	2.7982	2.82373	0.91
100.0	23.4281	23.67043	1.03
1000.0	201.831	203.71472	0.93
10000.0	1772.741	1787.7916	0.85

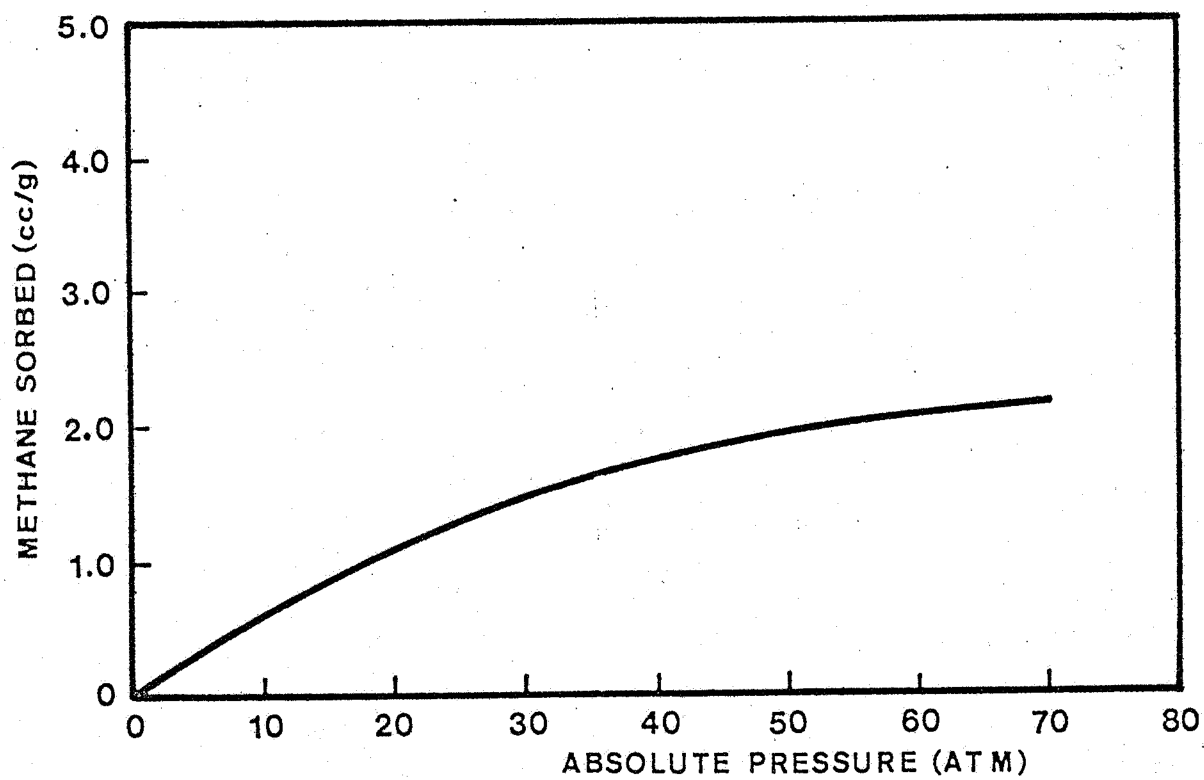

Figure 1 - Methane Sorption Isotherm

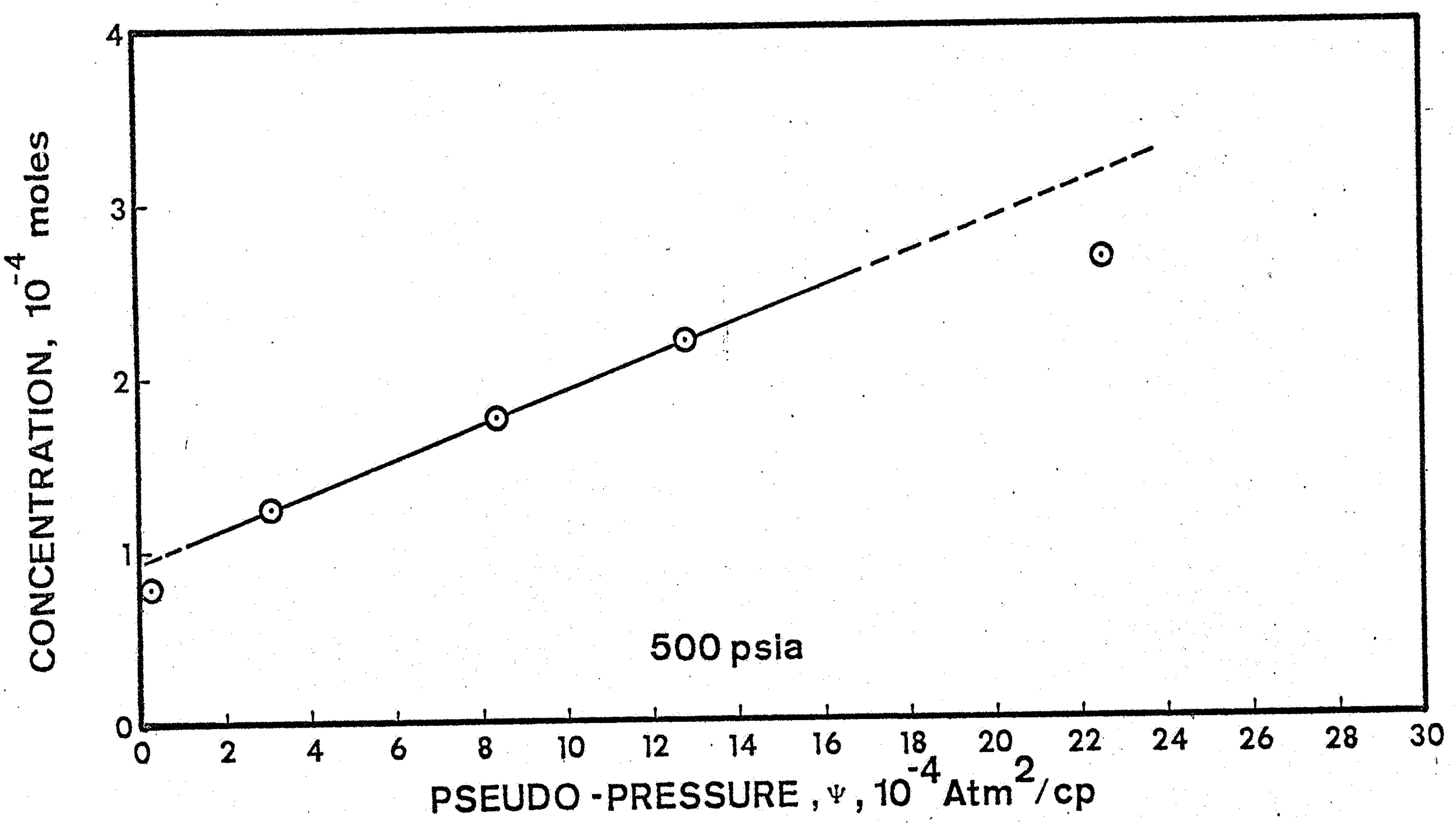

FIGURE 2. Methane Sorption Isotherm As A Function of Pseudo-Pressure

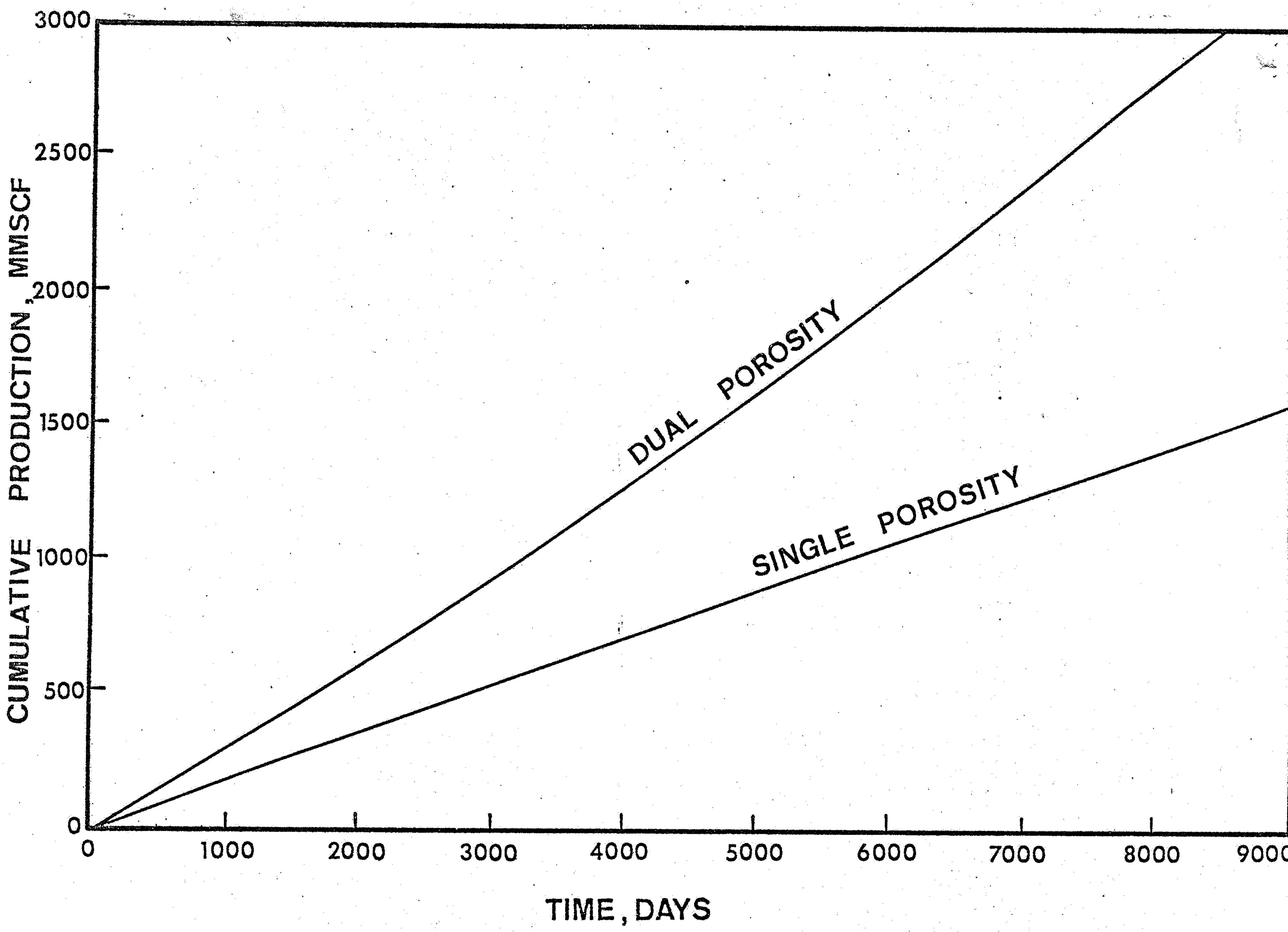

FIGURE 3. Cumulative Production Versus Time For A Single And Dual Porosity Gas Reservoir Producing At A Constant Pressure.

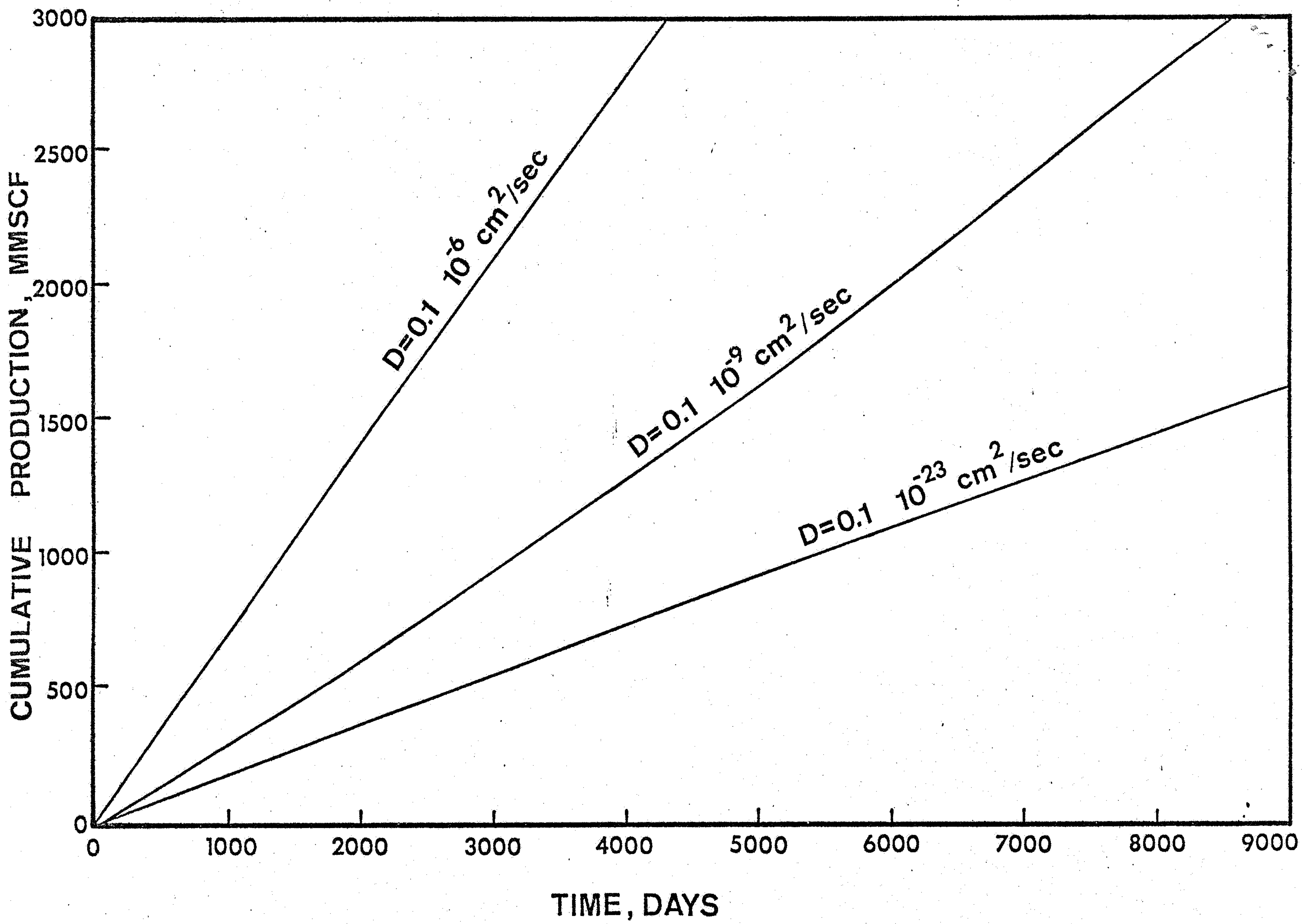

FIGURE 4. Cumulative Production Versus Time For Different Values of Diffusion Constant.

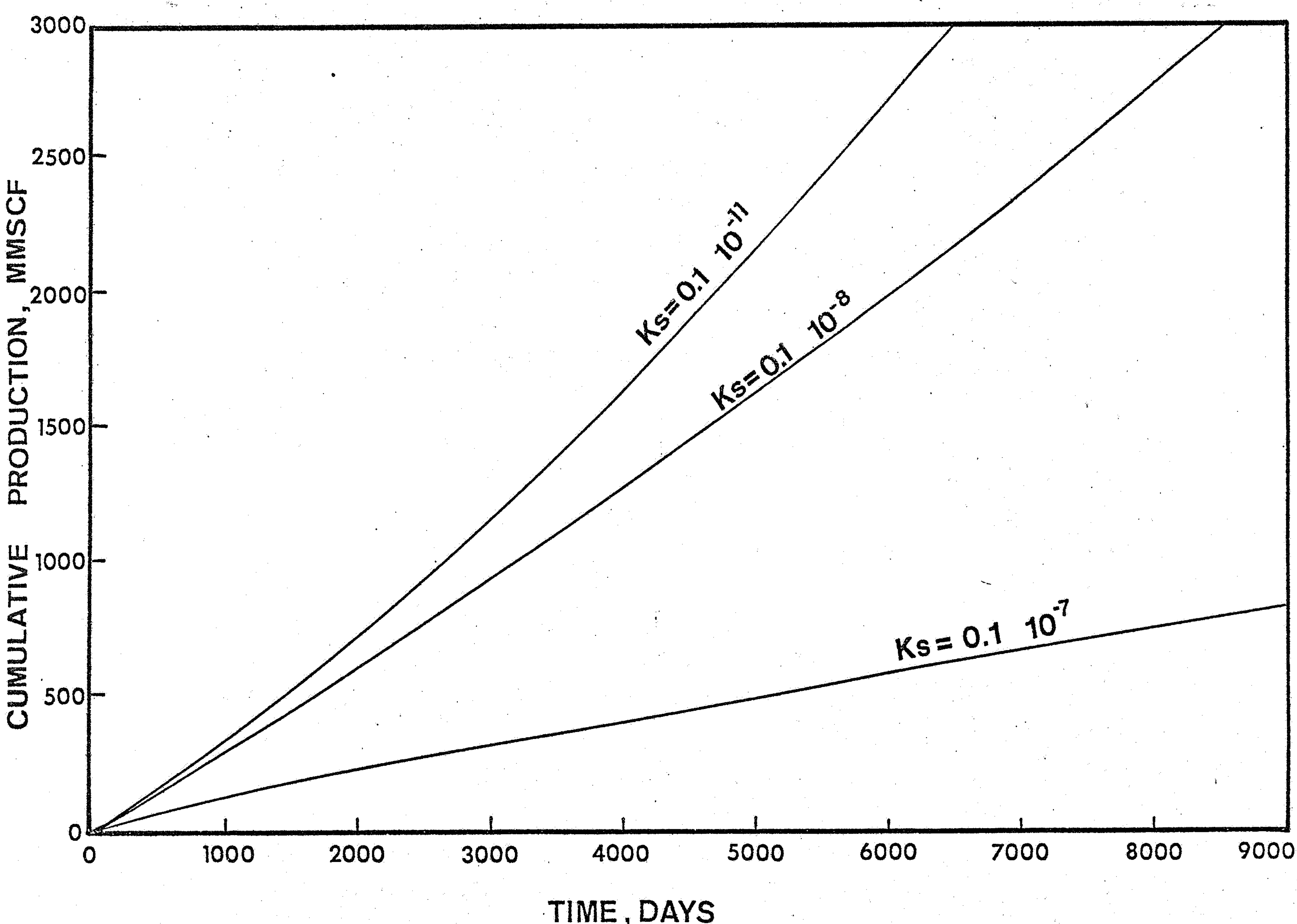

FIGURE 5. Cumulative Production Versus Time For Different Values of K_s.

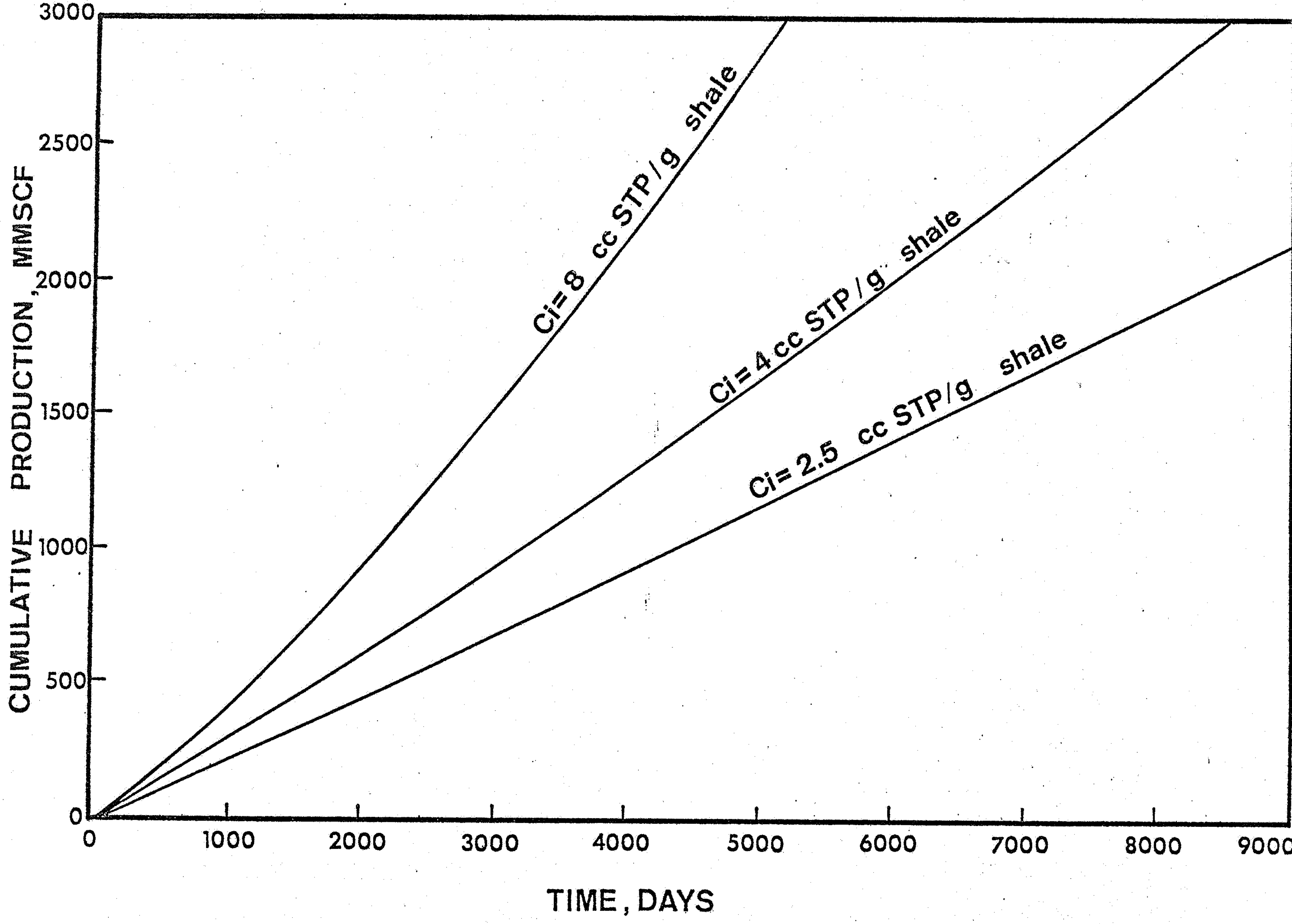

FIGURE 6. Cumulative Production Versus Time for Different Values Of The Initial Gas Concentration.

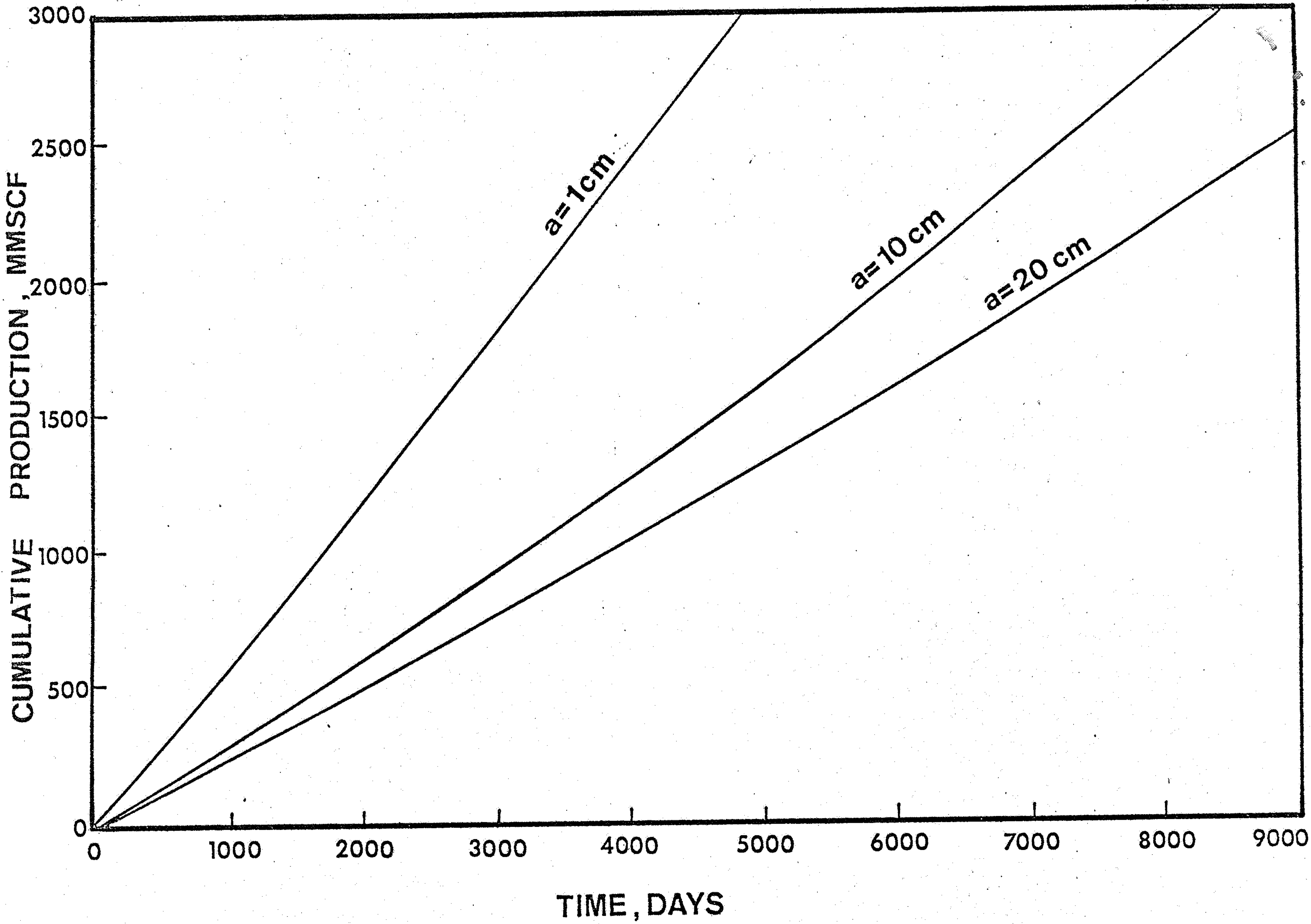

FIGURE 7. Cumulative Production Versus Time For Different Values Of Radius Of Sphere.

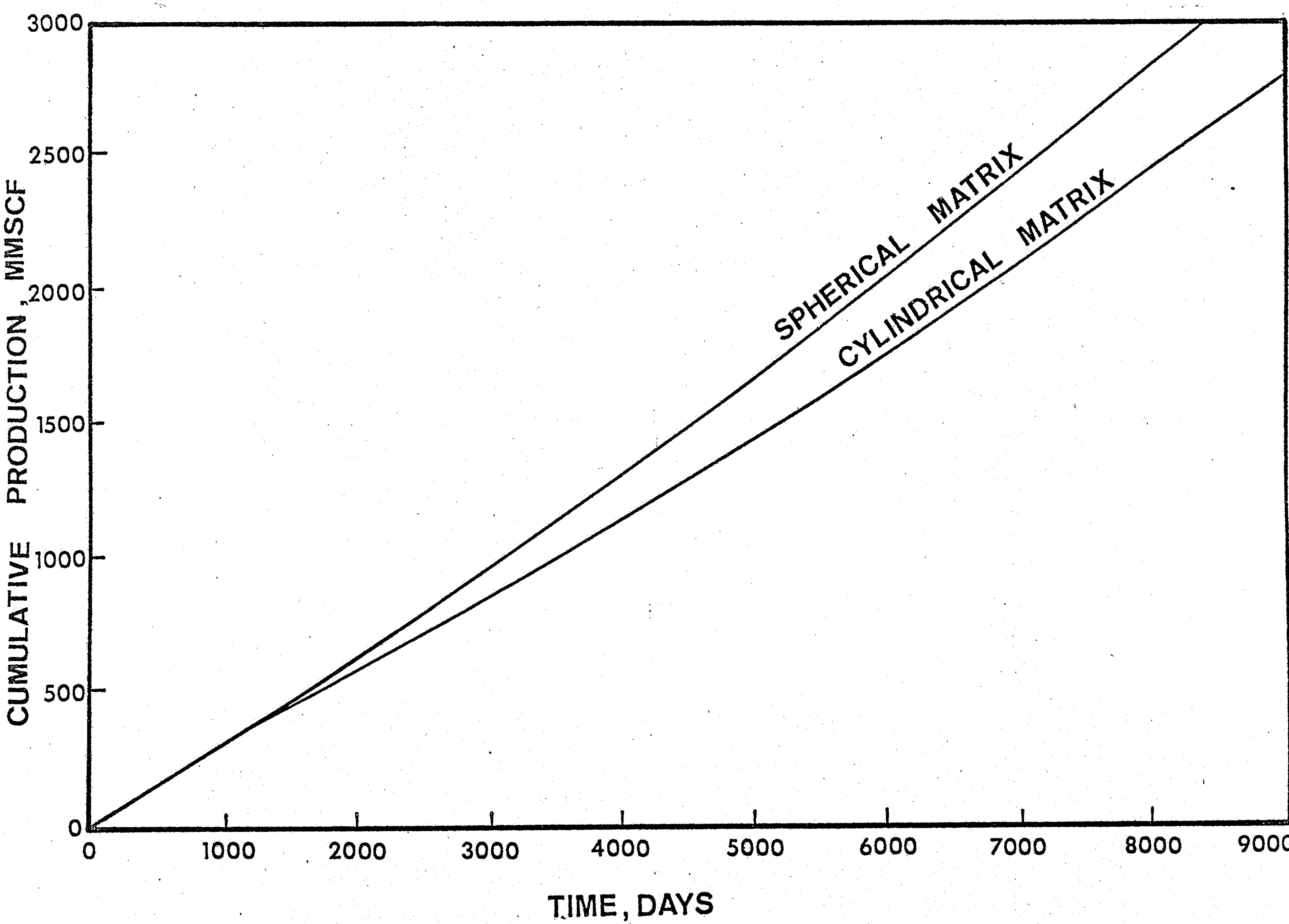

FIGURE 8. The Effect Of The Shape Of The Matrix On Production.