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SCALE-UP OF MISCIBLE FLOOD PROCESSES FOR HETEROGENEOUS RESERVOIRS

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1 Introduction

The current project is a systematic research effort aimed at quantifying relationships between process mechanisms that can lead to improved recovery from gas injection processes performed in heterogeneous Class 1 and Class 2 reservoirs. It will provide a rational basis for the design of displacement processes that take advantage of crossflow due to capillary, gravity and viscous forces to offset partially the adverse effects of heterogeneity. In effect, the high permeability zones are used to deliver fluid by crossflow to zones that would otherwise be flooded only very slowly. Thus, the research effort is divided into five areas:

- Development of miscibility in multicomponent systems
- Design estimates for nearly miscible displacements
- Design of miscible floods for fractured reservoirs
- Compositional flow visualization experiments
- Simulation of near-miscible flow in heterogeneous systems

The status of the research effort in each area is reviewed briefly in the following section.

2 Project Status

- **Developement of Miscibility in Multicomponent Systems**

  We are making progress on the creation of a systematic theory of miscibility development in multicomponent systems. The dispersion-free theory developed previously at Stanford shows that in any multicomponent displacement, the recovery behavior is determined by a small number of key tie lines that include the tie lines that extend through the initial and injection compositions, and one or more crossover tie lines. If any of those tie lines is a critical tie line, then the displacement is multicontact miscible. Thus the key to determining minimum miscibility pressure (or minimum enrichment for miscibility) is to find which tie line approaches the critical locus first as pressure (or enrichment) is increased. Graduate student Bruno Aleonard is currently investigating efficient algorithms for calculation of the critical locus.

The next step is to develop an algorithm to determine which tie line lies closest to the critical locus. Once the key tie line for miscibility is so identified, it should be possible to develop an efficient algorithm to determine minimum miscibility pressure for a multicomponent system.

- **Design Estimates for Nearly Miscible Displacements**

  The scaling theory developed last year has been extended to include the effects of layer ordering in scaling nearly miscible displacements in layered reservoirs. We found that gravity forces can offset the adverse effects of high permeability channels in some layered reservoirs if adequate vertical communication exists.

- **Design of Miscible Flood for Fractured Reservoirs**

  We have made significant progress in modifying the existing high pressure PVT equipment to conduct high pressure gravity drainage. We will soon be able to conduct high pressure gravity drainage in the presence of CO_2 in cores up to two feet in length.

  In this area, we have also made progress in developing the theory of three phase gravity drainage. Recent experimental and theoretical results suggest that we are close to being able to predict oil drainage rate and final recovery for a given system with good agreement between experiment and theory.

- **Flow Visualization Experiments**

  To understand more fully the visualization experimental results we have obtained, we are in the process of modifying a compositional simulator code to simulate the observations. A subroutine for calculating the phase behaviors of the fluid system used in our experiments is to be completed. The oil-water-alcohol systems used in the low pressure experiments are not modeled accurately by cubic equations of state, and hence an approach based on excess free energy models is being used.

- **Simulation of Flow in Heterogeneous Reservoirs**

  We have continued to investigate the streamtube approach as a numerical alternative to conventional finite difference simulators to be used in predicting near-miscible gas injection in heterogeneous reservoirs. We apply the streamtube approach to the two-phase immiscible case which is assumed to be well understood and for which many reliable

numerical methods have been developed. Our results have given us new insight on the applicability of the streamtube approach and, particularly, on the novel idea that proposes to map a one-dimensional analytical solution (Riemann solution) along the streamtubes to obtain a two-dimensional solution for a heterogeneous reservoir.

In the sections that follows, we summarize new results obtained by Ph.D student Marco Thiele concerning the streamtube approach.

3 Research Results

Previous research using streamtubes focused almost exclusively on the immiscible two-phase problem with an areal geometry. Principally, it is the weak nonlinearity of the immiscible two-phase problem (water-food) that allows for the assumption of constant streamtube geometries, which are almost universally applied in the published literature. A notable exception is *Renard* (1990) . Here the streamtubes are recalculated periodically, and the fluid is assigned to the new streamtubes using a much finer mesh than that upon which the tubes are calculated. In general though, the assumption of a fixed streamtube geometry is widely used and reinforced by the areal geometry since, by continuity, a streamline must start and end at a source. In the areal plane sources are points leaving little room for the streamtubes to change their shape during the displacement. Furthermore, except for *Hewett and Behrens* (1991), all studies in the literature consider a homogeneous reservoir to generate the streamtubes.

The physical and geometrical constraints imposed on the governing PDE then, are such that the geometry of the streamtubes becomes a weak function of the total mobility distribution. In light of this, the good matches reported by *Higgins and Leighton* (1962); *Higgins, Boley, and Leighton* (1964); *Martin and Wegner* (1979); and *Hewett and Behrens* (1991) are not all too surprising. Another way of accounting for the weak nonlinearities was proposed by *Martin and Wegner* (1979). They show that instead of updating the geometries of the streamtubes, the total volume of fluid injected can be made a function of the resistance of each tube. The change in volume of a streamtube is traded for a varying volume of fluid going down each tube, while keeping the actual streamtube geometry constant. Although the

approaches differ slightly, all show convincingly that streamtubes can successfully and inexpensively predict recovery for two-dimensional, areal waterflood problems.

In this work, the waterflood problem is approached differently and used to demonstrate the applicability of a Riemann method. Detailed descriptions of techniques for construction of solutions for multicomponent, two-phase flow problems have been presented previously (*Thiele and Blunt* (1993)). The domain is considered to be heterogeneous and the geometry to be cross-sectional. The streamlines are now no longer pinned down by two points in the domain as in the areal case. This makes them 'freer' to move and conform to the flow field allowing the nonlinearities to become more noticeable. But the real difference is in the mapping of the one-dimensional solution along each streamtube. In this work, the one-dimensional solution is treated as a solution to a Riemanm problem. This means that at the new time level the solution is not given by an integration from t_D to $t_D + \Delta t_D$, as in conventional time-stepping algorithms, but rather from 0 to $t_D + \Delta t_D$, where the initial condition at time $t_D = 0$ is characterized by a discontinuity at $x_D = 0$. This, of course, is the well-known initial condition for the Buckley-Leverett solution. Treating the one-dimensional solution as a Riemann solution means that each streamtube is treated as a true one-dimensional system on which the Buckley-Leverett solution is simply mapped repeatedly for different times.

In this approach, the underlying assumption is that the solution is indeed scalable by x_D/t_D along each tube. In simple words: the fluid entering a tube remains in the tube and can exit only by the outlet end. The validity of this assumption is considered in detail in the section that follows.

3.1 Validation of the Riemann Approach

The Riemann approach was tested by the following numerical experiment. Using a standard finite difference simulator (ECLIPSE), the velocity fields were stored for regular increments of dimensionless time. From each velocity field, the corresponding streamtubes were then calculated and used to find the saturation profiles by mapping a Riemann solution along the streamtubes. The saturation profiles obtained by this method were then compared to the saturation profiles obtained by

the direct Riemann approach proposed in this research, the underlying assumption being, of course, that the velocity fields obtained from the finite difference simulator are indeed 'correct'.

The data used to test the Riemann approach are shown in Fig. 1. Although the end-point mobility ratio is 10, the frontal mobility ratio is, in fact, only 1.36, resulting in a more stable displacement than suggested by the end-point value alone. A similar statement can be made for many waterfloods with 'reasonable' relative permeability curves: the frontal mobility ratio is usually of order 1 even though the end-point can be of order 10 or 100. The weak nonlinear behavior of the two-phase immiscible problem results from the relatively mild adverse mobility ratio.

The absolute permeability field is shown in Fig. 2. This field was derived from a finer 250x100 permeability field by taking a geometric average of each set of four blocks on the fine grid that equal one block in the 125x50 grid used here.

As is shown in Fig. 3, the pure Riemann approach agrees well with the mixed method (ECLIPSE velocity field + Riemann solution). Both are seen to be devoid of numerical dispersion, as expected, compared to the saturation profiles obtained directly from ECLIPSE. Fig. 3 displays profiles for dimensionless times $t_D = 0.2$ and $t_D = 0.4$. The upper row shows saturations profiles obtained directly from ECLIPSE. The middle row shows profiles obtained using the velocity from ECLIPSE (and consequently the streamtubes) but mapping the solution as a Riemann solution. The last row shows profiles obtained by using the Riemann approach only to solve the entire nonlinear problem.

The difference between the two methods is indeed small, suggesting that the nonlinearity of the velocity field is captured correctly by the direct Riemann approach. In fact, it is interesting to note that numerical dispersion causes a larger difference in recovery than the direct Riemann solution method. Fig. 4 shows the differences in recovery performance for the three methods. The recovery curve obtained from the mixed method (ECLIPSE velocity field + Riemann solution) lies above the recovery curve obtained directly from ECLIPSE. Because the velocity fields for these two recovery curves are identical, the difference must be attributed to numerical dispersion and the way in which the one-dimensional solution is mapped along the streamtubes.

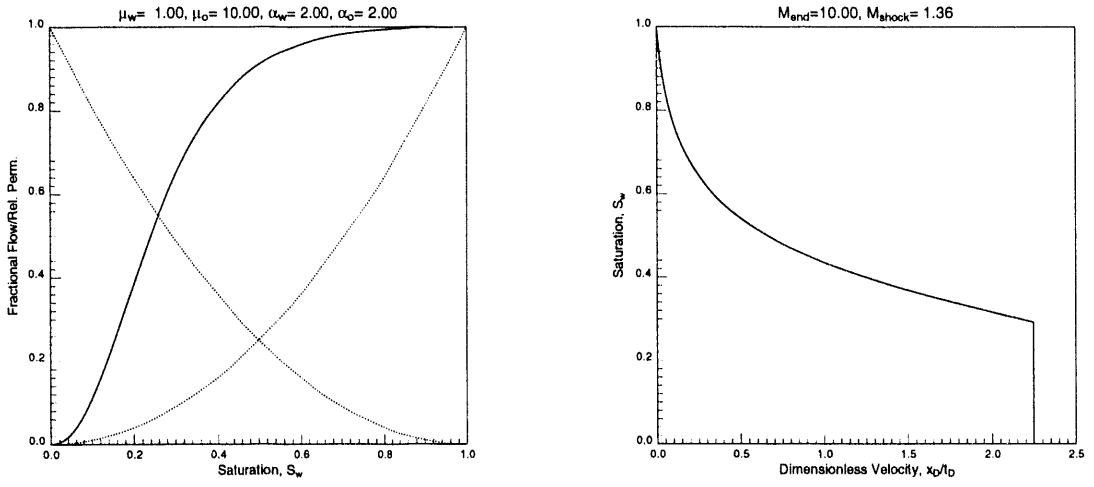

Figure 1: Relative permeability curves ($k_{rw} = S_w^2$, $k_{ro} = S_o^2$), corresponding fractional flow function for a viscosity ratio of 10 ($\mu_o = 10, \mu_w = 1$), and Buckley–Leverett analytical solution used for testing the Riemann approach. The mobility ratio at the shock front is $M_{\text{shock}} = 1.36$.

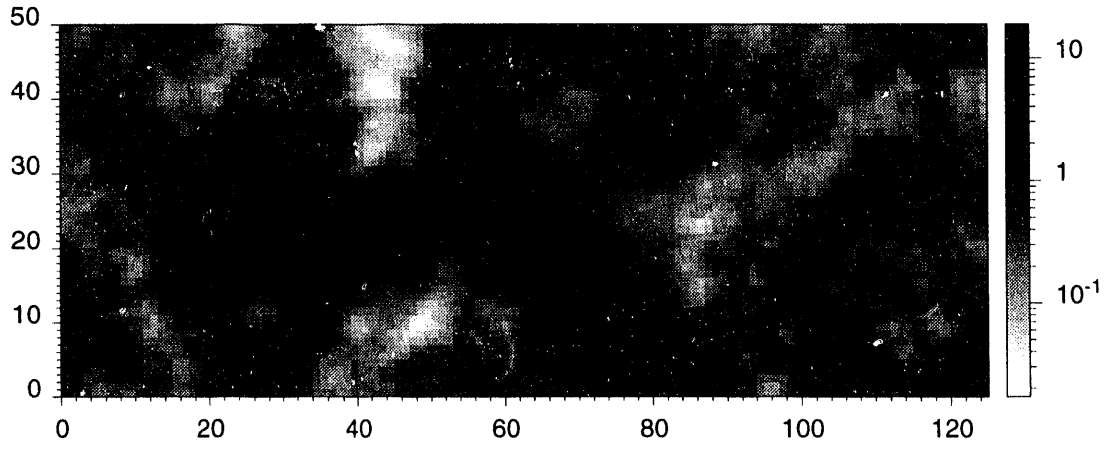

Figure 2: Permeability map with logarithmic scaling – (125x50 Grid).

That numerical dispersion may have the upper hand is suggested by the saturation profiles shown in Fig. 5.

Although this numerical experiment does not prove that the two-dimensional solution is scalable by x_D/t_D along streamtubes, it does suggest that the differences in the velocity field compared to a traditional finite difference approach is small and that numerical dispersion can lead to larger deviations than those caused by assuming a Riemann solution along streamtubes.

3.2 Convergence of the Riemann Approach

Before using the Riemann approach to investigate other immiscible cases, the issue of convergence is addressed in this section. In other words, at what rate must the streamtubes be recalculated in order to consider the solution converged? If the two-phase immiscible problem is indeed weakly nonlinear, then it may require fewer updatings of the pressure (or Ψ–field) than currently used in finite difference simulators and thus lead to a substantial speed-up. Finite difference simulators usually use a CFL (*Courant, Friedrichs, and Lewy 1928*) type stability criterion for the discretized hyperbolic conservation equation to determine when to resolve for the pressure field. In such methods the pressure field is recalculated at every time step.

In the streamtube approach, of course, the conservation equation is not discretized and therefore there is no CFL condition to worry about. The question that arises, instead, is how many times the streamtubes must be updated to consider the solution converged? This question is addressed by solving the previous problem repeatedly while increasing the number of streamtube updates over two pore volumes injected. Recovery curves for 1, 10, 20, 40, and 100 streamtube updates are shown in Fig. 6.

It is rather surprising to find that 20 solves are indeed sufficient to consider the problem converged over a range of two pore volumes. With only 20 solves, the new approach represents a reduction in computation time by two orders of magnitude compared to the 1000's of solves needed by a traditional finite difference simulator like ECLIPSE. Thus, two important points can be made here:

1. The displacement is indeed only weakly nonlinear. Although the

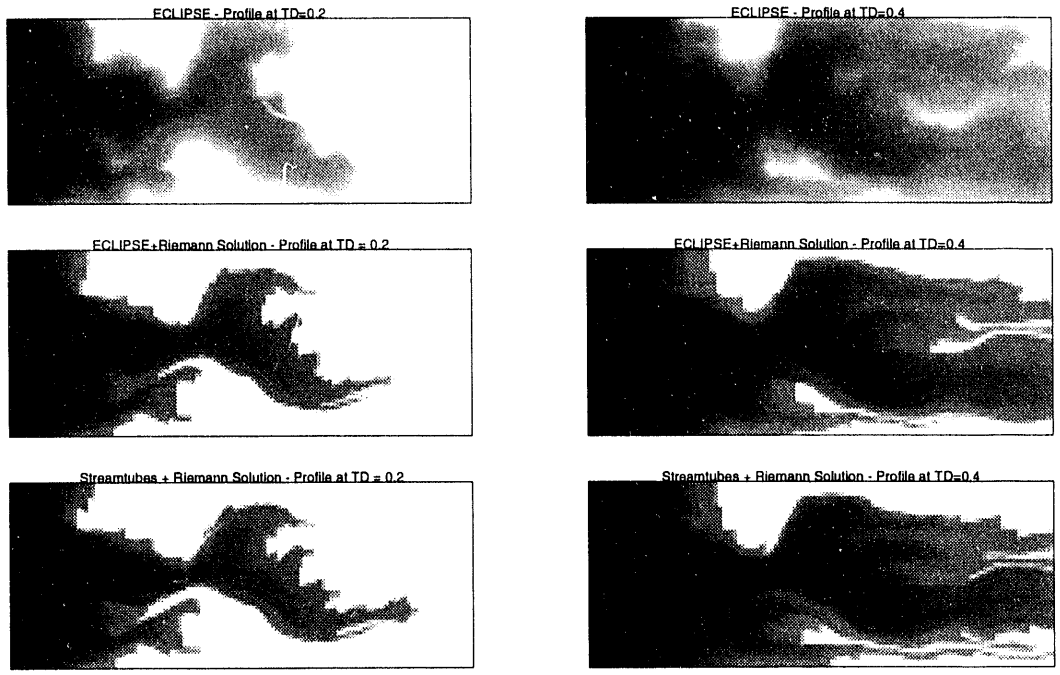

Figure 3: Saturation profiles at times $t_D = 0.2$ and $t_D = 0.4$. From top to bottom: profiles obtained directly from ECLIPSE; profiles obtained by using the velocity field from ECLIPSE but mapping a Riemann solution along streamtubes; profiles obtained by the method proposed in this work.

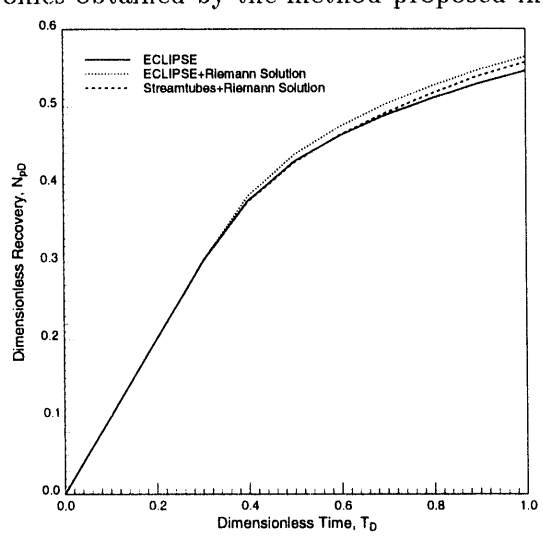

Figure 4: Recovery curves for the three different solution methods used to generate the profiles in Fig. 3.

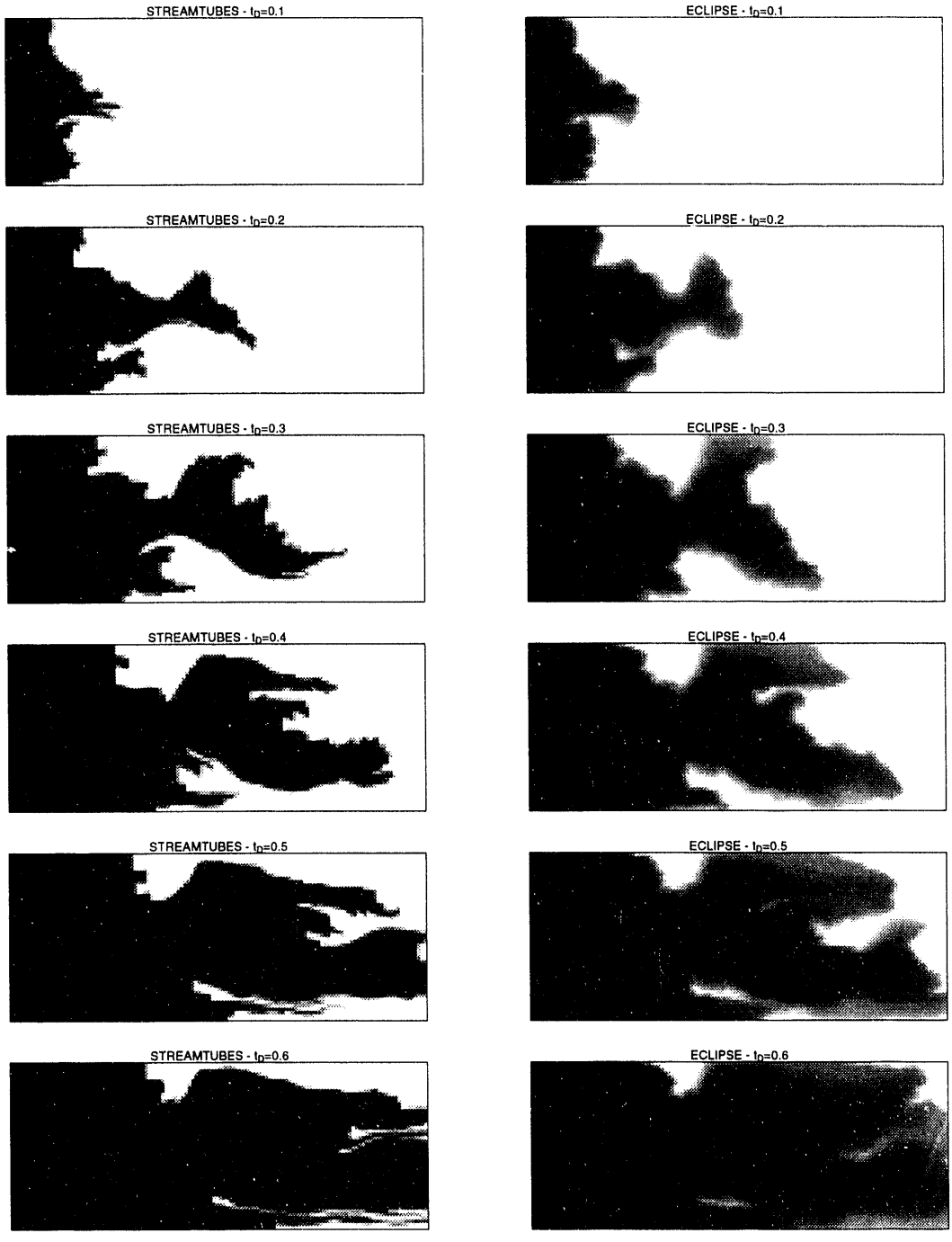

Figure 5: Displacement history at $\Delta t_D = 0.05$ intervals for a Buckley–Leverett problem with a fractional flow given by Fig. 1

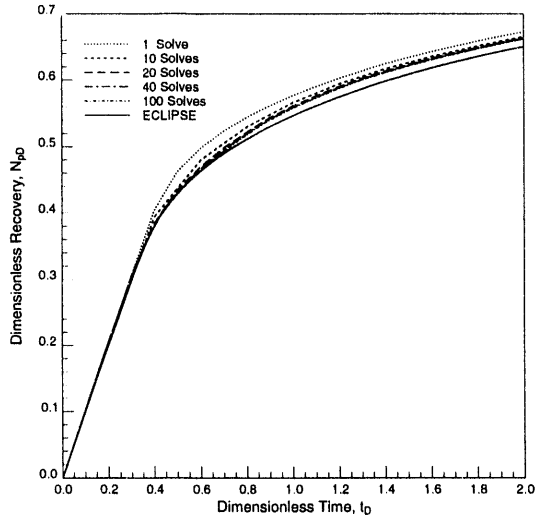

Figure 6: Recovery curves for for 1, 10, 20, 40, and 100 streamtube updates over two pore volumes injected ($t_D = 2$) showing that the problem can be considered converged if more than 20 updates are used.

end-point mobility ratio is 10, it is the shock-front mobility ratio of 1.36 that dictates the severity of the nonlinearity. Far fewer solves of the pressure field are required than used by traditional finite difference simulators leading to a two orders of magnitude speed-up.

2. The dominating factor in the displacement is not the viscous instability caused by the difference in viscosity of the fluids, but rather the reservoir heterogeneity and preferential flow channels. Since viscous instability is certainly the more difficult problem to solve, this is a very encouraging result.

3.3 Other Immiscible Solutions

This section consider other immiscible cases intended to verify further the streamtube method as well as to gain some physical insight. There are, of course, many parameters that affect the displacement efficiency and an exhaustive investigation would have to consider them all. Yet, since the scope of this research is to understand miscible displacements, only two parameters are investigated here: (1) The end-point mobility

ratio and (2) the correlation length of the reservoir. Except where noted, the relative permeabilities are assumed fixed and given by $k_{rw} = S_w^2$, $k_{ro} = S_o^2$.

End-Point Mobility Ratio

The end-point mobility, defined as

$$M_{\text{end}} = \frac{\lambda_J}{\lambda_I} = \left(\frac{k_r}{\mu}\right)_J \left(\frac{\mu}{k_r}\right)_I = \frac{\mu_o k_{rw}}{\mu_w k_{ro}} \quad , \tag{1}$$

where the subscripts I and J refer to initial and injected conditions respectively, is simply varied by increasing the resident fluid viscosity, in this case that of the oil. As is shown in Fig. 7 and Fig. 8, the changing end-point mobility ratio seems to affect the rate of convergence only, with the $M_{end} = 1$ case requiring one solve, the $M_{end} = 3$ less than ten, the $M_{end} = 5$ approximately ten, and the $M_{end} = 10$ twenty solves as mentioned earlier. It is interesting to note that the converged recovery curves all compare in the same way with the recovery curves obtained from ECLIPSE. As discussed in the previous discussion on the validity of the Riemann approach, the differences can be attributed to numerical dispersion caused by the finite difference formulation in ECLIPSE. A more surprising result is that ECLIPSE consistently underestimates compared to the recoveries obtained from the streamtube approach –surprising because it is usually assumed that numerical dispersion increases recovery, since it mitigates viscous instability thereby reducing viscous fingering and early breakthrough. One possible explanation in this case is that numerical dispersion smears the shock front and thus reduces its effectiveness in recovering the oil ahead of it. Since the streamtube actually uses an analytical solution along each streamtube, the piston-like recovery mechanism remains intact and predicts a higher oil recovery from the reservoir. In fact, the true physical answer may lie somewhere between the two, but these results are significant because, for the first time, the effects of numerical dispersion on recovery are quantifiable.

Data for another example are shown in Fig. 9. In this case the end-point relative permeability of the oil phase is $k_{ro}|_{S_o=1} = 0.25$. Keeping the same exponents as before and assuming an oil-to-water viscosity ratio of ten ($\mu_o = 10, \mu_w = 1$) leads to an end-point mobility ratio of

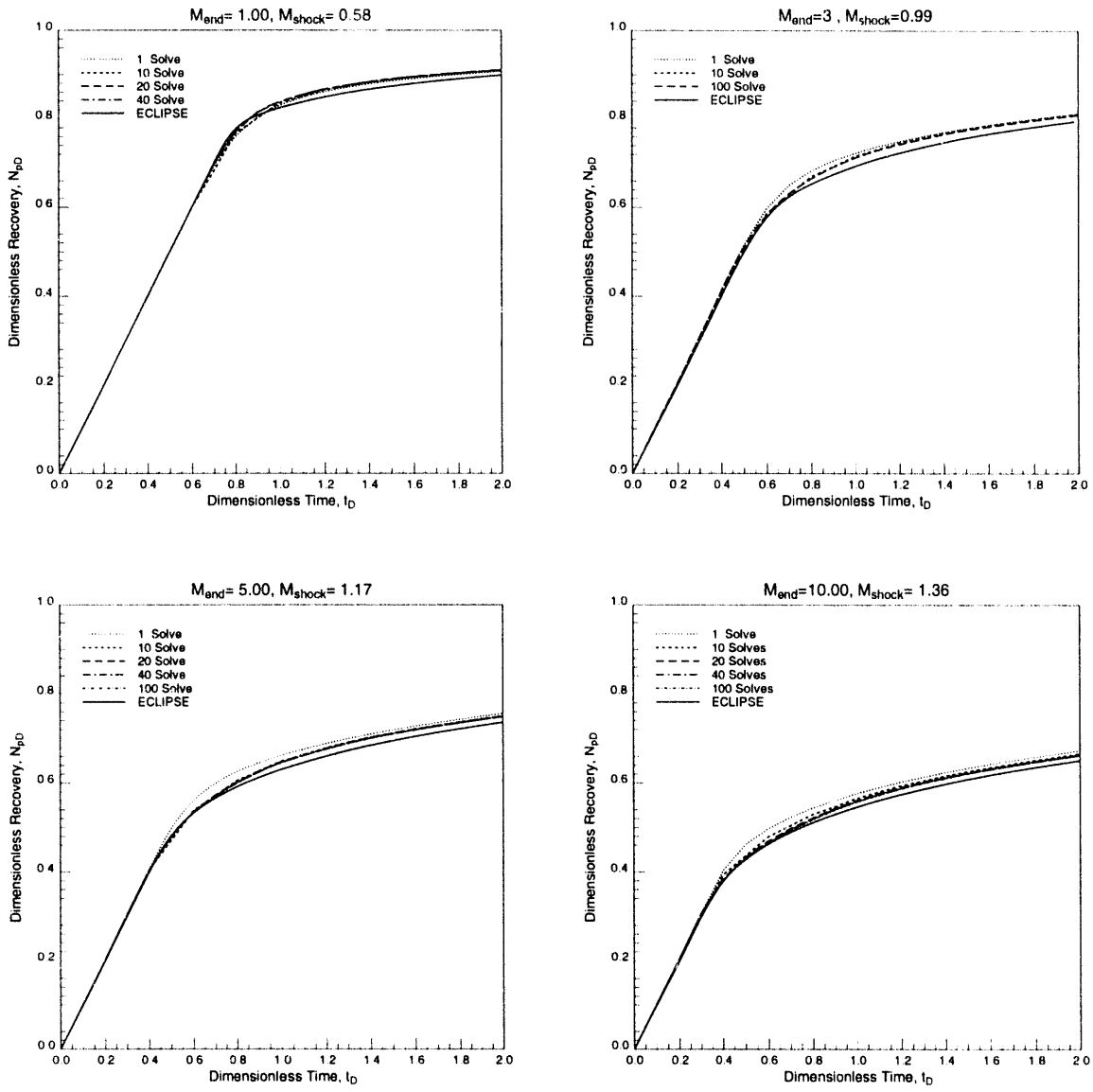

Figure 7: Recovery curves for for M=1, 3, 5, and 10 showing convergence of the streamtube approach and comparison with recovery curves obtained from ECLIPSE.

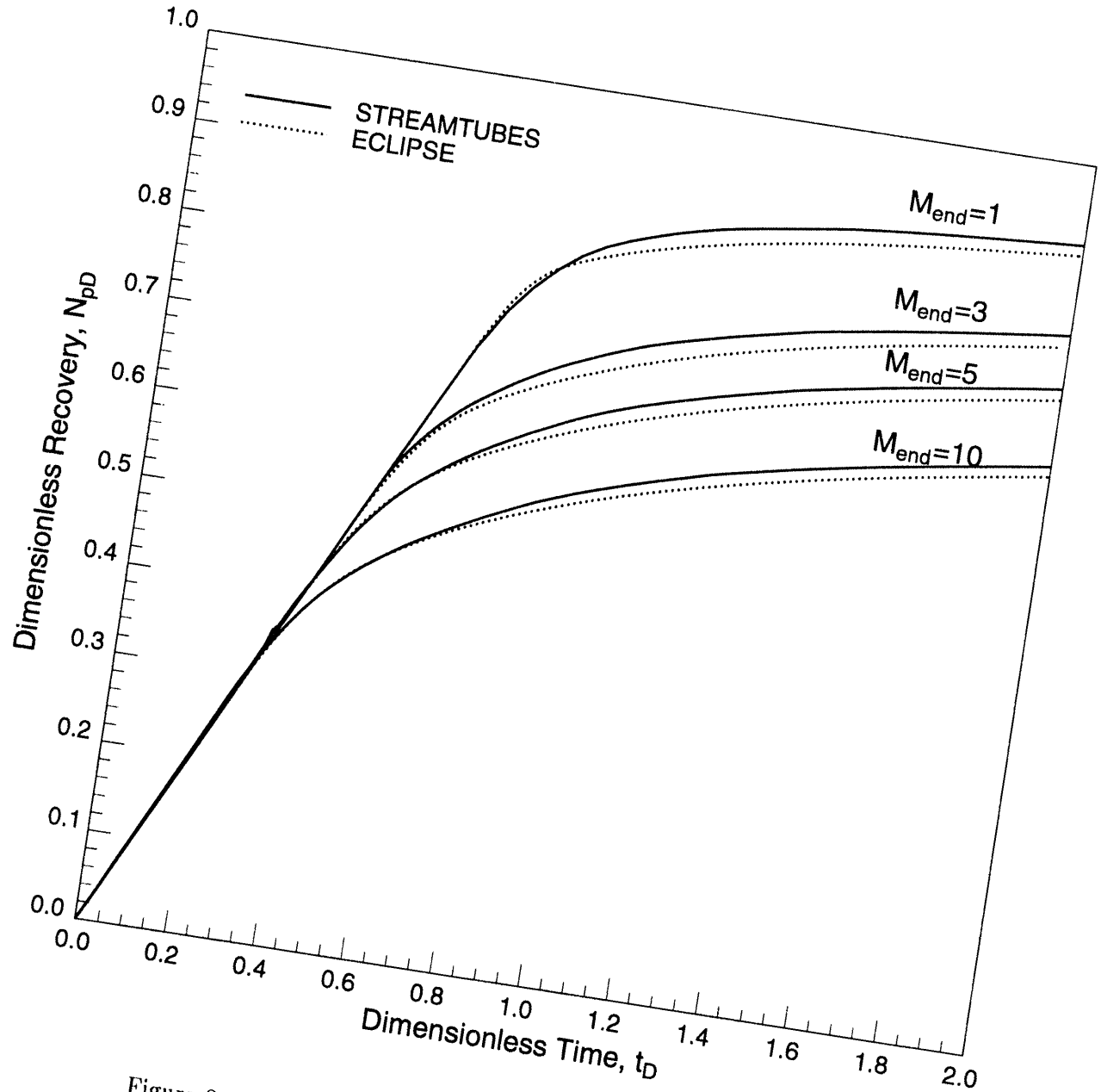

Figure 8: Summary of converged recovery curves.

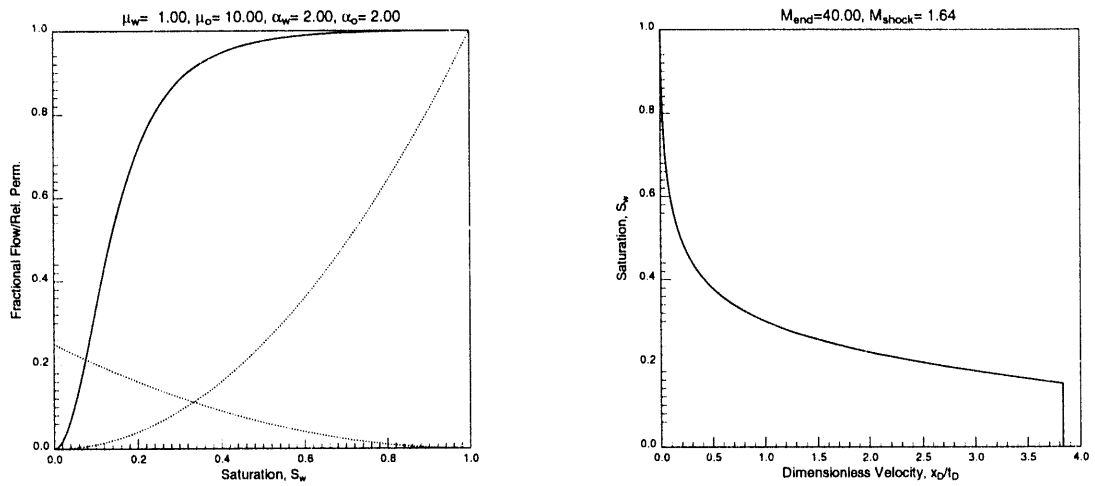

Figure 9: Relative permeabilities, fractional flow, and one-dimensional saturation profile for $M_{\text{end}} = 40$. The shock-front mobility ratio is $M_{\text{shock}} = 1.64$. ($k_{rw} = S_w^2$, $k_{ro} = S_o^2, \mu_o = 10, \mu_w = 1$).

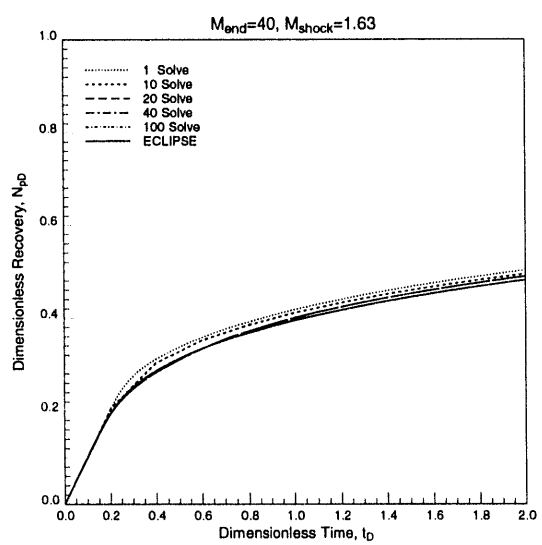

Figure 10: Recovery curves for the data shown in Fig. 9.

$M_{\text{end}} = 40$. But even in this case, as the recovery curve in Fig. 10 demonstrates, the streamtube method is able to capture the nonlinearity within 20 updates.

Reservoir Heterogeneity

To understand the impact of reservoir heterogeneity on recovery, the $M_{end} = 10$ one-dimensional solution of Fig. 1 was used to find recoveries up to two pore volumes injected for the three permeability fields shown in Fig. 11. All three permeability fields have four orders of magnitude variation in absolute permeability and differ only in the correlation length. The recoveries are shown in Fig. 12.

As in the end-point mobility case, the streamtube method is able to give some interesting insight that may have not been observable with a standard finite difference simulator. For example, the recovery curves suggest that the degree of the nonlinearity of the problem is actually dependent on the reservoir heterogeneity present. A permeability field with a very short correlation length tends to mitigate the nonlinearity (see case with $\lambda_c = 0.02$) while longer correlation lengths allow for a stronger nonlinear behavior (see case with $\lambda_c = 0.6$). Short correlation lengths promote the growth of many fingers, but no single finger is able to grow into a dominant flow channel that will characterize the recovery curve. As soon as a finger begins to grow beyond the field's correlation scale there is a high probability that it will encounter a low permeability region and slow down. Therefore, the nonlinearity is not as noticeable in the geometry of the streamtubes since all streamtubes expand and contract many times over the duration of the displacement and end up, on average, to behave like streamtubes from the unit mobility case. The correct recovery curve can be obtained by simply solving for the streamtubes once and mapping the appropriate Buckley-Leverett solution. The success of thet approach is is clearly shown by the recovery curves for $\lambda_c = 0.02$ in Fig. 12 which practically fall on top of each other.

As the correlation length increases the nonlinearity becomes more noticeable, and, in fact, for a $\lambda_c = 0.6$ the converged solution is reached at about 40 updates. The long correlation length now produces a clear path of least resistance between the inlet and outlet, which the fluid has no problem finding. A dominant flow channel is created and early

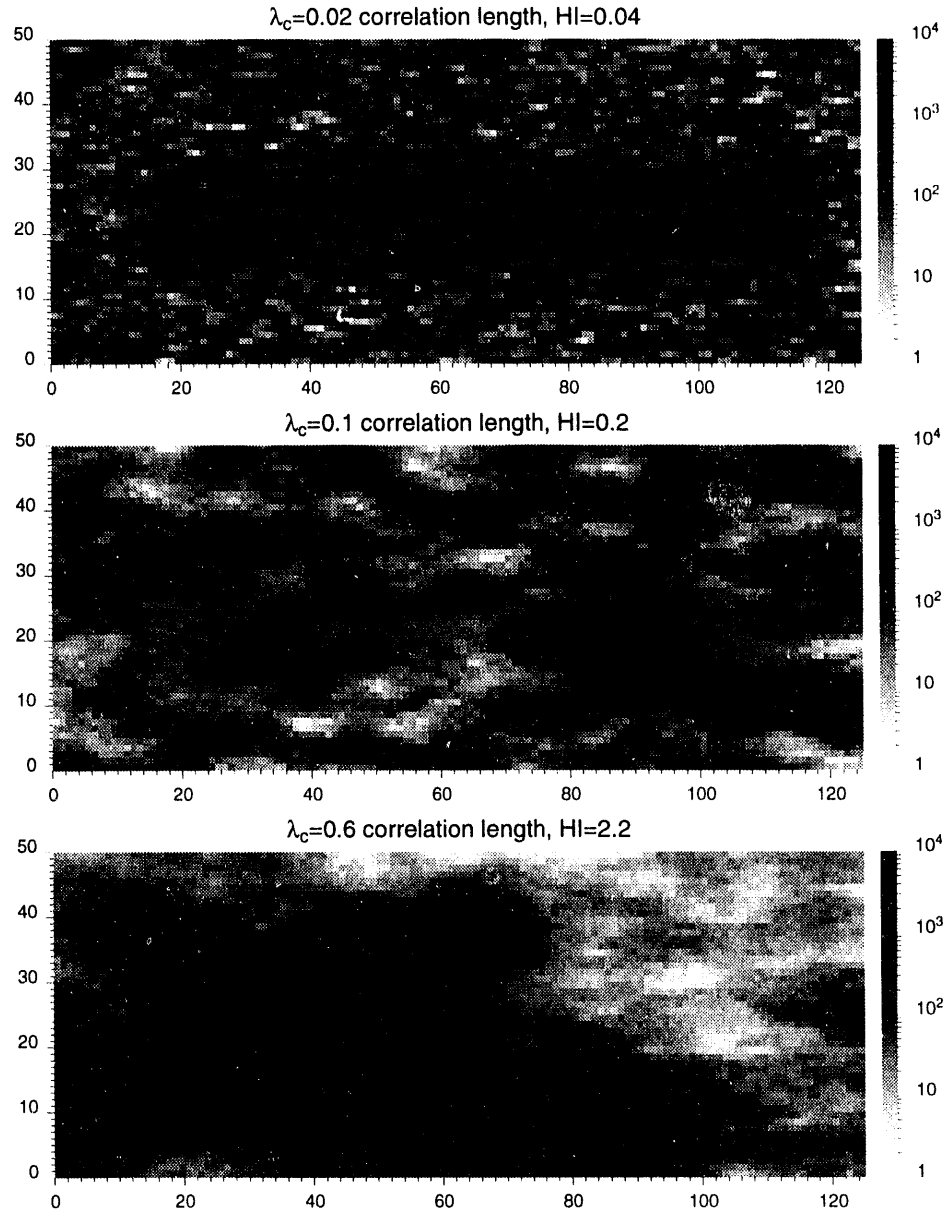

Figure 11: Permeability fields differing only in correlation length. Correlation lengths from top to bottom: 0.02 (HI=0.04), 0.1 (HI=0.2), and 0.6 (HI=2.2) of reservoir dimensions.

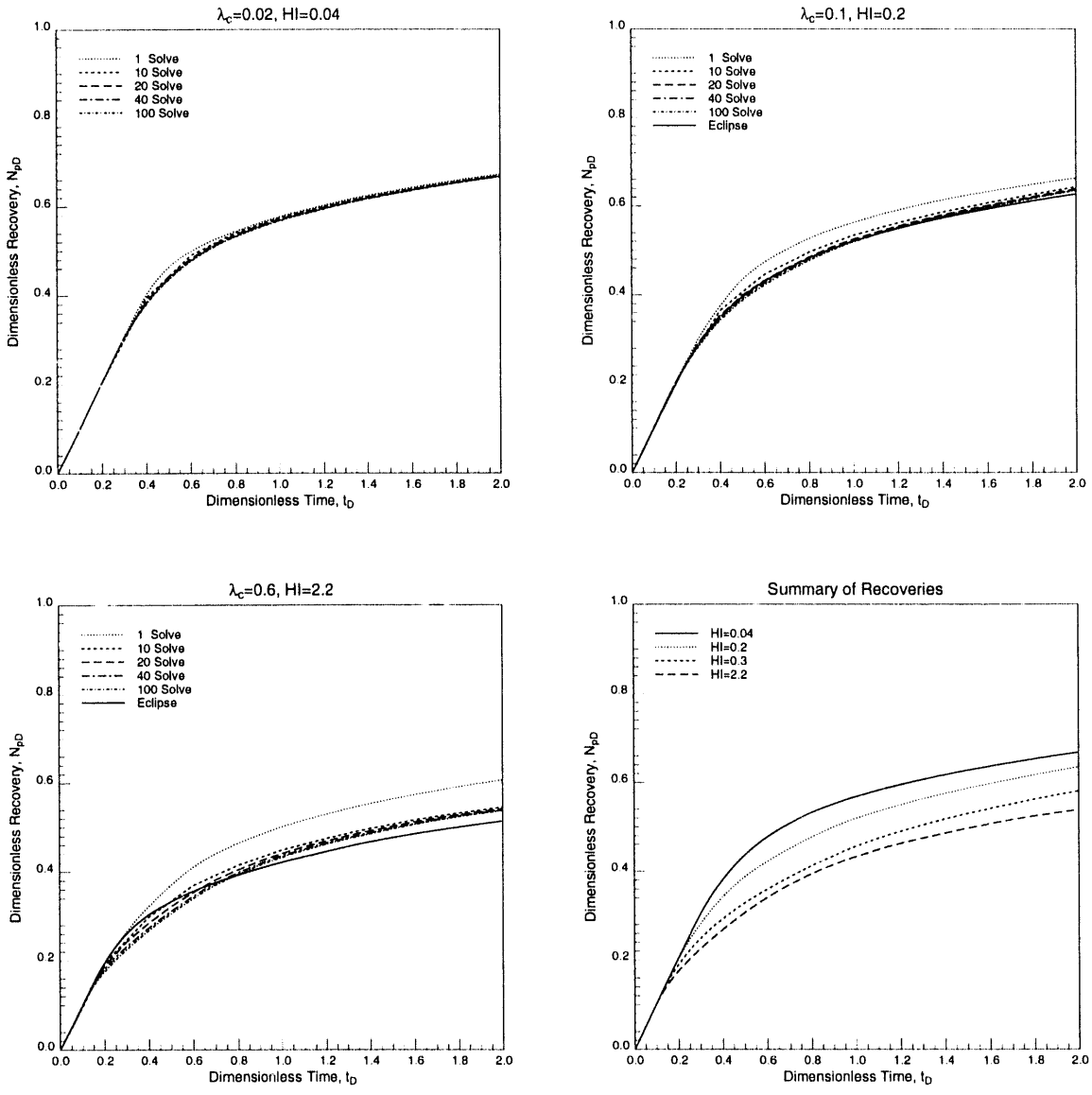

Figure 12: Recovery curves for permeability fields of Fig. 11 showing convergence of the streamtube approach and comparison with recovery curves obtained from ECLIPSE.

break through occurs. As a result, the nonlinearity of the problem is accentuated, meaning that the injected phase increases its total velocity along the path until it reaches the outlet end (this increasing total velocity *IS* the nonlinearity). Capturing this continuous increase in total velocity is, in fact, what requires the more updates of the streamtubes compared to the shorter correlation length cases. It should be emphasized at this point that 'more' is to be understood within the context of streamtubes, but still compares very favorably to 1000's of solves a traditional finite difference simulator would require.

3.4 Conclusions

(1) Two-phase problems for two-dimensional heterogeneous reservoirs can be solved efficiently by mapping a one-dimensional Buckly-Leverett solusion onto streamtubes.

(2) The Combination of Riemann solutions and streamtubes leads to solutions that are devoid of numerical dispersion.

(3) Computations with the Riemann streamtube approach are orders of magnitude faster than comparable finite difference computations.

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