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PREDICTION OF FRACTURE EXTENT BY
SIMULATION OF GAS WELL PRESSURE
AND PRODUCTION BEHAVIOR

By

W. K. Sawyer
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Date Published--October 1976

Morgantown Energy Research Center
Morgantown, West Virginia

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W. K. Sawyer,¹ J. C. Mercer,² and K-H. Frohne³

ABSTRACT

This report describes the use of transient gas reservoir simulation techniques for the purpose of evaluating fracture geometry in two different types of formations. Extensive efforts were made to determine fracture length and reservoir permeability by history matching well pressure tests for one well in a sandstone formation and one well in the Devonian Shale formation. In both cases all available information was used regarding fluid and formation properties. For each of these wells field data was available which consisted of both well test and production performance.

In the sandstone well it was also desired to determine why the wells initial production during drilling operations (approximately 7 MMSCFD) decreased after an extensive hydraulic fracturing operation. Different combinations of grid-block sizes and different fracture representations were used in an effort to characterize the permeability distribution and fracture extent. Two different combinations of fracture lengths and formation permeability distributions were found to reasonably match the well pressure history. The high initial productivity was explained by the fact that a high permeability zone (5-10 md) had to be used in an unfractured zone to match the well test performance.

In the Devonian Shale well fracture length, formation permeability, and porosity were varied extensively until good agreement was obtained between measured and simulated well test and production data. With the effective height constant only one combination of these parameters resulted in good agreement. Fracture length and permeability values were consistent with the volume of sand injected and core analysis results. However, the effective gas porosity had to be reduced to a very small value to match both the well test and production data. This suggested the possibility of either a low porosity natural fracture system or a very high liquid saturation.

¹Research Mathematician

²Mathematician

³Petroleum Engineer

It is concluded that numerical simulation can be a valuable tool in evaluating the critical parameters in low permeability stimulated gas reservoirs. Moreover, simulation can be used whenever well test data is not amenable to conventional or type-curve analysis. Finally, by observing if the parameters determined from the well test match also give good agreement when simulating the production history, an extraordinary level of confidence can be placed in the results.

INTRODUCTION

With an ever decreasing supply of natural gas reserves it is becoming more and more important to be able to develop and evaluate low permeability gas reservoirs. Over the past 15 years hydraulic fracturing has become and remained the most widely used means of stimulating marginal oil and gas formations. Consequently, the evaluation of fractured wells producing from tight gas reservoirs is currently of tremendous importance to the overall energy picture.

Numerous papers have appeared in the literature concerning the effects of vertical fractures on well productivity. Prats, Hazebroek, and Strichler (12)⁴, McGuire and Sikora (9), Scott (18) and Russell and Truitt (14) were among the first to do a detailed analysis of the effect of vertical fractures on reservoir behavior. Later Van Everdingen and Meyer (2) and Morse and Van Gonten (11) presented excellent papers on the same subject but for different well or reservoir conditions. More recently, Raghavan, Cady, and Ramey (13) and Gringarten, Ramey, and Raghavan (4-5) have provided new analytical solutions which are very useful for short-time or type-curve analysis. The results of all these papers, however, apply rigorously only to a slightly compressible fluid of constant compressibility and viscosity.

Two of the earliest papers on gas well test analysis in fractured wells are those of Wattenbarger and Ramey (19) and Millheim and Cichowich (10). More recently several papers have appeared regarding numerical simulation of gas flow in fractured reservoirs (e.g. Lemon, Patel, and Dempsey (8), Holditch and Morse (6-7), Crafton (1), and Sawyer and Locke (17)).

In this study appropriate plotting routines were combined with a general purpose gas reservoir simulator (17) to develop a technique for determining induced fracture length and effective formation gas permeability from well test and/or production data. The method is basically a curve matching procedure whereby simulated and actual bottom-hole pressures

⁴Underlined numbers in parentheses refer to items in the list of references at the end of this report.

are compared. Virtually any type of rate or pressure data may be used including periods in which neither is controlled. Thus the method does not depend on a constant rate or the attainment of pseudo-steady state as do some analytical methods.

Using all available log and/or core analysis data the known formation and fluid properties are evaluated and inserted as fixed input data into the simulator. Ranges are decided upon for the unknowns (e.g. frac length and permeability) and a matrix is set up for varying all unknowns over a finite number of values within the range of each variable. Each well test is then simulated for the entire matrix with plots produced showing well test data and simulated data on the same graph. In this manner unlikely combinations can quickly be eliminated by visually comparing the shapes of the simulated and actual well data curves. A unique set of values can usually be determined rather quickly if two or more types of performance data are available.

Two examples of this technique are presented in which both well test and production data were available. In both cases fracture length and formation permeability were determined which reasonably explained the well performance.

ACKNOWLEDGMENTS

The authors wish to express their sincere thanks and appreciation to David Locke and Hal Koerner, petroleum engineers at the Morgantown Energy Research Center, for their helpful advice and stimulating comments during the course of this work.

DESCRIPTION OF WELL A

Well A is a wildcat well in Venango County, Pa., drilled in the Queenston Sandstone to a total depth of approximately 6720 feet. It was completed and hydraulically fractured in early 1973 and put on production in November of that same year. While drilling, a substantial volume (7 MMSCFD estimated) of natural production was encountered. However, after an extensive fracturing operation production was significantly less than the above estimated value. Table 1 contains a summary of the frac treatment and figure 1 shows gamma ray and tracer logs following the frac treatment.

As a result of the low productivity a testing program was implemented in an effort to characterize the productivity and reserves of the well. Modified isochronal, drawdown, and buildup tests were conducted. The modified isochronal test indicated an absolute open flow potential of only 5

TABLE 1. - Fracture treatment on well A

<u>Stage</u>	<u>Depth</u>	<u>Perforations</u>	<u>Acid (gal)</u>	<u>Water (gal)</u>	<u>Sand (lb 20/40)</u>	<u>Rate (bbl/min)</u>	<u>Pressure (psi)</u>
1	6586-93	30	600	2570		2	6000
2	6544-51	8					
	6530-40	11	400	11,820	11,000	18	5225
3	6512-13	2					
	6505-07	2					
	6494-98	5					
	6484-89	6	400	14,200	15,000	19	4780

Table 2. - Producing intervals, well A

<u>Interval</u>	<u>Zone</u>	<u>Net Pay</u>	<u>ϕ (%)</u>	<u>S_w (%)</u>	<u>ϕ_{eff} (%)</u>
A	I	6484-6491			
	II	6494-6499	12	9.21	25.0
B	III	6505-6508			
	IV	6512-6514			
	V	6536-6540			
	VI	6545-6550	14	4.55	19.7

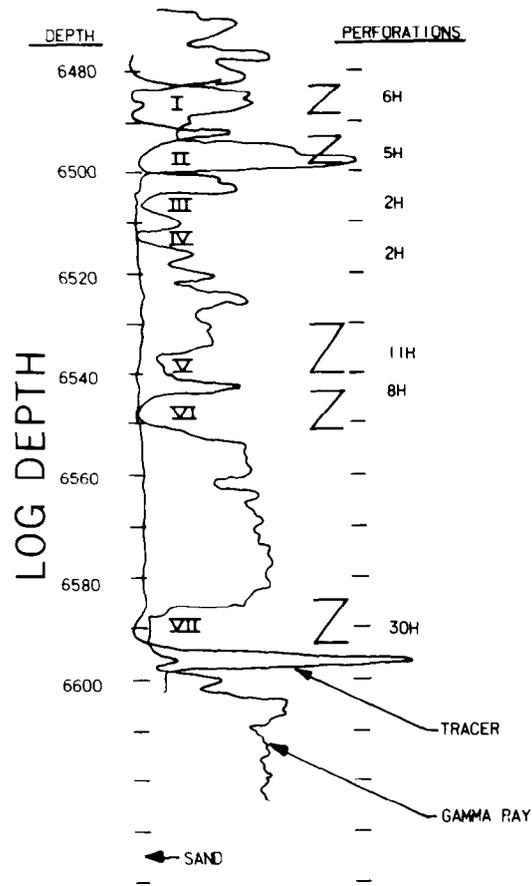


FIG. 1 - GAMMA RAY AND TRACER LOG - WELL A

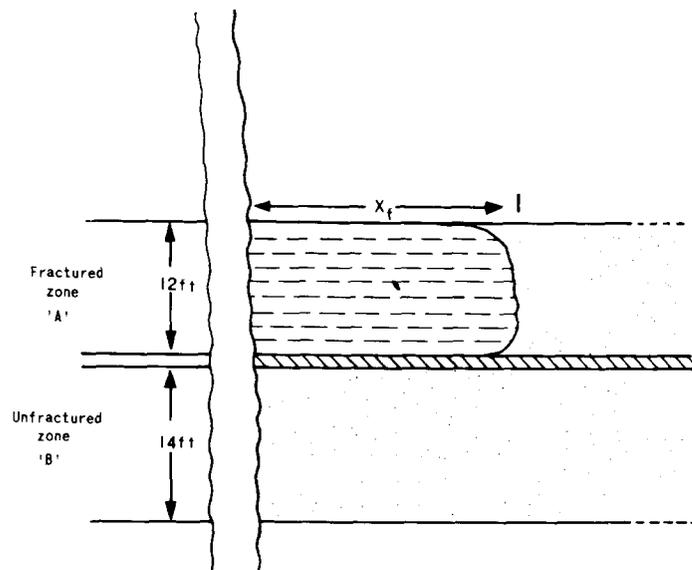


Fig. 2 - EFFECTIVE PRODUCING ZONES - WELL A

MMSCFD. The drawdown and buildup tests were difficult to analyze by conventional methods because of changing slopes but gave a permeability on the order of 1 md and a negative skin factor of about 3.9 indicating a significant improvement due to the fracture. The extrapolated shutin pressure was about 1800 psi compared to the discovery pressure of 1954 psi. The two point estimation of reserves was only 65 MMSCF.

NUMERICAL SIMULATION RESULTS--WELL A

On the basis of the conventional well test analysis it was concluded that a network of highly permeable fractures in conjunction with a very low permeability matrix was providing the production. However, the reason for the substantially lower production after the fracture could not be explained. In an effort to delineate fracture extent and explain the reason for the anomaly in production behavior the gas reservoir simulator was used to history match the well pressure behavior during the testing program.

Based on gamma ray and tracer logs it was concluded that the effective producing interval consisted of a fractured zone and an unfractured zone as depicted in figure 2. Table 2 gives a breakdown of these two intervals according to the zone numbers given in figure 1. Zone VII was not included because (1) the log analysis report indicated a high probability of oil and (2) the gamma ray log indicates a very possible shale streak where the fracture penetration is located.

Using the effective gas porosities given in table 2 and the gas analysis data in table 3 a three-dimensional model was set up to simulate gas flow in the fractured and non-fractured intervals of figure 2. Hereafter, this will be referred to as Model I. Initially reservoir boundaries were selected at sufficiently large distances so that no boundary effects would be present during the simulation of the well tests. Fracture length and formation permeability were varied to obtain the best match between simulated and actual well pressure behavior.

Figure 3 and 4 show the actual and simulated data for the modified isochronal, drawdown, and buildup tests. Figure 3 is the best fit that could be obtained to the isochronal test data by carefully and systematically studying pressure difference and varying permeabilities (both in the x and y-directions) and fracture length. A somewhat better match to the drawdown and buildup data than shown in figure 4 was obtained for slightly different permeabilities but in these cases the isochronal match was not quite as good.

TABLE 3. - Gas analysis--well A

<u>Component</u>	<u>Mole Fraction</u>
Nitrogen	0.250
Methane	0.9515
Ethane	0.0218
Propane	0.0014
Iso-Butane	0.0001
N-Butane	0.0002

TABLE 4. - Maximum production rate--well A

<u>Time (hours)</u>	<u>q (MMSCFD)</u>
.0166	13.28
.166	8.874
.250	7.599
3.0	7.269
8.0	6.789
13.0	6.597
18.0	6.469
24.0	6.355

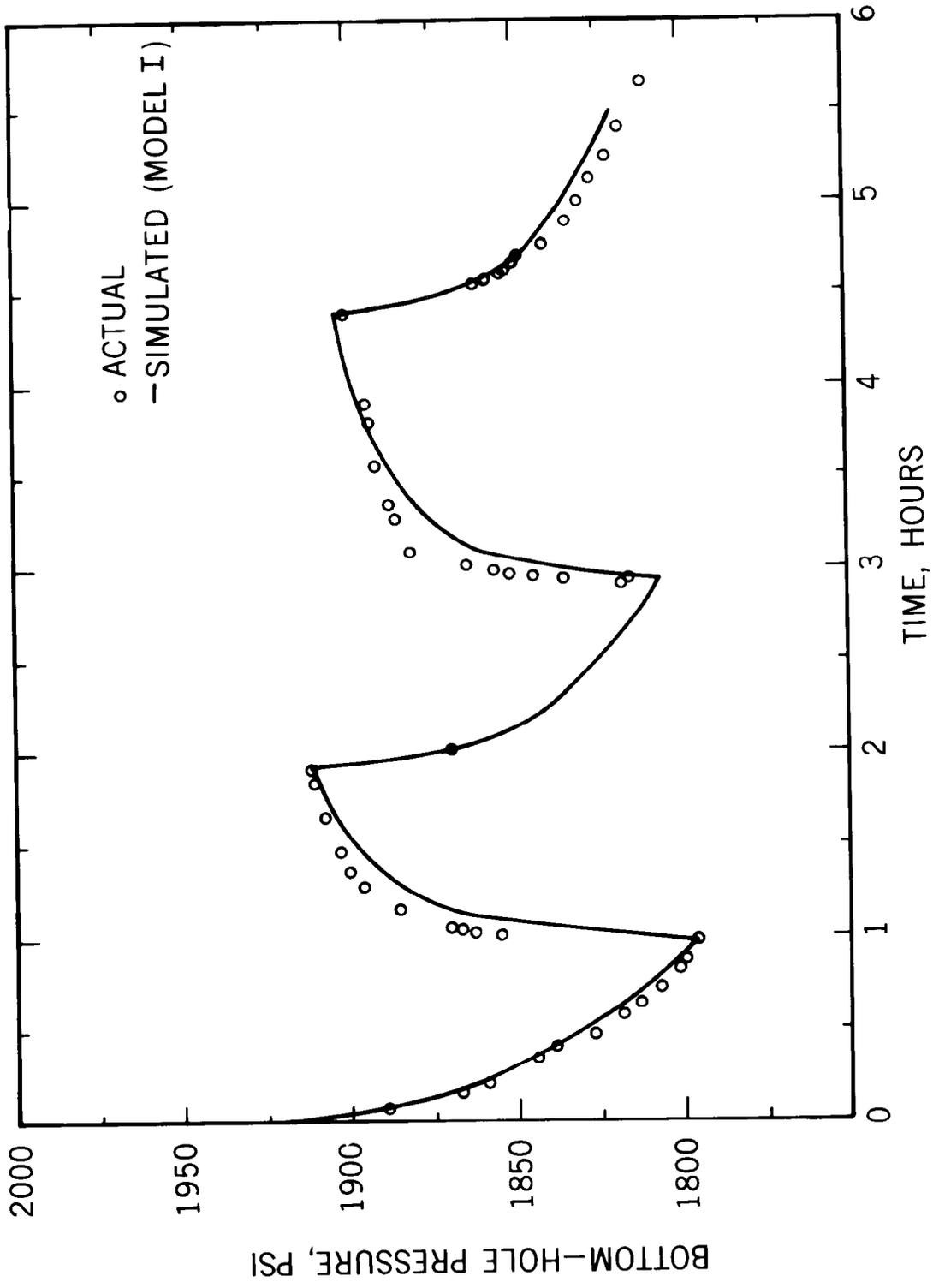


Fig. 3 - MODIFIED ISOCHRONAL TEST, WELL A

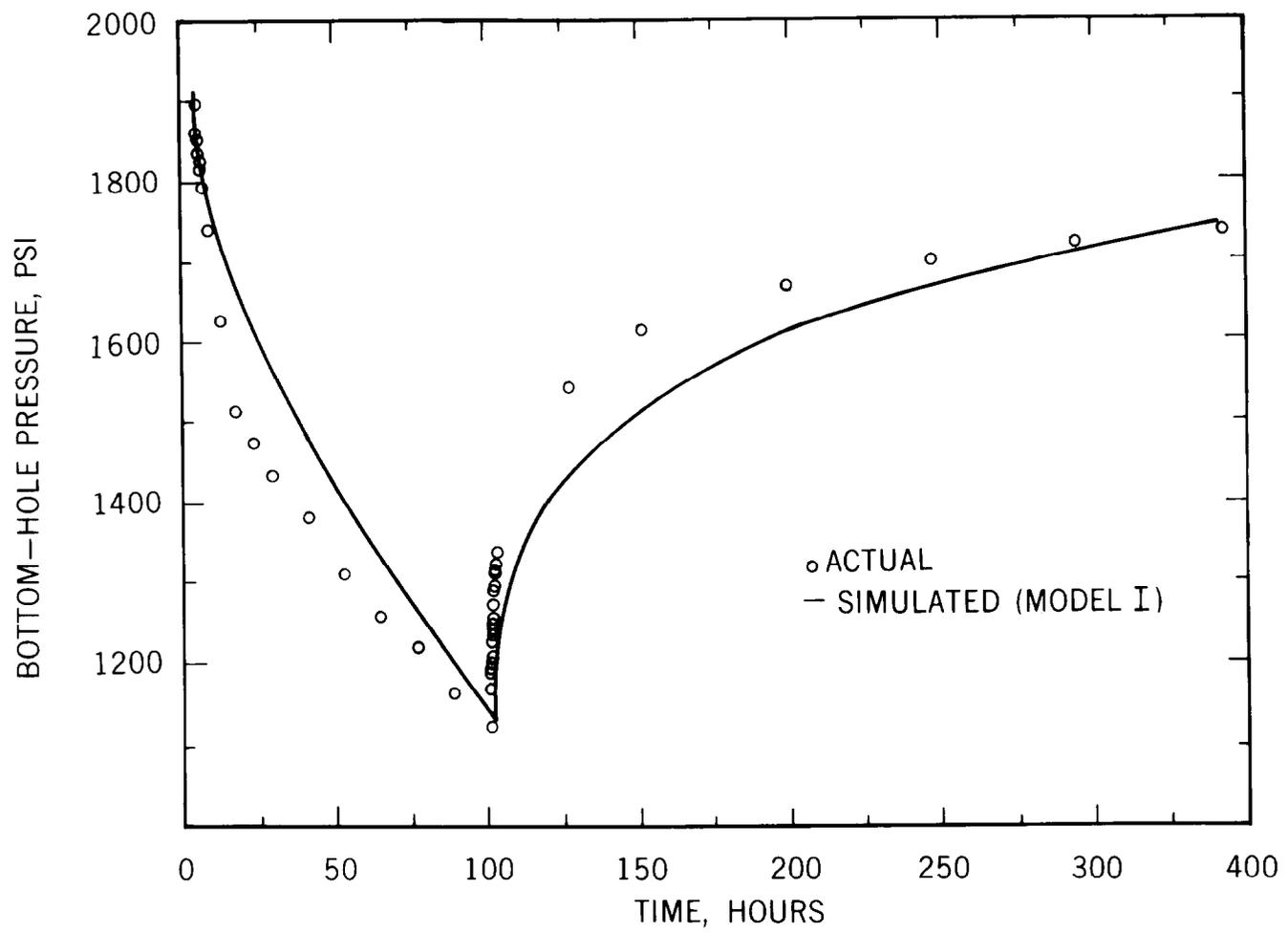


Fig. 4 - DRAWDOWN AND BUILDUP TESTS, WELL A

The initial runs with Model I were made with homogeneous permeability and the calculated pressures were drastically different from the measured values. To obtain the matches in figures 3 and 4 a zone of high permeability (5-10 md) had to be used in a 90 x 330 feet region about the wellbore in the unfractured interval. The fracture radius used was 400 feet and the permeability in the fractured interval was found to be only about 0.1 md. This is significant because the initial high productivity (estimated during drilling) can now be explained. That is, when the highly permeable (10 md) zones were being penetrated, the high initial pressure resulted in a high productivity. To quantitatively determine the magnitude a simulation run was made with a 14 feet, 10 md unfractured zone. For $P_i = 1954$ and $P_{wf} = 500$ the results are given in table 4. Thus we can indeed explain the initial estimated production of 7 MMSCFD. Moreover, due to the relatively small extent of this "highly permeable" strata and the fact that the fracture did not penetrate this zone, it is easy to understand why this well turned out to be much less profitable than expected.

Model I represented the fracture by a small grid block (0.5 feet) with a permeability of 12 to 6 darcys. This representation was shown to be valid by a separate study which is described in the appendix.

However, even with the 0.5 feet grid block Model I turned out to be quite expensive to run because of the third dimension. Thus, it was decided to also try a very simple, inexpensive model (Model II) to simulate both the well tests and the production behavior. A very coarse grid was used as shown in figure 5. Table 5 summarizes the reservoir properties used in both models and figures 6 and 7 give the actual and simulated well pressures using Model II. This "match" was considered to be reasonable; thus Model II was also believed to be an adequate representation of the reservoir. Figure 8 shows the average monthly rates during the wells first year of production and figure 9 shows the simulated and actual pressure behavior during this period. The average pressure decline is seen to be adequately described. Thus while Model I was sufficiently detailed to explain the anomaly in production behavior, Model II, although very simple, was more desirable to simulate long-term behavior of the reservoir.

DESCRIPTION OF WELL B

Well B was rotary-drilled to a depth of 986 feet and cored to 1001 feet in the Devonian Shale near Youngstown, Ohio in the spring of 1975. Log analysis indicated three zones over a 100 foot interval approximately 10 feet in thickness which were essentially silty sandstone. Both log and core analysis are given in reference 3.

TABLE 5. - Summary of reservoir properties used in simulation models for well A

	Model I	Model II
Size, ft	960 x 165 x 26	930 x 270 x 26
Effective porosity	.037, .069	.05
Permeability, md	.03 - 10.0	.005 - .6
Fracture radius, ft	400	100
Fracture block permeability, darcies	12 - 6	-
Initial gas in place, MMSCF	122	178

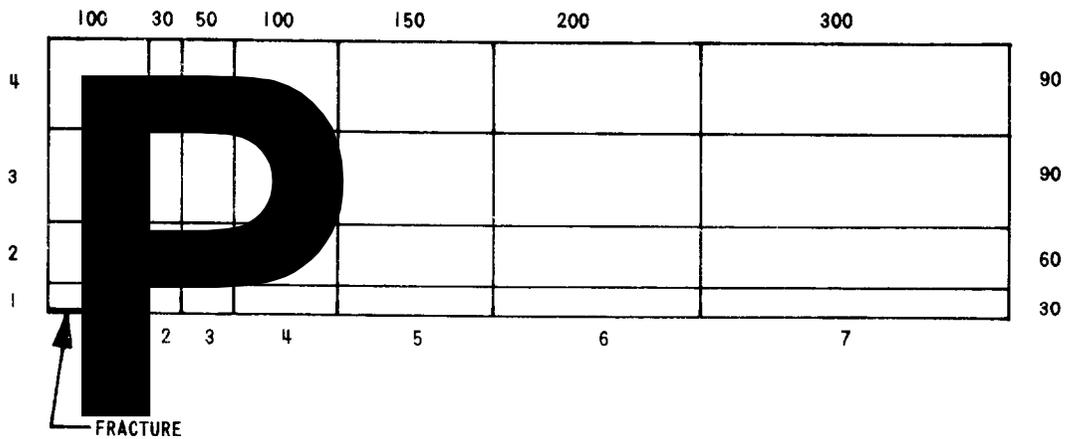


Fig. 5 - FINITE - DIFFERENCE GRID
USED FOR MODEL II - WELL A

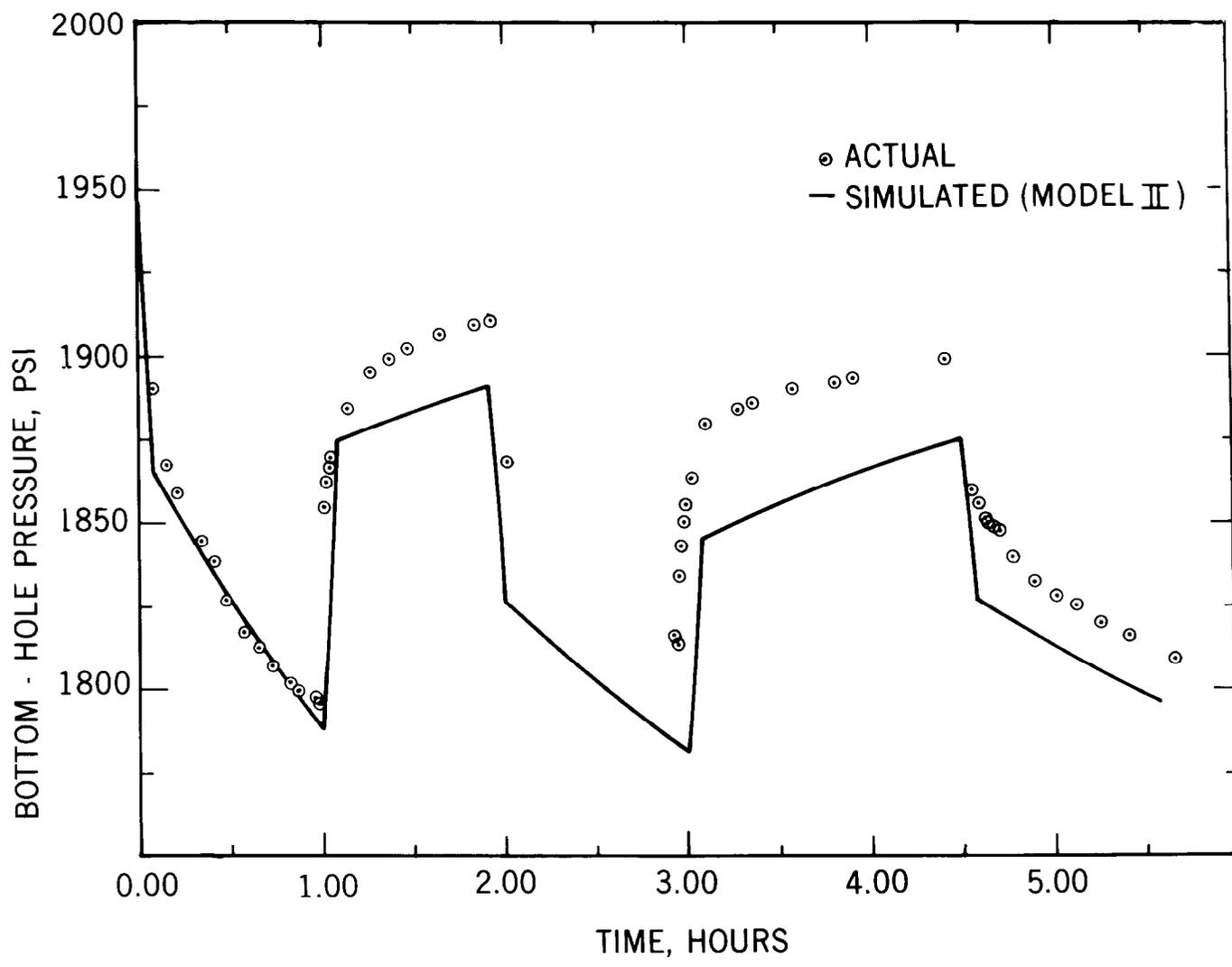


Fig. 6 - MODIFIED ISOCHRONAL TEST, WELL A

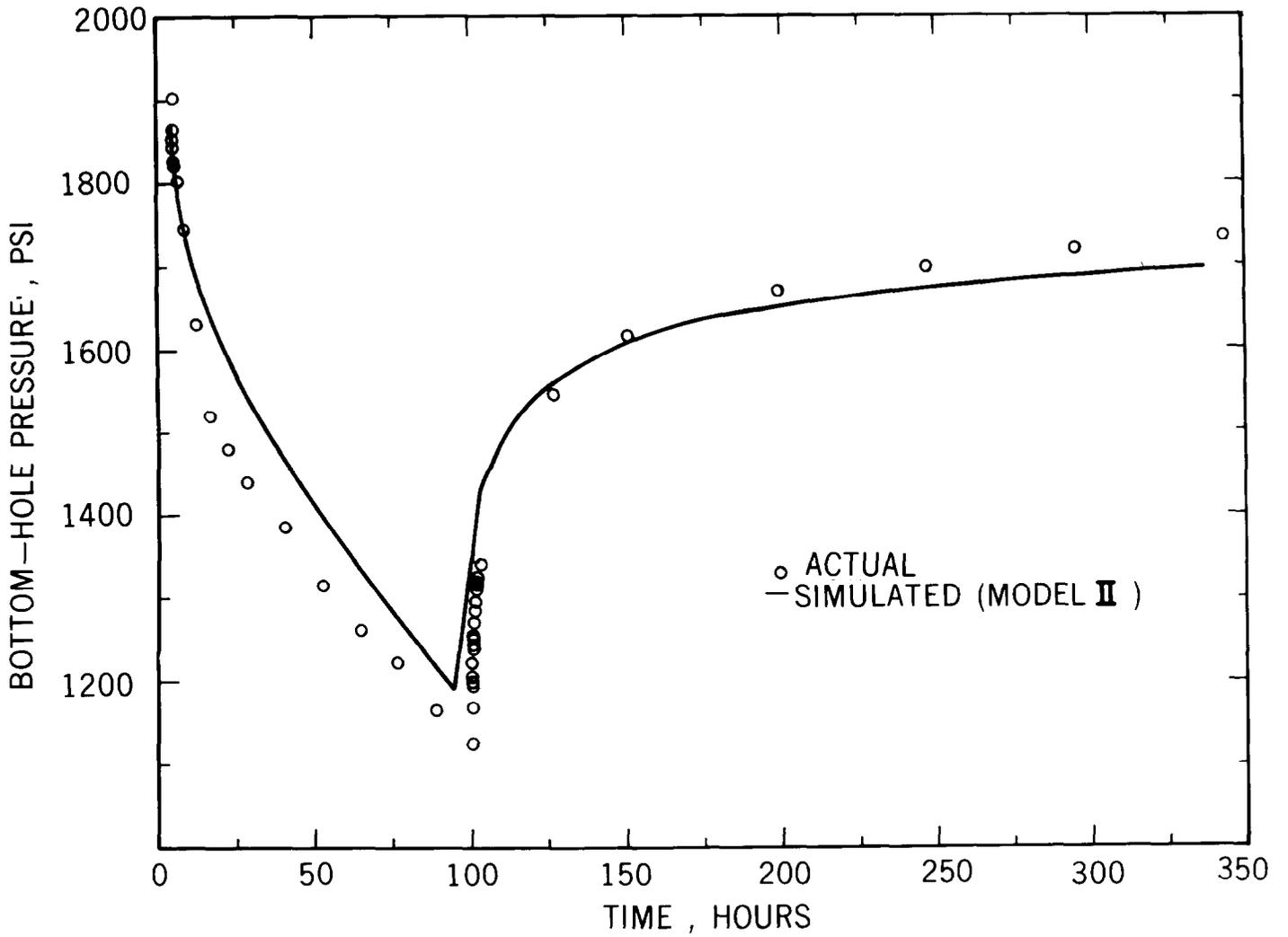


Fig. 7 - DRAWDOWN AND BUILDUP TESTS, WELL A

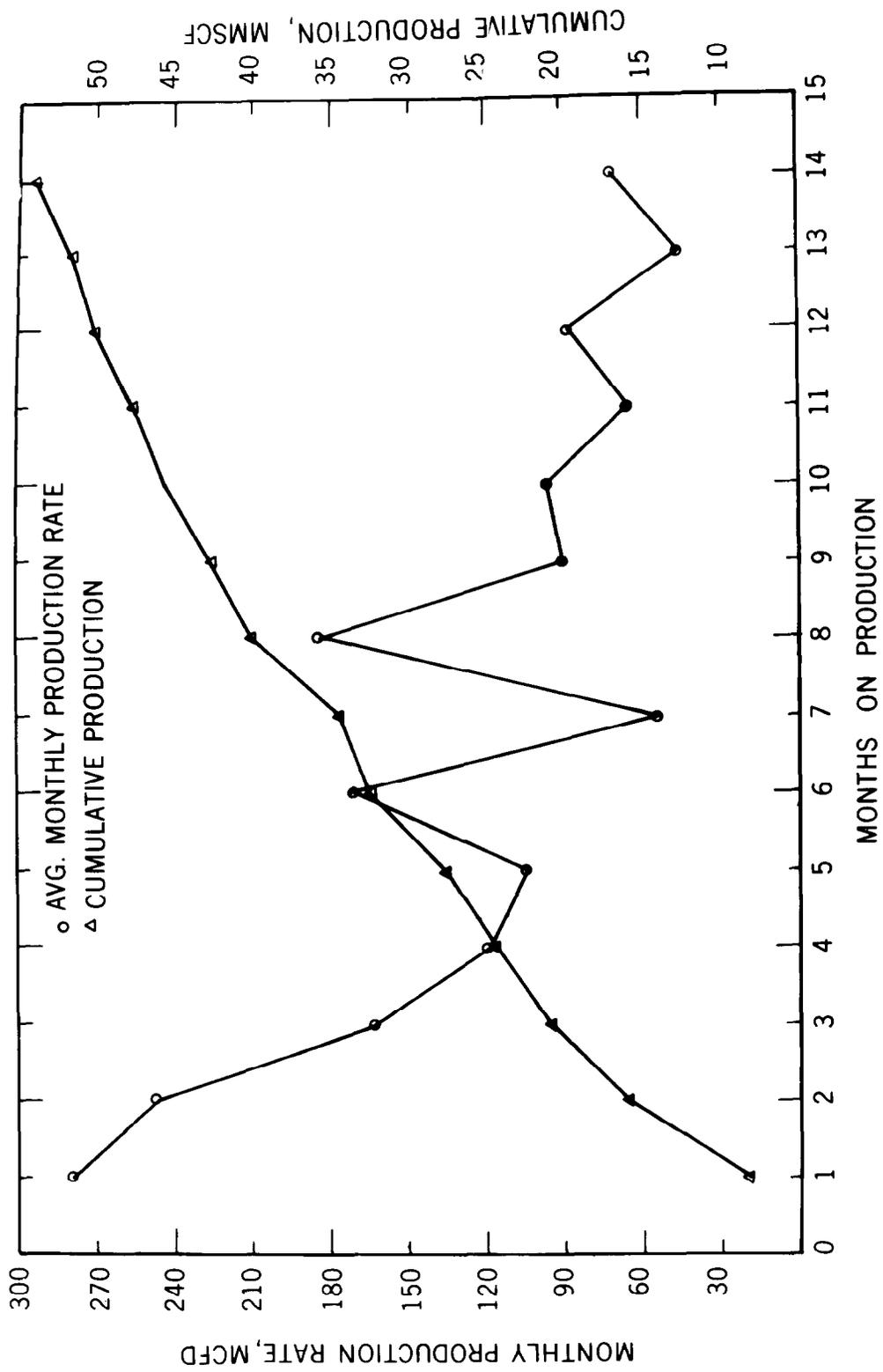


Fig. 8 - FIRST YEAR PRODUCTION DATA—WELL A

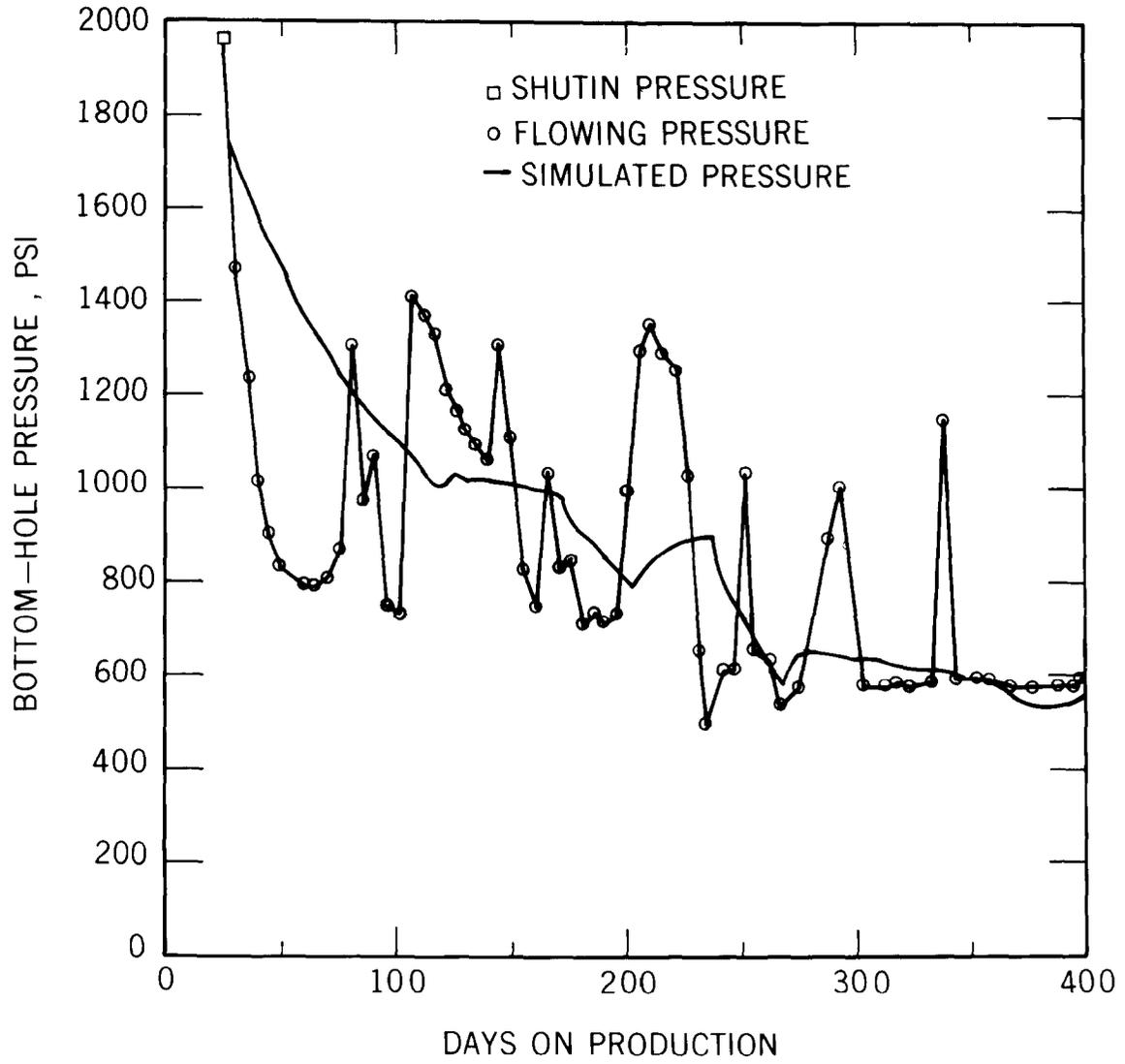


Fig. 9 - PRESSURE HISTORY DURING PRODUCTION—WELL A

A string of 4.5 inch casing was run and cemented in place, and the silty zones were simultaneously fractured using an experimental foam process (3). Subsequently a modified isochronal test was conducted followed by an open flow period of two hours to obtain a sufficient pressure range for a buildup curve. The well was shut in for about 10 days and then put on production into a gas line with a surface pressure of 90-100 psi.

NUMERICAL SIMULATION RESULTS--WELL B

As no other wells were active in the area during the well tests and the initial production period a large rectangular area was used for simulation purposes, with it being necessary to model the behavior in only one quadrant. The fracture was represented by a 0.5 foot grid block with a 5 darcy permeability (see appendix).

Core analysis indicated an average permeability of about 0.1 md and an average porosity of 6.4 percent over the gross fractured interval. However, log data before the frac job showed a total liquid saturation of about 75 percent thus reducing the effective gross interval gas porosity to 1.6 percent. Using these values it was attempted to match simulated and actual well test data. The results are summarized in figures 10 and 11. (In all subsequent figures X_F = frac radius, K = permeability, md, and PHI = porosity, percent.) It was discovered that a short fracture had to be used with a formation permeability of 0.25 md to get a reasonable match for the isochronal test (fig. 10E). However, as shown in figure 11E the agreement between the simulated and actual well pressures during the buildup was not good. Moreover, decreasing k or increasing x_f did not improve the match (figs. 11C and 11F). Also at any of the three permeabilities given, increasing x_f from 150 feet to 400 feet resulted in less agreement. Thus it was concluded that for $h = 30$ feet no combination of $x_f > 150$ feet and $k < .25$ md would give a reasonable match.

Next it was decided to use a calculated fracture radius based on a width of 0.1 inch and the volume of sand injected (432 cu ft). Also it was decided to lower the effective gas porosity as the first few months of production data showed an unexpected rapid pressure decline and it was known that a lower permeability would not match the well test data. Hence x_f was increased and ϕ was reduced by keeping ϕx_f^2 constant as determined from $\phi = .016$ and $x_f = 150$ feet. The new values were $x_f = 864$ ft and $\phi = .00048$. Adjusting k it was found that a satisfactory match still could not be obtained.

Subsequently a simple correction factor was calculated for x_f based on the assumption of linear flow during the isochronal

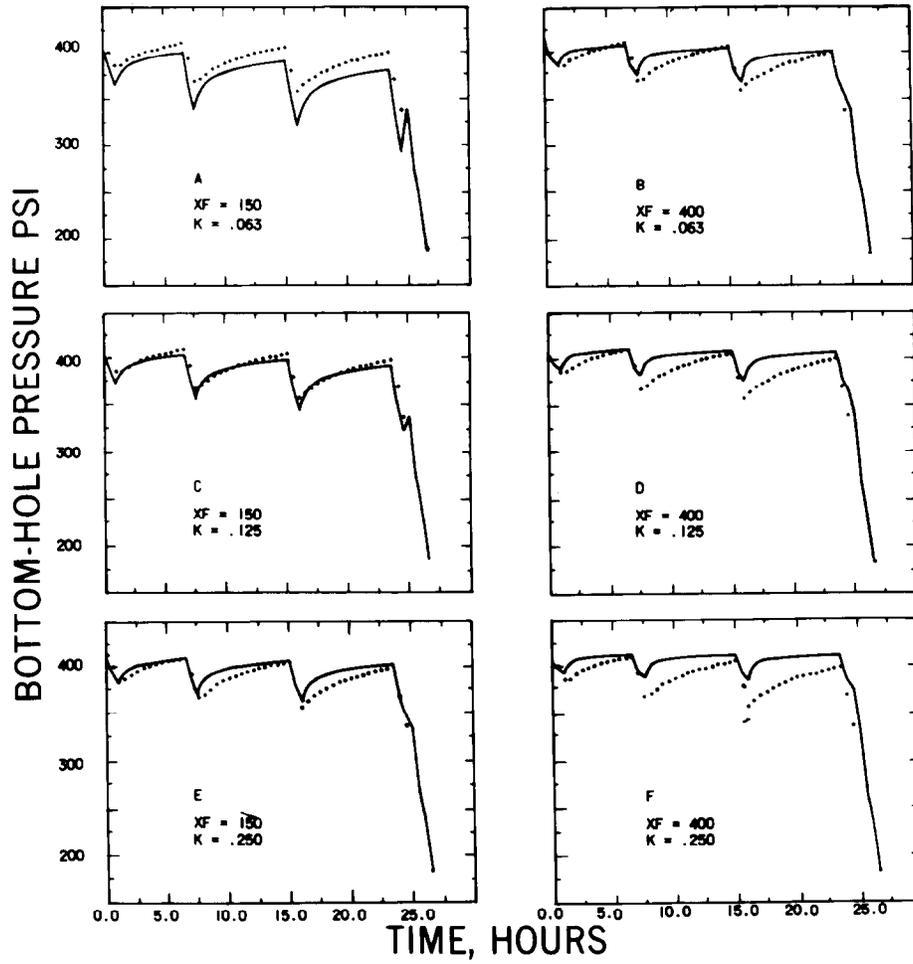


Fig 10 - ISOCHRONAL TEST SIMULATIONS
WITH PHI = 1.6 PERCENT

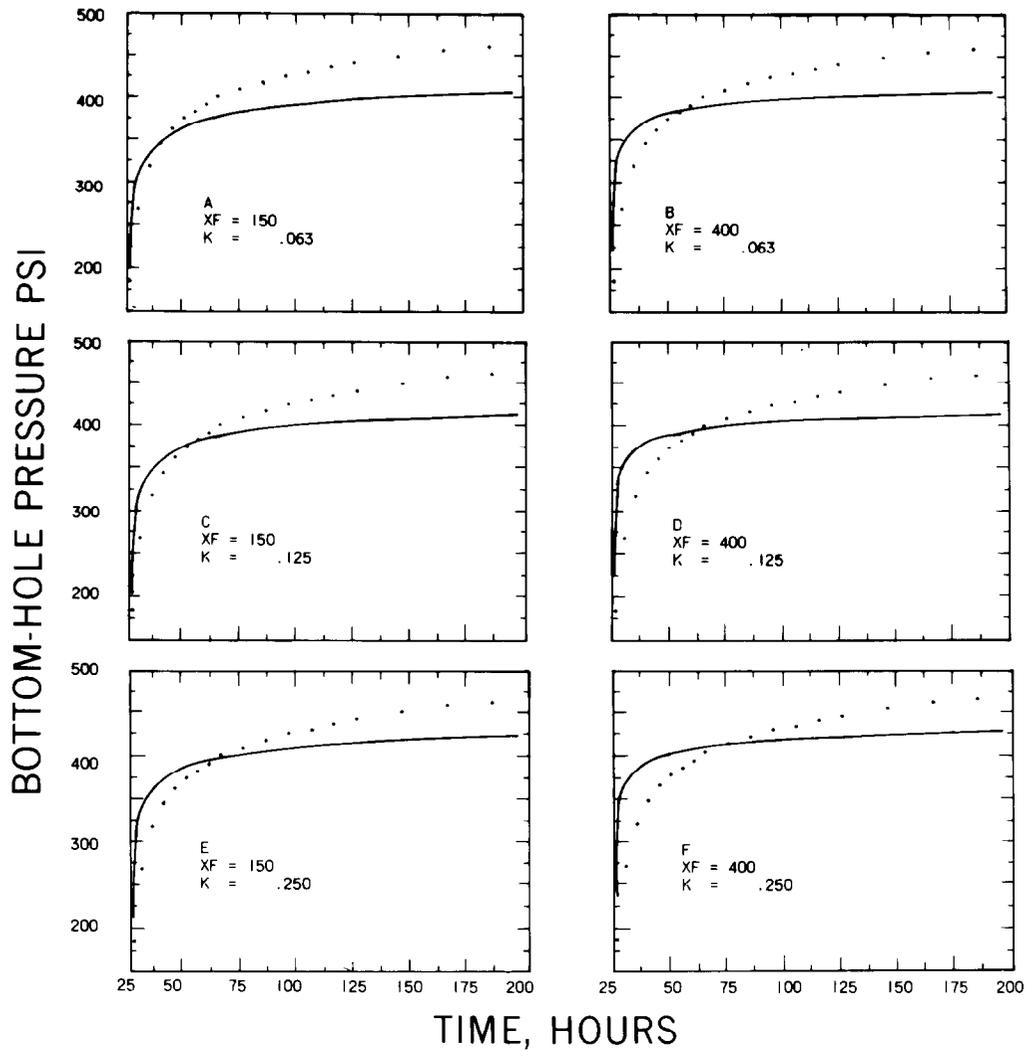


FIG. 11 - BUILDUP TEST SIMULATIONS WITH PHI = 1.6 PERCENT

flow periods. In this case the pressure drop in the fracture is directly proportional to the fracture length. Using a new x_f calculated from the second flow period of the isochronal test a good match for all available data (isochronal buildup, and production) was obtained by reducing both k and ϕ . The conditions for this match were

$$\begin{aligned} h &= 30 \text{ feet} \\ \phi &= .00025 \\ k &= .125 \text{ md} \\ x_f &= 628 \text{ feet.} \end{aligned} \quad (1)$$

Figures 12A and 12B show the good correlation between the calculated and actual well test data. Also the production behavior is fairly well explained by these parameters as shown in figure 12C.

The very low gas porosity could be explained by a high permeability (100-300 md) fissure system with a very high liquid saturation. This would result in a very low gas porosity and also a very low relative gas permeability. This situation is supported by the fact that some liquid accumulation (both oil and water) occurred in the wellbore during production.

Another feasible explanation of the abnormal low porosity is a natural fissure system outside the range of the log tool. For example, consider a system of parallel microfissures oriented normal to the fracture axis. The porosity and permeability of such a configuration are given by Sawyer (15) as

$$\begin{aligned} \phi &= N_f W_f & (2a) \\ k &= 53.76 \times 10^9 N_f W_f^3 & (2b) \end{aligned}$$

where N_f = number of fissures/inch, W_f = fissure width in inches, and k = permeability in md. For example, if $N_f = 2$ and $W_f = 10^{-4}$ then

$$\begin{aligned} \phi &= .00020 & (3a) \\ \text{and} \quad k &= .108 \text{ md,} \end{aligned}$$

which are very close to those found to best match the field data.

A third possibility that might explain the observed behavior is the following:

- (a) effective height less than 30 feet
- (b) high porosity (6%)
- (c) very short fracture (\leq 150 feet)
- (d) very small reservoir

This situation was not investigated but is believed unlikely as there are other wells in the area showing the same lithology.

To further investigate the possibility of a low gas porosity a sensitivity analysis was conducted in which x_f , k , and ϕ were varied about the values given by (1). The results are given in figures 13-18. In each case the "best fit" is shown in the center for comparison. A careful study of these illustrations shows that increasing or decreasing any of the three parameters makes either the isochronal or buildup correlation worse. There are only three cases in which a parameter change resulted in a good match to one of the well tests.

To further study these cases the production period was simulated for each of them. Figure 19 shows the results for the isochronal, buildup, and production periods. While the production pressure decline for the shorter fracture (fig. 19F) may be acceptable we see that for the lower porosity case (fig. 19C) the production pressures fall too low. Also for the shorter fracture, higher porosity situation the production pressures are too high (fig. 19I). Thus, since the isochronal match is unacceptable for the short fracture (fig. 19D) and the production match is questionable we have determined (for a particular reservoir size) a combination of x_f , k , and ϕ which will explain both the well test and early production behavior. Referring again to figures 10-12 and 13-18 we have also shown that for $\phi > .001$ no combination of x_f and k in the ranges

$$\begin{aligned} 150 &\leq x_f \leq 800 \\ .06 &\leq k \leq .25 \end{aligned}$$

will give a good match to the well test data.

Of course with very low porosities boundary effects become significant. The reservoir size used throughout this study was approximately 3200 feet by 3800 feet. Boundary effects were small during the well tests with $\phi \geq .001$ but had some influence on the buildup and a large effect on the production behavior for $\phi = .00025$. This however further substantiates the possibility of a low porosity system of natural fissures. That is, if the boundaries were extended to a distance sufficient to remove all boundary effects the porosity would have to be lowered even more to give the same match to the production data.

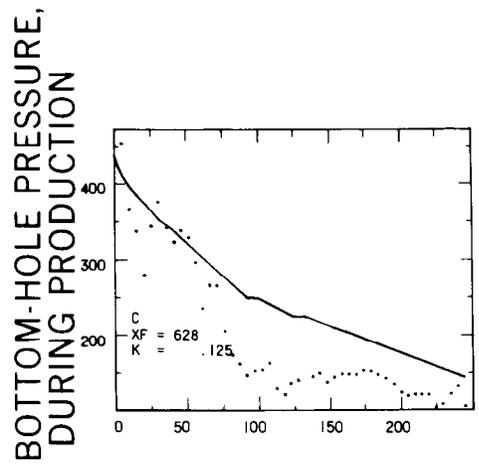
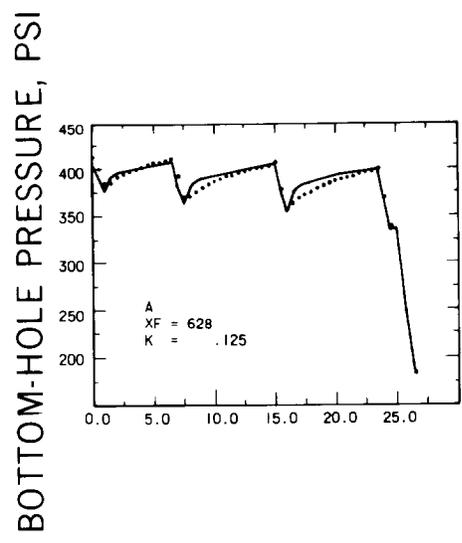
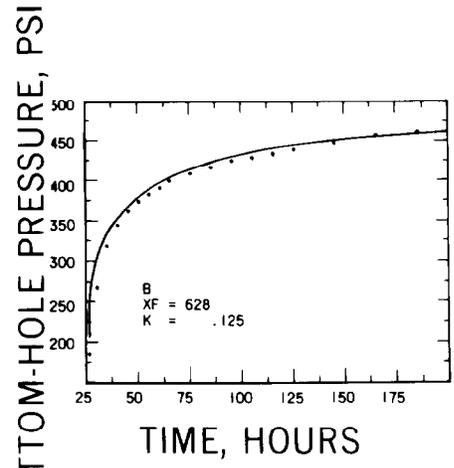


Fig. 12 - BEST CURVE MATCH FOR WELL TEST AND PRODUCTION DATA, $\Phi = .025$ PERCENT

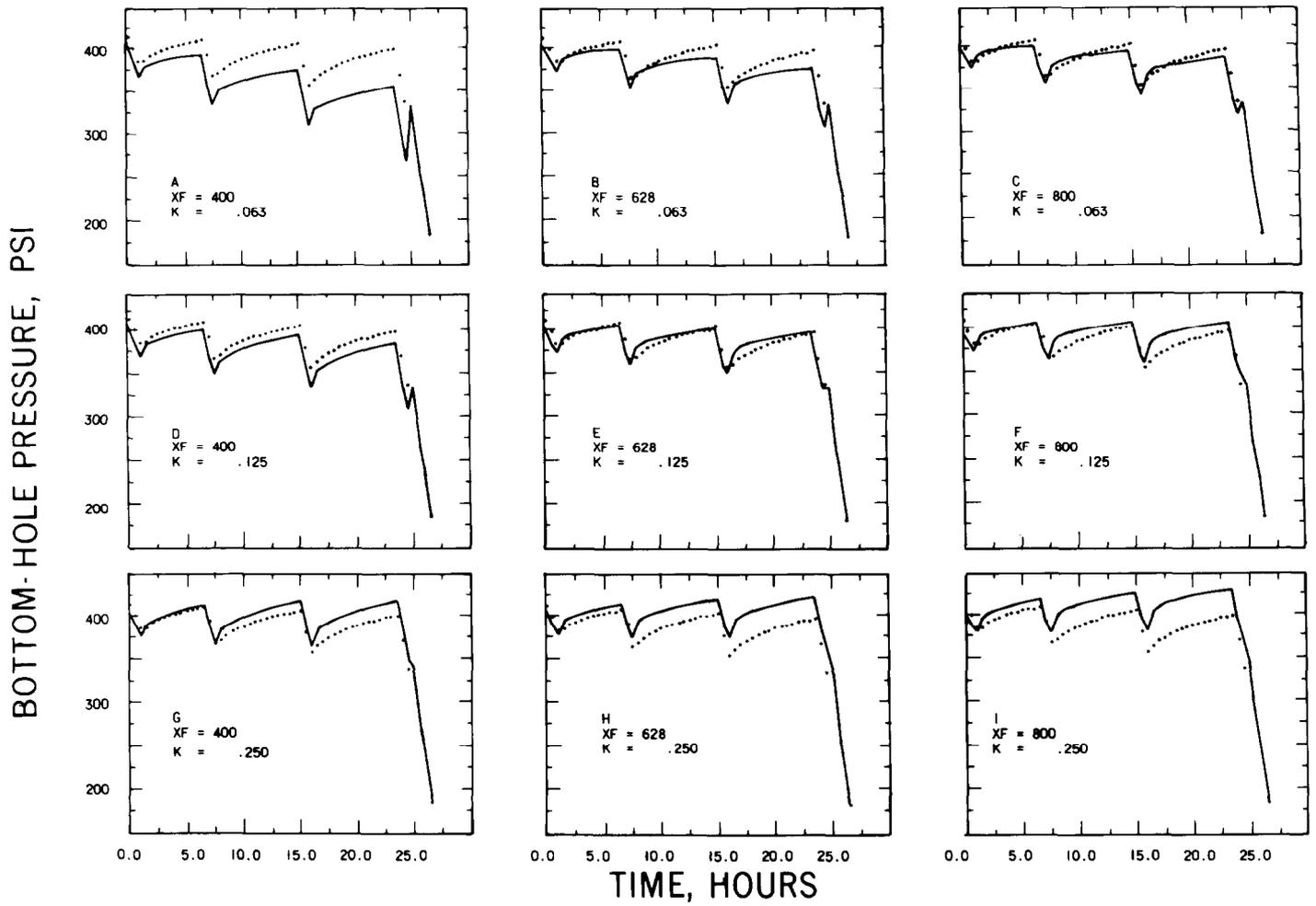


Fig. 13 - FRACTURE LENGTH -
PERMEABILITY STUDY, ISOCHRONAL TEST, PHI = .025 PERCENT

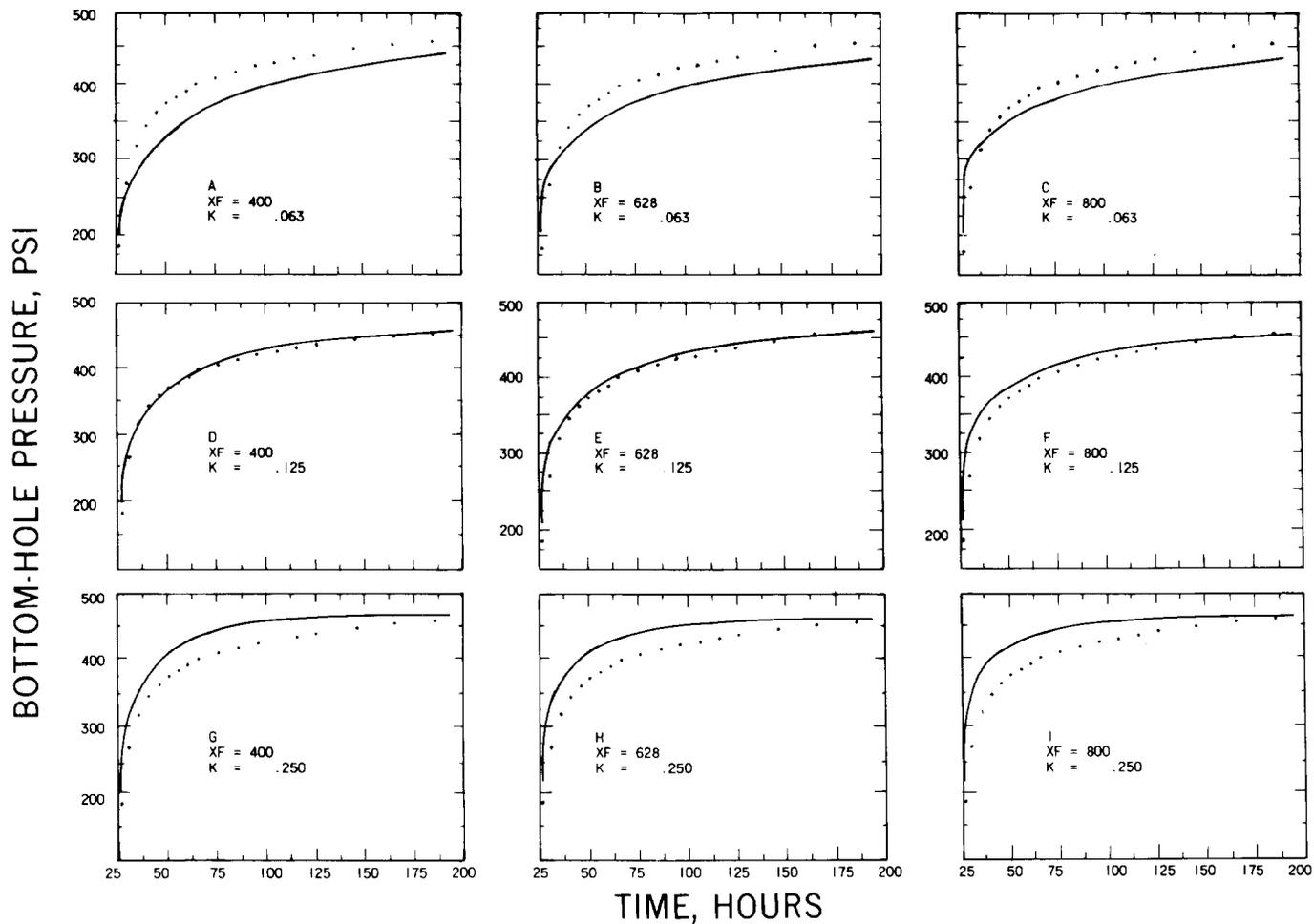


Fig. 14 - FRACTURE LENGTH - PERMEABILITY STUDY, BUILDUP TEST, PHI = .025

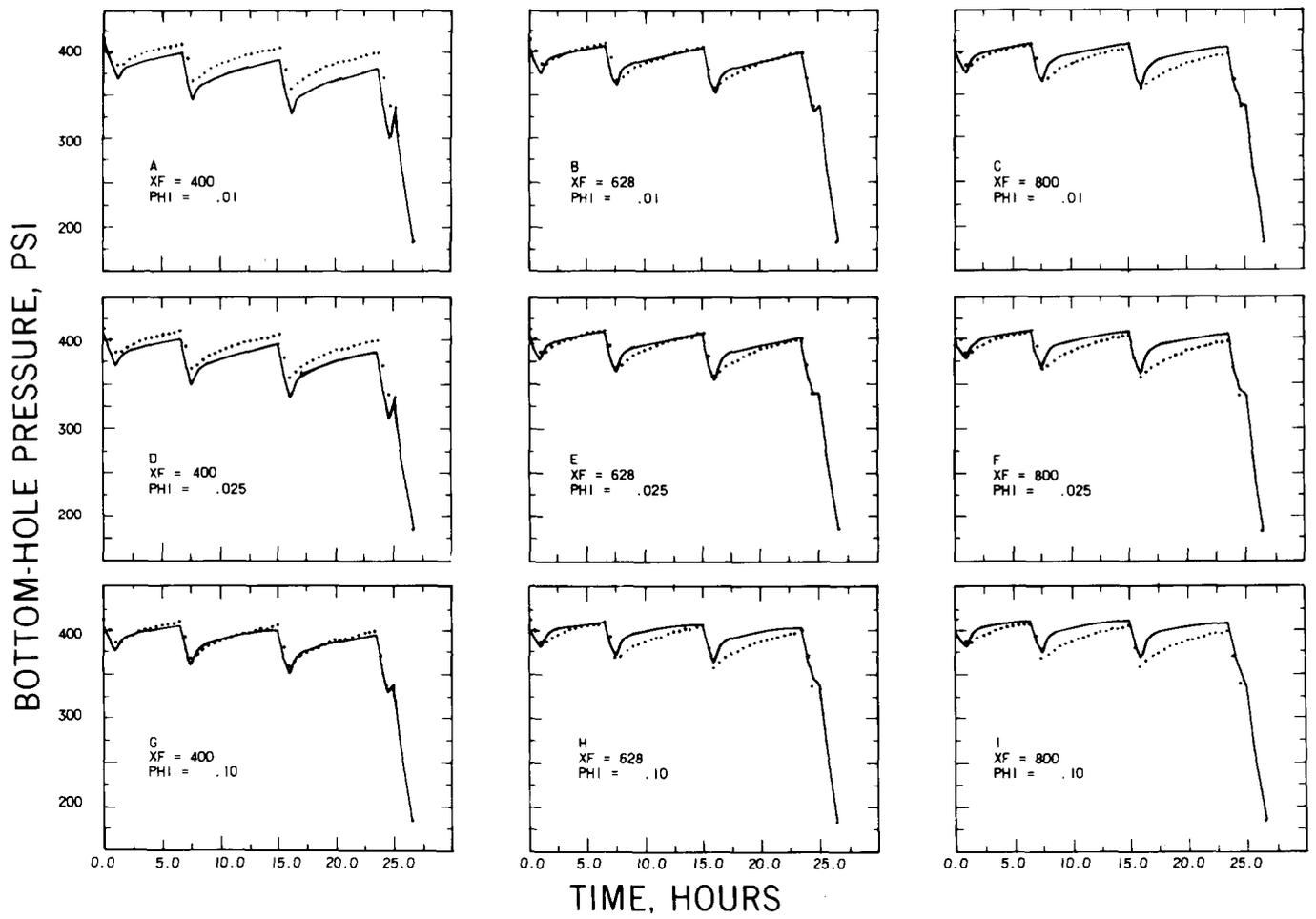


Fig .15 - FRACTURE LENGTHS - POROSITY STUDY, ISOCHRONAL TEST, K = .125 md

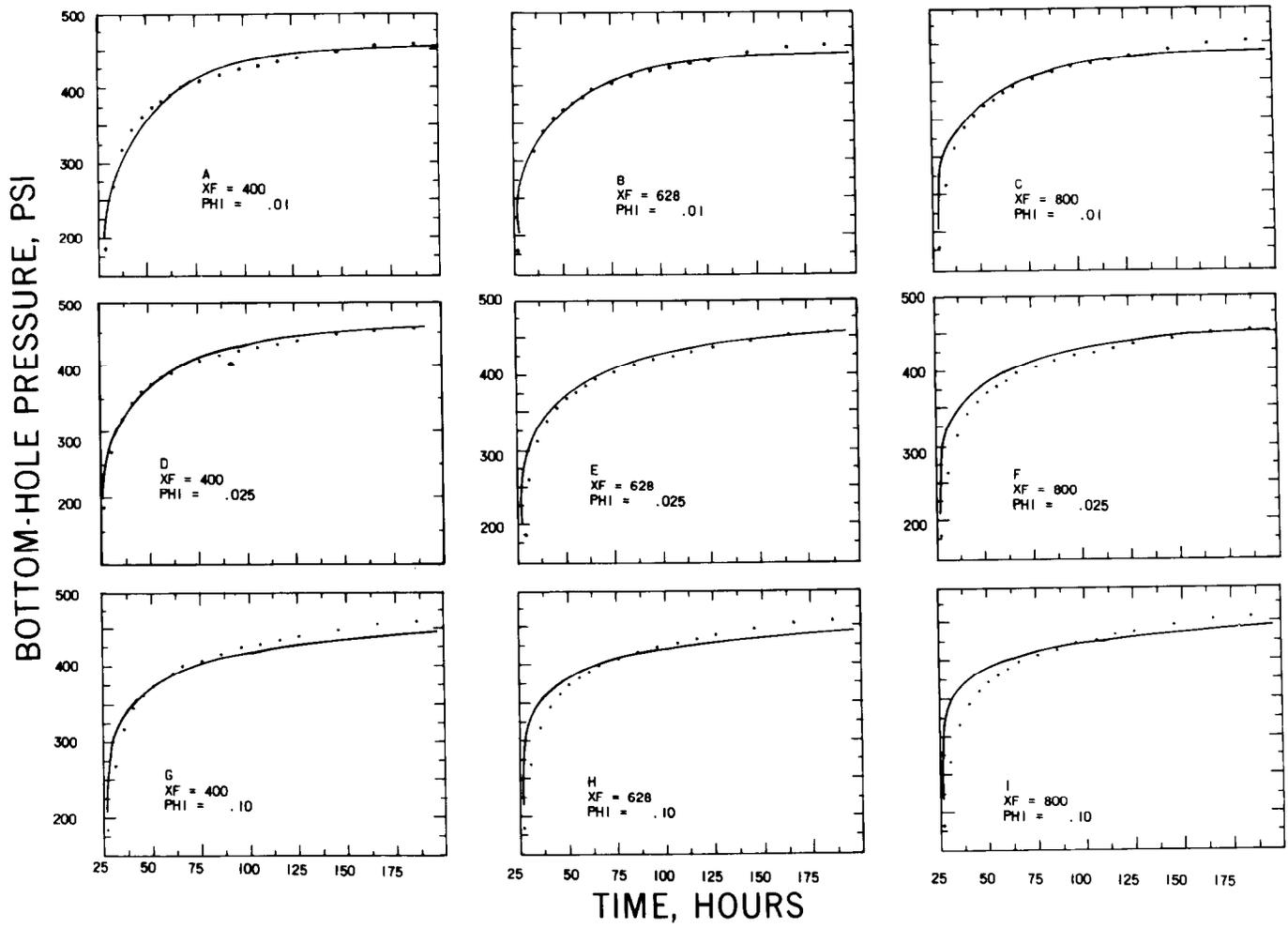


Fig. 16 - FRACTURE LENGTH - POROSITY STUDY, BUILDUP TEST, K = .125 md

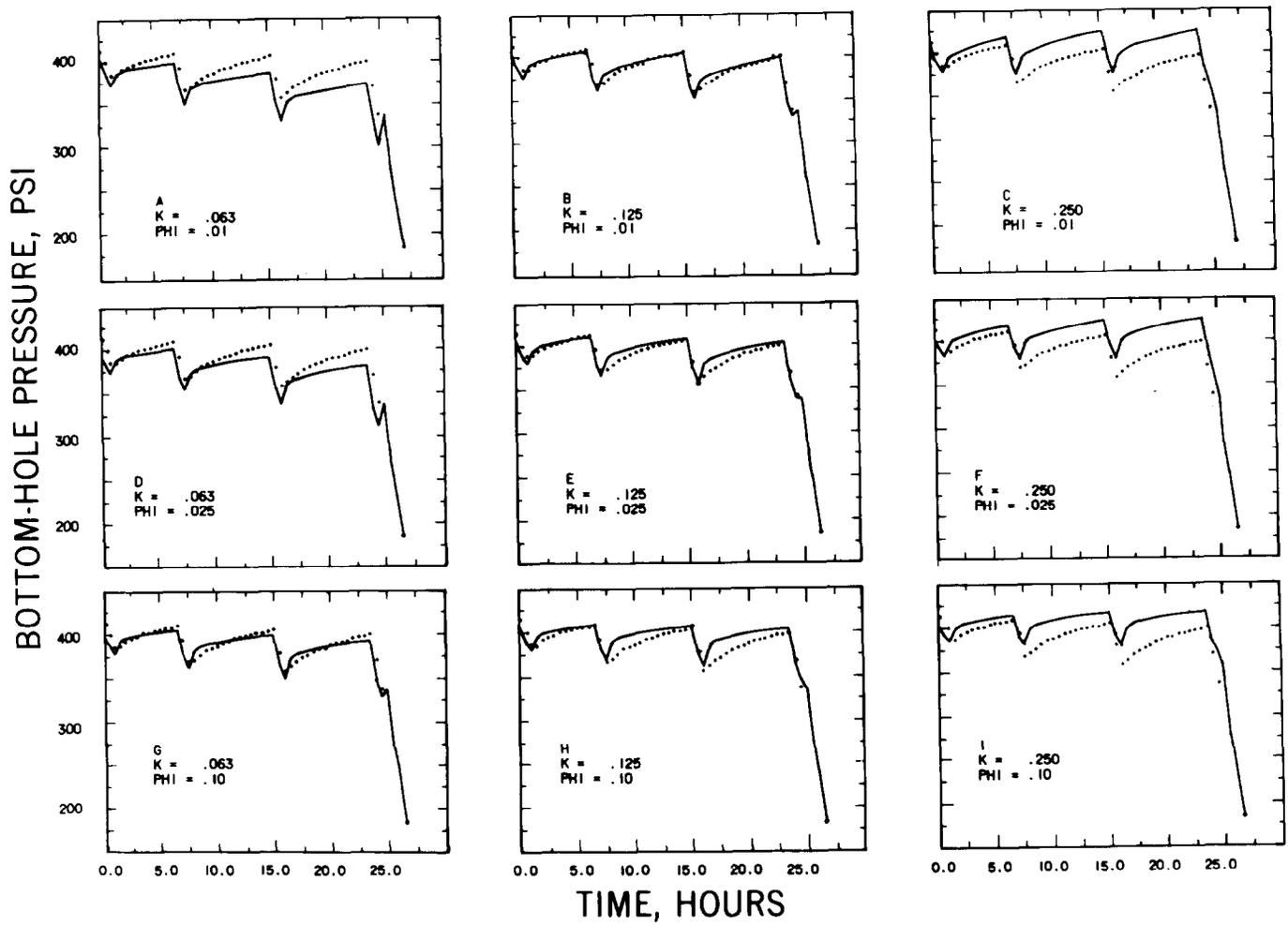


Fig. 17 - PERMEABILITY - POROSITY STUDY, ISOCHRONAL TEST, XF = 628 ft.

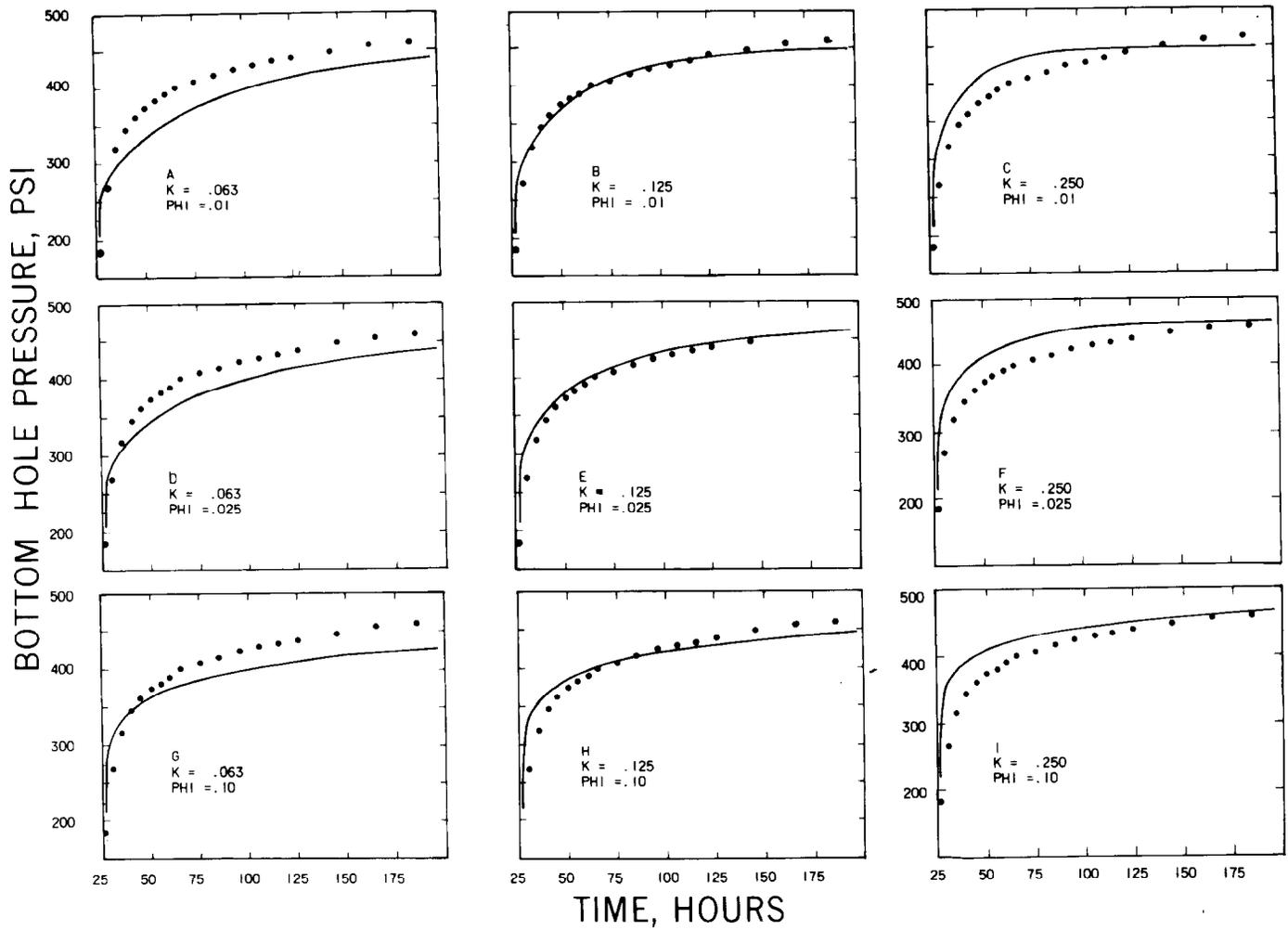


Fig. 18- PERMEABILITY - POROSITY STUDY, BUILDUP TEST, $X_F=628$ ft.

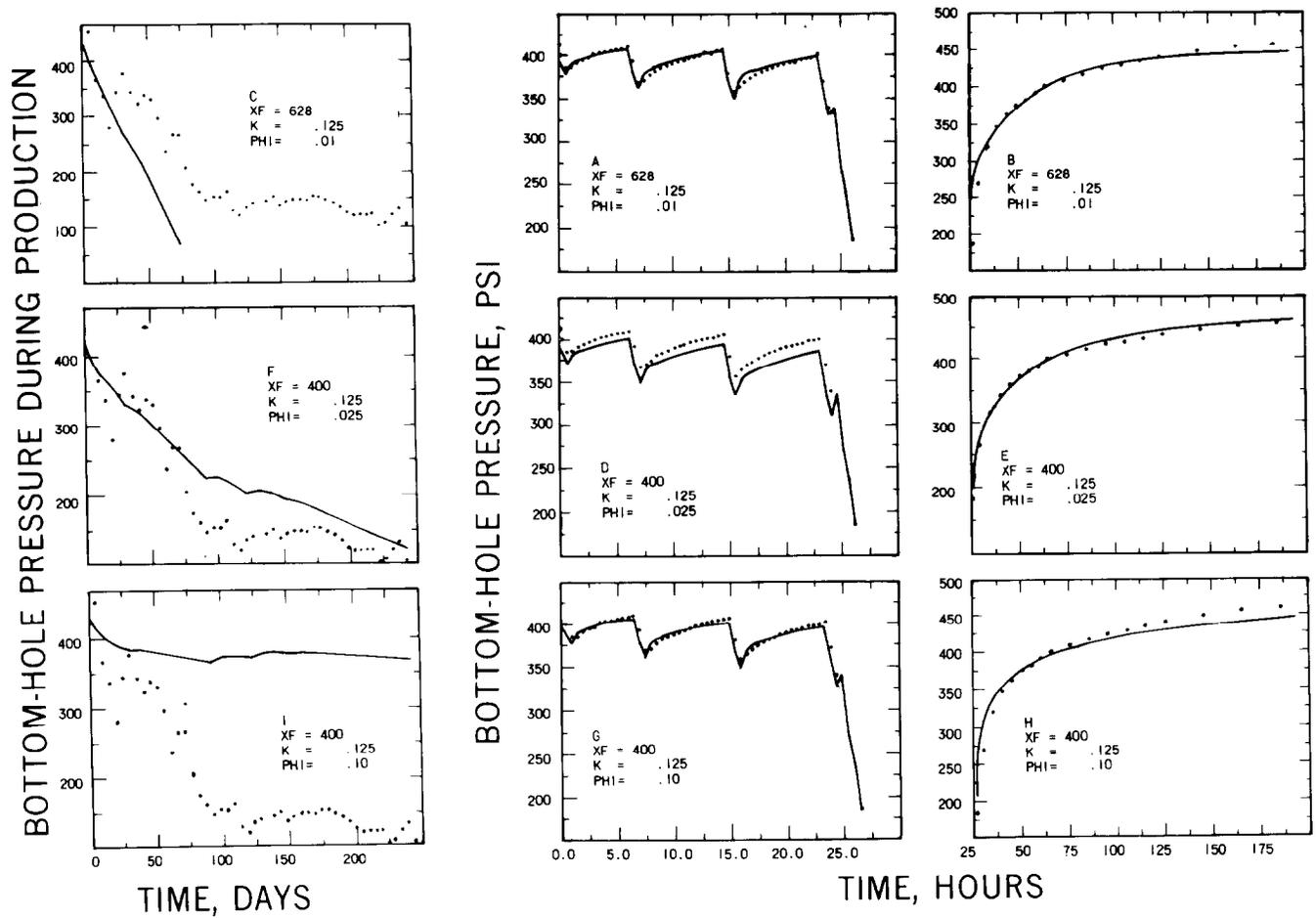


Fig. 19 - CASES OF GOOD CURVE MATCH FOR ONE WELL TEST

DISCUSSION

During the course of the sensitivity analysis on Well B it was realized that it is much faster and possibly cheaper (based on total computer time required) to set up a matrix of reservoir parameters and run all combinations at the outset. This can be done in one day as opposed to several weeks of trial and error runs. In addition, being able to see the effect of a change in several parameters makes it possible to quickly narrow down the range of possible combinations which will result in a match to the well performance data. Of course one well test is insufficient to determine a unique set of values (or range of values). However, from the experience obtained during the course of this work, it is believed that three types of well performance data (e.g. two different well tests and production data) will uniquely determine the unknown reservoir parameters to within an acceptably small range.

CONCLUSIONS

Numerical simulation can be a valuable tool in evaluating the critical parameters in low permeability fractured gas wells.

The curve-matching technique developed herein has wider application than conventional or type-curve analysis in that a constant rate test is not required and virtually any type of well performance data may be used. Also anomalies in production behavior due to stratification or heterogeneity may be explained by a careful study of how permeability and fracture length effect well pressure behavior.

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APPENDIX

What is the basis for representing a fracture of very small width (e.g. 0.1 inch) by a 6 inch block to conserve computer time? And why was 5 darcys used as the x-direction block permeability? To answer these questions two independent studies were done. In the first study two seven day drawdown tests were simulated both for constant rate and constant pressure. The fracture was simulated first using its actual width and permeability (0.1 inch and 10 darcys) and then using a 3 inch grid block containing the fracture with an effective permeability of .333 darcys as given by the relationship for parallel beds

$$k_{\text{eff}} = \frac{k(\Delta y - W_f/2) + k_f W_f/2}{\Delta y}$$

where

Δy = total width of grid block

W_f = fracture width

k = formation permeability

k_f = fracture permeability.

For the constant pressure case the calculated production rates for the two fracture representations agreed within 3 percent after the first few time steps and within 0.4 percent at the end of the simulation. For the constant rate case the calculated bottom-hole pressures for the two cases agreed within 0.7 percent the first time-step and within 0.4 percent at the end of the run. Thus it was concluded that the "effective permeability method" of representing a fracture is justified.

In the second study a drawdown test was simulated in a hypothetical reservoir with 1 md permeability. A fracture was simulated by a 6 inch block with an x-direction permeability of 5 darcys. The length of the fracture was calculated from a log-log plot of the flowing bottom-hole pressure data using type-curve analysis. The calculated length from the uniform flux match was within 2.5 percent of the actual length used in the simulator. Considering that the 5 darcy 6 inch block represents a very high, but not infinite, conductivity fracture this agreement was considered very good. Based on this result the value of 5 darcys is adequate to represent a high conductivity fracture when formation permeabilities are equal to or less than 1 md.