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MOBILITY CONTROL FOR CO₂ INJECTION

**Final Report
May 15, 1981-January 31, 1986**

**By
John P. Heller**

August 1986

Performed Under Contract No. AC21-81MC16426

**New Mexico Petroleum Recovery Research Center
New Mexico Institute for Mining and Technology
Socorro, New Mexico**



**National Petroleum Technology Office
U.S. DEPARTMENT OF ENERGY
Tulsa, Oklahoma**

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Principal Investigator

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Prepared for
U.S. DEPARTMENT OF ENERGY
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I. EXECUTIVE SUMMARY

This report gives an overall description of a Mobility Control Test of CO₂-Foam, performed in Pennzoil's Rock Creek Field of Roane County, West Virginia. The test did not produce any oil and was unfortunately not as definitive as had been wished. In particular, it did not give an unequivocal answer to the major question that had been asked--"What is the effectiveness of such a simultaneous injection of dense CO₂ and surfactant solution in suppressing viscous instability of the displacement front in order to improve the performance and efficiency of CO₂ floods?"

Despite this relative silence with respect to the most important question, the test did yield some results which can be of use to the industry. The first of these is that it is possible to inject dense CO₂ and surfactant solution simultaneously into a regular injection well. The reduced injectivity can be interpreted as due to reduced mobility of CO₂-foam, as an injection fluid with an apparent viscosity (in this geometry) about 2 1/2 times that of water.

The second test result of some interest is that the simultaneous injection of surfactant with the CO₂ was apparently successful in retarding the formation of viscous fingers and early breakthrough of CO₂ into a producing observation well only 75 feet away. A total of 8000 reservoir barrels of dense CO₂ were injected (simultaneously with about 2500 bbl of surfactant solution), with no massive CO₂ breakthrough experienced at the observation well.

The report gives details on related research and operating projects, on the design of all features of this test, on the chronology of injection and sampling, and on the test results. In addition, three extensive appendices give descriptions of related computer calculations of expected flooding patterns, of results to be expected from the Point Dilution Method (an operational test which was not performed due to an unavoidable change in field plans) and in the distribution of tracer concentration in the formation after a period of radial flow from the injection well.

II. INTRODUCTION

Though long-term trends in the Nation's supply of petroleum are masked by major fluctuations, such as the discovery of large individual oilfields and the variations of market conditions, it was generally recognized after 1973 that the average slope was downward, and that intermittent but ever more serious shortages of liquid fuel could not be avoided in the future. To alleviate or to postpone these shortages until some advanced technology provided a solution, it became apparent that efforts to produce a larger proportion of the oil in known fields might be the most effective way to slow the decrease of reserves.

Although waterflooding has been the recognized means of secondary recovery and has been widely applied for years, it has not been practical in all fields and has everywhere displaced significantly less oil than had been retained by the formation after primary production. Thus, there was a substantial **tertiary recovery** or **enhanced oil recovery (EOR)** goal. Several distinct EOR methods have been conceived and developed in industry, university and government laboratories. EOR research performed under sponsorship of the United States Department of Energy (DOE) has become increasingly important in the last decade, with several different research projects concerned with each of the recognized EOR methods. These methods are each best adapted to a different set of field circumstances, so that in most cases the most promising EOR process could be chosen by consideration of various characteristics of the reservoir [Taber & Martin, 1984].

In the fairly common circumstances where an oil contains enough light ends [Orr & Silva, 1985], where the formation is deep enough [Heller & Taber, 1986], and where a source of CO₂ is available, best prospects for additional oil recovery could come from the use of dense CO₂ as an injection fluid. The required CO₂ density, of 35 to 55 lb/cu ft, is obtained by compressing (or maintaining) the CO₂ at a suitably high pressure. The minimum formation depth requirement is a result of the need to avoid fracturing the rock while achieving the high density. Such overpressuring would open new and undesired routes for the injected fluid that might represent irreparable damage to the formation around the injection well.

If the CO₂ density is sufficient, the displacement process develops a "miscible transition zone" during the flow, in a manner first described by Hutchinson and Braun [1961] for displacements by light hydrocarbon fluids. A "developed miscibility" front, like that from a "first contact miscible" fluid, is capable of displacing a much larger fraction of the oil than is an immiscible fluid--as much as 95% or more, as opposed to 40 to 60%. Thus, despite its higher cost, CO₂ can be a more profitable displacement fluid than water. The mechanism by which this high recovery occurs, as well as further examples in the case of oil displacement by CO₂, have been studied by a number of researchers who have considered the special case of mixtures of crude oils and CO₂ [Holm, 1959; Holm, 1976; Orr & Silva, 1983; Yellig & Metcalfe, 1980].

Unfortunately, this capability for high recovery efficiency does not usually occur in large three-dimensional structures like reservoirs, in which the flow velocity is not uniform. The high efficiencies can be demonstrated only in specially designed laboratory tests where the flow is contained in a tube of such small diameter that the average flow through the packed sand or

glass beads simulating the reservoir rock is essentially one-dimensional. While these so-called slim tube tests are invaluable for verifying the required density in a CO₂ flood, they cannot be expected to model the effects of non-uniform flow in reservoirs.

The non-uniformities in the flow velocity, which can be viewed as the fundamental cause of displacement inefficiency, are themselves the result of several factors. Not much can be done to modify the velocity variations that follow from the well-to-well nature of the flow, or that result from the natural heterogeneity of the reservoir. A third cause of flow non-uniformity and displacement inefficiency, however, is within reach of the reservoir engineer. This is the frontal instability caused by an unfavorable mobility ratio.

Whenever a less mobile fluid is displaced from a permeable medium by a more mobile one, the displacement front becomes unstable, for the simple reason that the rate of energy dissipation is lower when the front develops "fingers" than when it remains smooth and perpendicular to the average flow velocity. The dominant spacing or wavelength of the fingers, their rate of growth and their influence on other processes going on in the reservoir are the subjects of continuing study. It is apparent, though, that the growth of these instabilities is a cause of displacement inefficiency that can be reduced by lowering the mobility of the injected fluid. In principle, a further benefit that can be attained by thickening the displacement fluid is to preserve the identity of distinct zones or slugs of injected fluids. Such slugs are often required as buffer zones between incompatible fluids.

In the case of CO₂ floods, two thickening methods have been investigated in a Department of Energy sponsored research project [Heller & Taber, 1983] at the New Mexico Petroleum Recovery Research Center (PRRC). The first of these methods requires the creation of a "foam-like dispersion" of dense CO₂ and surfactant solution [Heller, Lien & Kuntamukkula, 1985]. This mixture, which can be called CO₂-foam, requires a greater pressure gradient to flow through the rock at the same rate as does CO₂ alone. The mobility reduction attained by this means is apparently dependent on both the flow rate and on characteristics of the rock. Until the flow mechanism is more fully understood, the use of CO₂-foam will require specialized laboratory studies aimed at the particular reservoir.

A second thickening method that is still under study involves the solution in the dense CO₂ of a suitable polymer, to increase its viscosity directly [Heller et al., 1985]. While this method has the potential advantage that no water needs to be introduced into the reservoir at the displacement front, it will require further research before it is ready for field use. In fact, no such "direct thickeners" are yet available for CO₂.

This report concerns a field trial of the first of these methods of thickening CO₂ for mobility control. The DOE project under which it was performed was made possible by two relevant parallel developments. One of these was the PRRC project referred to above, on the Development of Mobility Control Methods for CO₂ Floods. The other development was the previous collaborative work between the DOE and Pennzoil Exploration and Production Company, to operate a conventional CO₂ pilot flood in the Rock Creek field in Roane County, West Virginia.

These two developments are briefly described in the next two sections of this report.

III. DOE - PENNZOIL PROJECT IN THE ROCK CREEK FIELD

This oilfield was discovered in 1906 and has been produced since that time--first by primary means and sporadically in the past five decades by gas recycling. It is located in south central West Virginia, about 25 miles from Charleston. The field contains about 11,000 productive acres. Maps are shown in Fig. 1. Primary production by solution gas drive was relatively inefficient: only about 10% of the original oil in place (OOIP) was produced. Six different secondary recovery projects, utilizing three different methods, have been implemented in the field. The first and only successful project among these utilized low pressure gas recycling. That project was first installed in 1935 and was extended in 1972 when Pennzoil purchased a new portion of the field. Over the years, the recycling method has been responsible for recovering another 10% of the OOIP.

Among the unsuccessful secondary recovery attempts were three separate pilot waterfloods initiated in the '50's and '60's. None of these were able to move enough oil to be economically feasible--probably by reason of the high water saturation of about 50% and a high relative permeability to water causing correspondingly high produced water cuts. Following these failed waterfloods, a steamflood was tried in the late 60's. This was also unsuccessful, apparently because of high heat loss and low injectivity.

Pennzoil considered the possibility that CO₂ flooding might be effective in recovering some of the estimated field residual of 150 MMBO. The possible suitability of CO₂ was indicated by the reservoir characteristics shown in Table 1, particularly by the low temperature and sufficient depth of the Big Injun sand, and by the high gravity of the oil. The latter aspect was reinforced by Pennzoil's slim-tube tests and by oil analyses performed at the New Mexico Petroleum Recovery Research Center [Silva et al., 1982]. These led to the conclusion that enough pressure margin would exist for operation of a CO₂ flood above a minimum miscibility pressure (MMP) of about 1000 psi (6.9 MPa). After further reservoir study, and the selection of a suitable pilot area, Pennzoil and DOE entered into a cost-sharing agreement for a CO₂ displacement program in the area indicated on Fig. 1. This area included adjoining ten-acre, five-spot patterns and thirteen surrounding water injection wells, as shown in Fig. 2.

Initial pressurization, by water injection into the backup injection wells, was commenced in October 1976 and supplemented in April 1977, with water into the six pattern injectors. Injection history is shown in Fig. 3. Water injection was continued as field pressure rose and then limited to a rate just sufficient to maintain surface injection pressures at 1000 psi (6.9 MPa). Production over this period is shown in Fig. 4. During the latter six months of this preparation for the CO₂ flood, in the last half of 1978, there was a short-lived increase of oil production from the two center producers, from less than one BPD up to the 50 BPD range. Total oil production during this "waterflood response" was about 11,300 bbl. After both patterns were essentially watered out, CO₂ injection was started in February 1979. Original plans called for continuous CO₂ injection, but field problems and CO₂ availability caused fluctuations in rate, so that most of the CO₂ was injected during four periods, the last of which ended in February 1982. During all of this time, water was injected into the six pattern injectors when they were not taking CO₂. The production time delays--of the waterflood production kick after initial fillup,

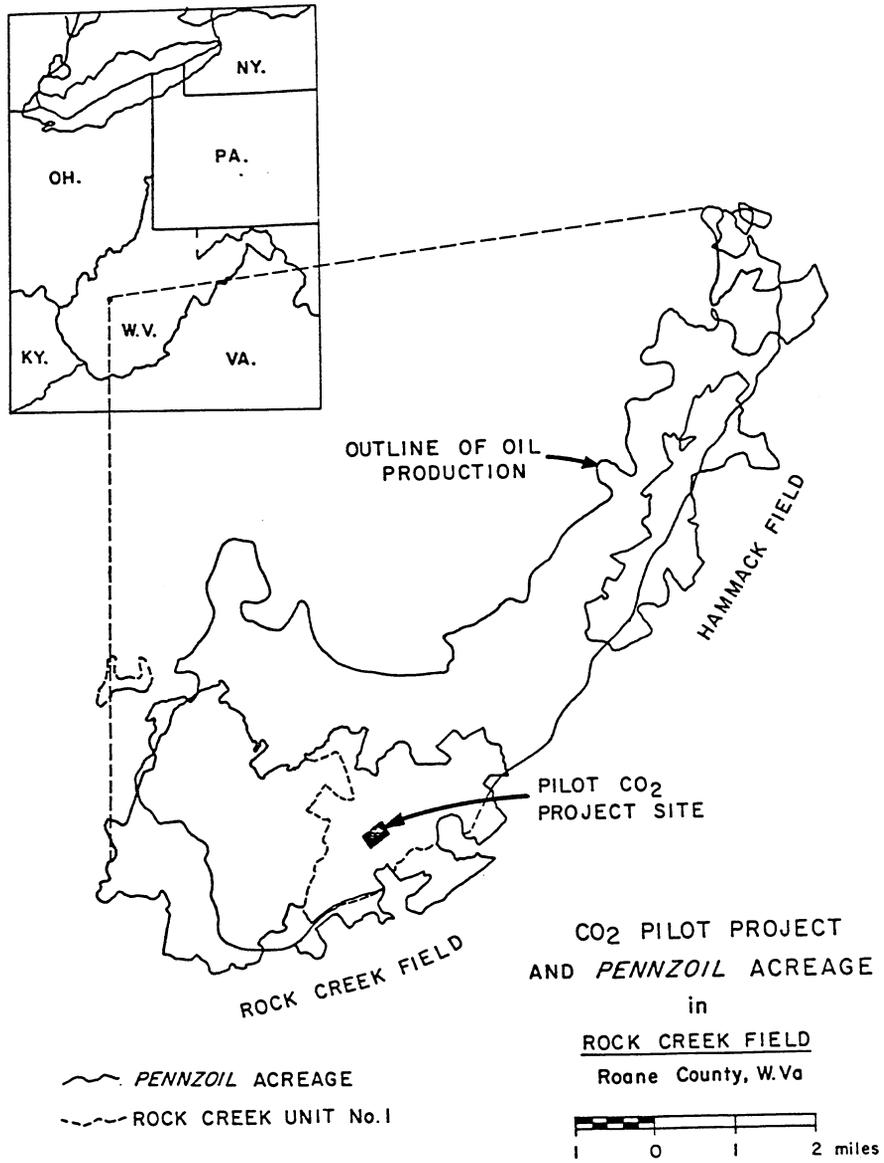


Fig. 1. Geographic location map of Rock Creek field.

TABLE 1
 ROCK CREEK CO₂ PILOT
 Roanne County, West Virginia

Big Injun

Reservoir Properties

Pilot Area, Acres	19.65
Oil Gravity, °API	43
Oil Saturation Prior to Waterflood, %	34.4
Original Oil in Place, STB	380,708*
Oil In Place, STB (as of Dec. '83)	243,426*
Oil Type	Paraffinic
Oil Viscosity, cp	3.2
Rock Type	Sandstone
Depth, ft	1975 Below Surface (-1037 from Sea Level)
Thickness, ft	24.6
Porosity, %	21.7
Temperature, °F	73
Permeability, md	21.5
Salinity of Formation Water	180,000 ppm Chlorides
Pore Volume, Res. bbl.	813,780*
Water Saturation, %	50

*Calculated on basis of thickness shown.

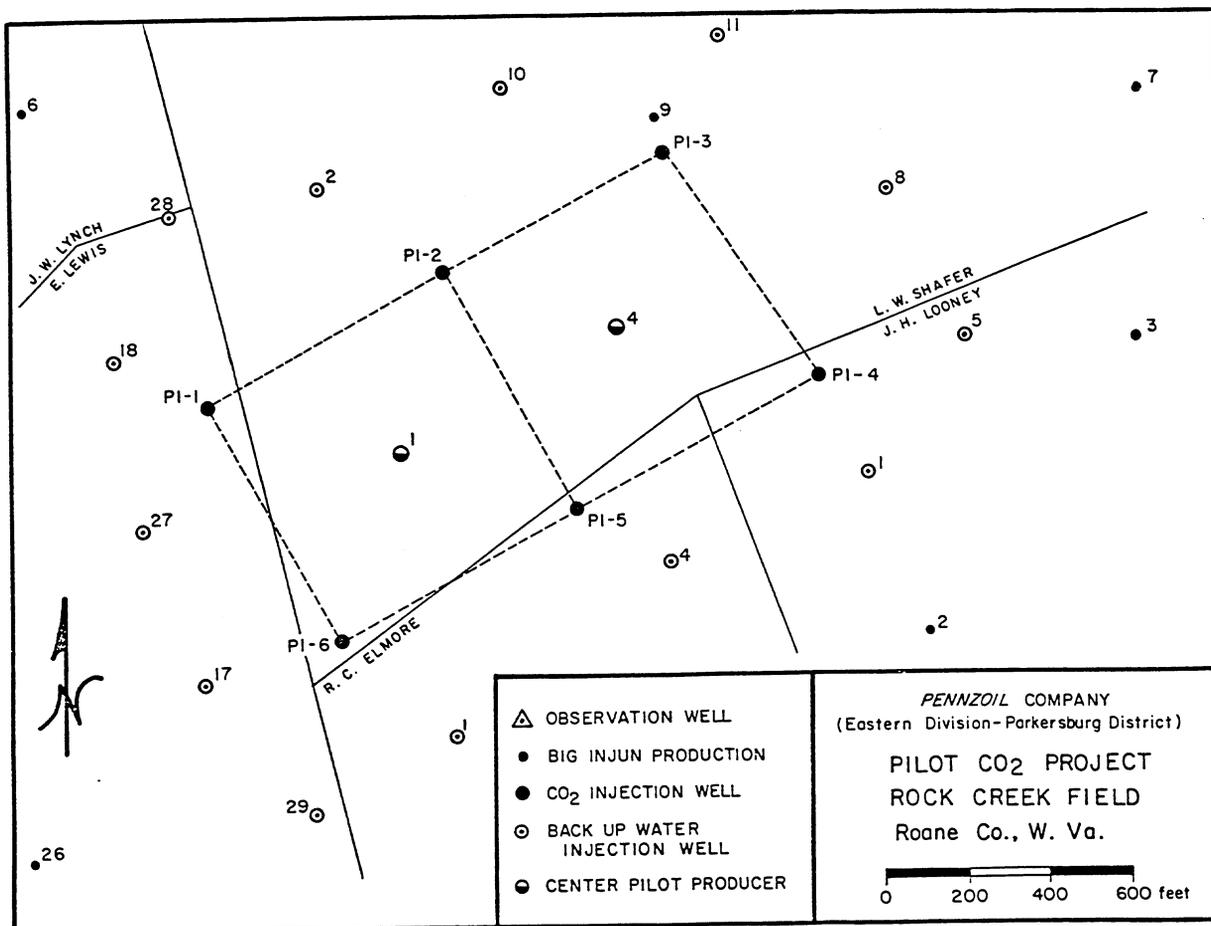


Fig. 2. Early development of preliminary CO₂ flood.

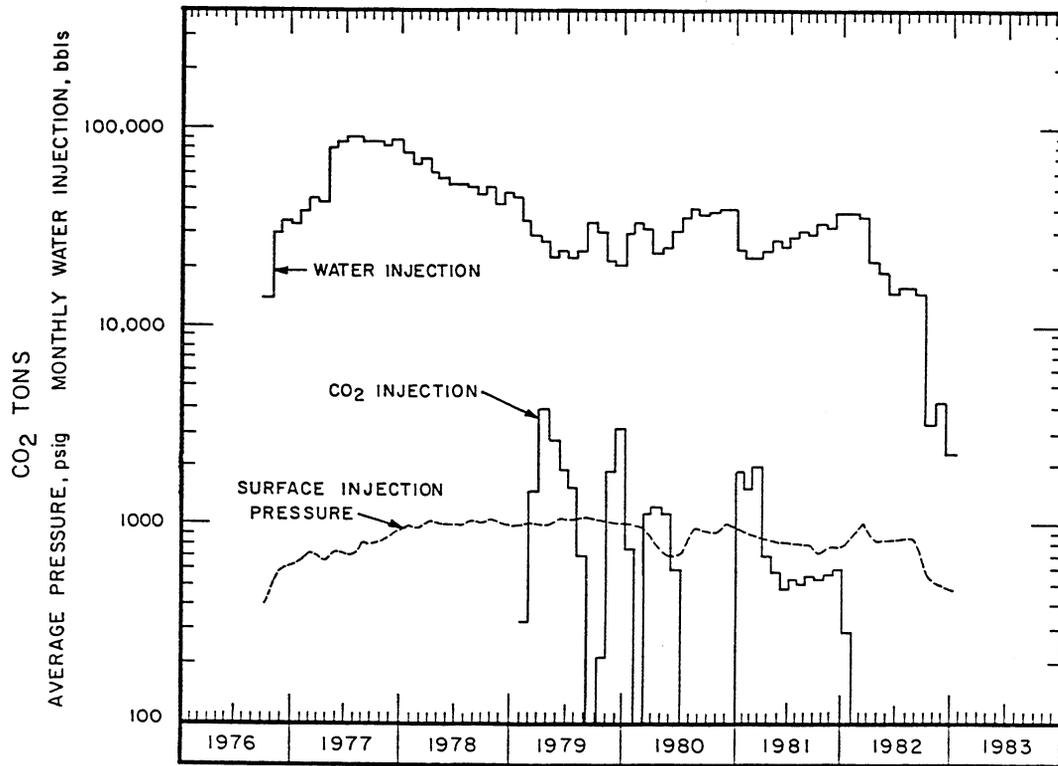


Fig. 3. Injection history of Rock Creek preliminary CO₂ flood.

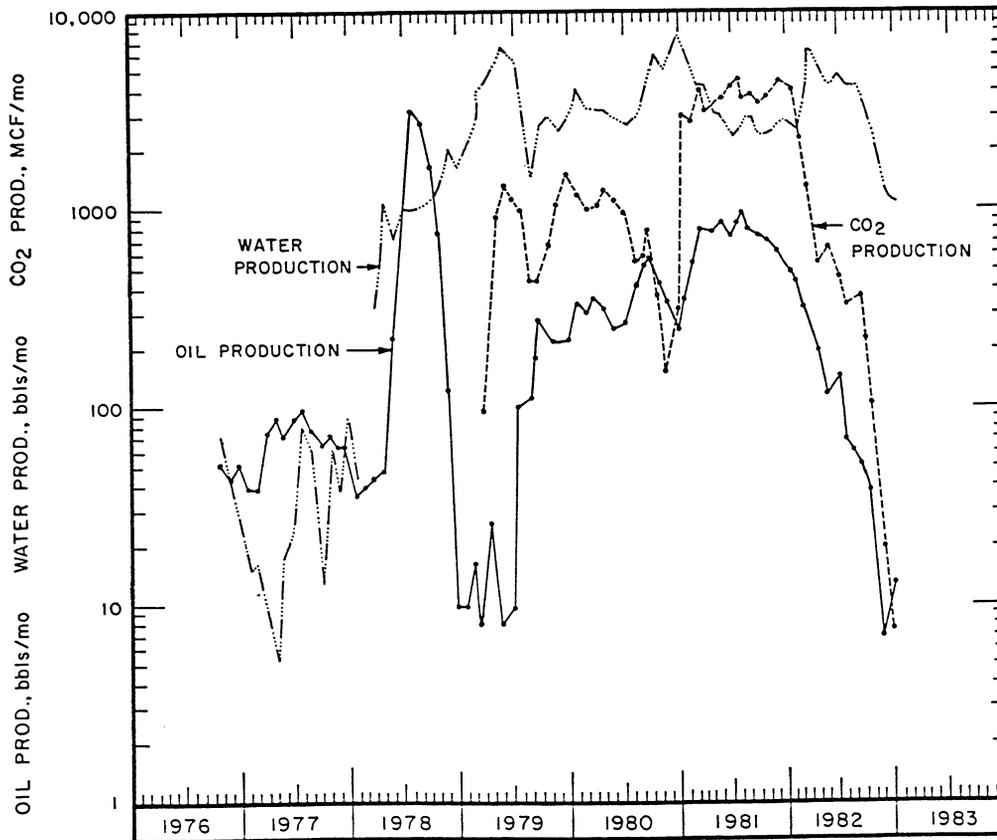


Fig. 4. Production history of Rock Creek preliminary CO₂ flood

and of the CO₂ production after start of CO₂--are evident through comparison of the graphical histories of injection and production in Figs. 3 and 4.

During the latter part of the CO₂ flood shown in Figs. 3 and 4, two new injectors (PI-7 and PI-8) and a producer (OB-1) were brought into operation. These three wells opened a "mini-pilot" region in the southeastern corner of the existing patterns (as shown in Fig. 5), and demonstrated (by a substantial quantity of additional oil recovery through OB-1) that areal sweep in the original five-spots was far from complete.

Additional evidence was also obtained concerning the effectiveness of the CO₂ flooding performed by Pennzoil in its joint project with DOE. This new information resulted from the drilling of a new well, designated OB-2, at a point 75 feet from the injector PI-2, in the direction of the producer LWS-4. OB-2 was pressure-cored through the Big Injun sandstone. This enabled most of the recovered core to be brought to the surface under pressure and frozen. This is done in order to prevent redistribution of residual oil during the trip to the surface; subsequent extraction of the core samples then is able to give a reliable measure of the oil saturation at depth.

The results of these tests are presented in Fig. 6. They show that in the upper twelve feet of the Big Injun, the oil saturation had been reduced to an average value of about 4.5% of the porespace. This is a good indication that, in selected zones at least, dense CO₂ does as good a job of oil recovery as is indicated by laboratory slim-tube tests. In the lower zone of the pay, the oil saturation remains high--though perhaps not as high as it had been left by the preliminary waterflood. The difference in permeability of the rock in the middle 12 feet from that in the upper 12 feet is not enough to explain the difference in sweep. Neither is gravity, since the probable density of the CO₂ in this region (near the injector), about 0.75 g/cc, was quite close to that of the oil.

In fact, the large difference between the oil saturations of these zones seems to be convincing evidence that the sweep by CO₂ is markedly influenced by frontal instabilities, and that mobility control measures are indeed necessary to increase the efficiency of oil recovery by CO₂.

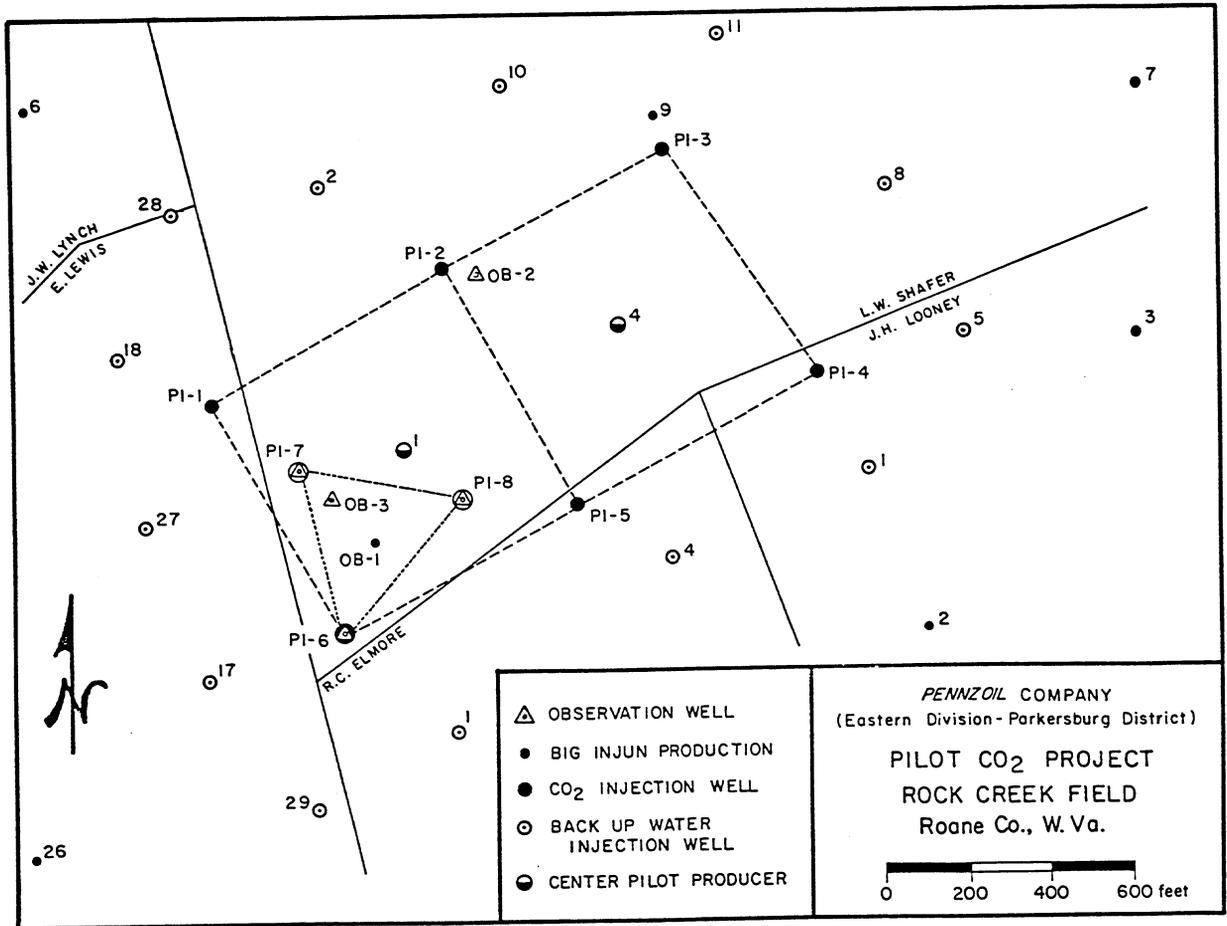


Fig. 5. Supplementary development of "Mini Pilot" area (defined by PI-7 and PI-8, and OB-1)

OB-2 PRESSURE CORE

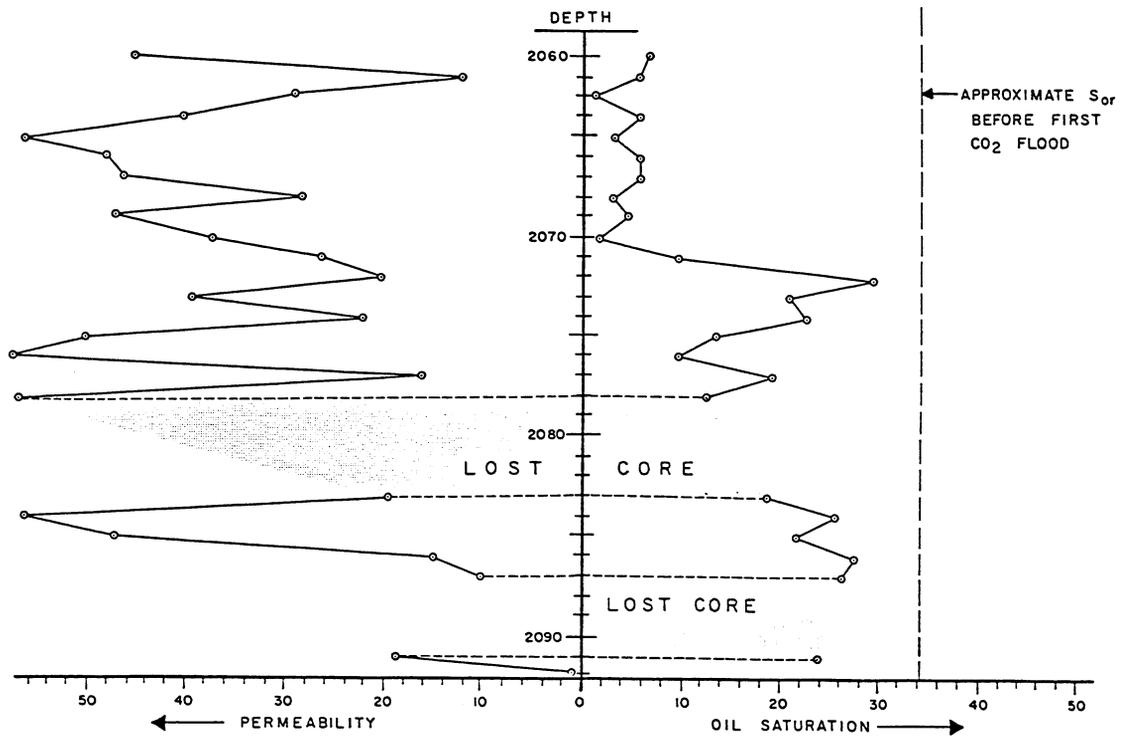


Fig. 6. Permeability and post-flood oil saturation in well OB-2.

IV. PRRC PROJECT TO DEVELOP MOBILITY CONTROL FOR CO₂ FLOODS

This previously mentioned project, entitled "Development of Mobility Control Methods to Improve Oil Recovery by CO₂", was jointly funded by the U.S. Department of Energy (under Contract No. DE-AC21-79MC10689), and by the State of New Mexico (New Mexico Energy Research and Development Institute, Contract No. EMD-3307). There was some overlap in time between work on that project and this one, and expectations that the time could be used to enhance the outcome of both projects was fulfilled. The earlier research on the development of mobility control additives had led to concepts for the use of CO₂-foam by continuous injection with a minimum surfactant usage rather than by means of a preliminary slug of more concentrated surfactant solution followed by CO₂. The interactive work also led directly to the specification of the three particular choices for surfactant to be used at Rock Creek. In addition, its close association with field problems stimulated by work on this foam injection project was beneficial in maintaining a proper motivation and direction for the earlier and more general work on the development of mobility control additives and methods in CO₂ floods.

The research project consisted of four parts. The first of these was the preparation of a comprehensive literature survey on the background, needs, and current practice of mobility control in CO₂ and related floods [Heller & Taber, 1980].

The second task of the project was the development of experimental designs, and the construction of a large core flooding test for the assessment of the influence on recovery of various developed mobility control additives and procedures. This work resulted in the development of the so-called instrumented core, an experimental technique by which detailed electrical conductivity could be measured as a function of the longitudinal dimension along the core. This was attained by the use of closely spaced voltage electrodes emplaced along one side of the core. The electrical potentials between adjacent pairs of these electrodes were measured during short pulses of current (alternating in direction) that were sent through the core from gold-plated screen-wire current electrodes at each end. From these measurements, and from similar readings across a standard resistor in the main current circuit, calculations could be made of the electrical resistance of the 6.6 mm thick of the slab of core defined by the voltage electrodes in question. These calculations, and those for each such slab, in turn, were made by a TERAK microcomputer which was also programmed to operate in sequence the proper solid-state switches for system operation. With the usual assumption of uniformity of saturation distribution within a slab, the computed electrical resistance results were translated into values of average slab conductivity. When brine composition is known, such conductivity readings are good indications of the aqueous phase core saturation. During a coreflood with CO₂, when brines with an unknown amount of dissolved CO₂ were present, the measurements were ambiguous, however.

The instrumented core also had provisions for the measurement of pressure gradient at four locations along its length. Straddling each of these four imaginary planes was a pair of pressure taps. These were 1/16th-inch stainless steel tubes that penetrated the epoxy sealing coat around the core, communicating directly with the fluids in the rock, to make possible a differential pressure measurement to indicate flowing mobility. The eight

tubes from these taps were brought to one end of the core where they emerged from the overburden pressure chamber through an end-plate bolted to a flange on the container. Outside the pressure chamber, the tubes led to four differential pressure transducers. The voltage outputs from these transducers were read by the same analog-to-digital converter that sent to the computer information from the conductivity electrodes. These arrangements for both conductivity and pressure measurements are shown in Fig. 7.

The purpose of the core instrumentation was to provide data during CO₂ floods operated both with and without mobility control additives. The measurements could be interpreted to yield the length of the transition zone during the displacement. This information would give indirect evidence of the effectiveness of the mobility control agents in the suppression of viscous fingers during the flood. Because the instrumentation was designed only to measure the changing conditions of flow along the length of the core, it was incapable of any direct measurements of the variations of flow velocity or fluid compositions in directions perpendicular to the average flow. The experimental difficulties and expense of possible three-dimensional observation methods (such as the use of computerized axial tomography) were deemed excessive, so the one-dimensional instrumented core seemed to be a useful partial solution to the assessment problem.

Unfortunately, this part of the project did not produce the kind of results desired. During the period that the instrumented core was operating, there had not yet been any mobility control additives developed. Due to the complex construction and to several poor design choices, the operation of the experiment was intermittent, with research being frequently interrupted by failure of the overburden pressure pumps or the opening of cracks in the epoxy coat. The latter accidents would allow the overburden fluid (an oil) to be forced into the core, necessitating extensive cleaning and repair. Nevertheless, work on the instrumented core did provide useful data on an unprotected CO₂ flood (one performed without mobility control), and on the moving conductivity profile during a miscible flood of one brine by another. These experiments, and details of the construction and measurement design, are described in the Second Annual Report of this project [Heller & Taber, 1981].

The third and fourth parts of the Research Project 10689 were concerned with the actual development of mobility control additives and procedures. The additives were "CO₂-foam" and "Direct Thickeners."

CO₂-foams are foam-like dispersions of dense CO₂ and surfactant solution. They are "foam-like" in that the continuous phase, the surfactant solution, is present at a low volume fraction. The major volume fraction--75% or more--is the non-aqueous and discontinuous phase which in this case is liquid or dense supercritical CO₂. In such a dispersion, the non-polar fluid is confined to bubbles or cells which are crowded together so closely that little independent motion is possible, even in a relatively large channel. In porous rock where the porespace is comparable in size with the "foam" cells, or even smaller, the flow is retarded principally by the necessity for motion of the lamellae that separate cells or packets of CO₂. Such a lamella consists of double film that encloses a thin layer of surfactant solution. The solution contains both free surfactant molecules and micelles that are hollow assemblages of oriented surfactant molecules, with their non-polar ends directed inward. These micelles represent a dynamic supply of the surfactant

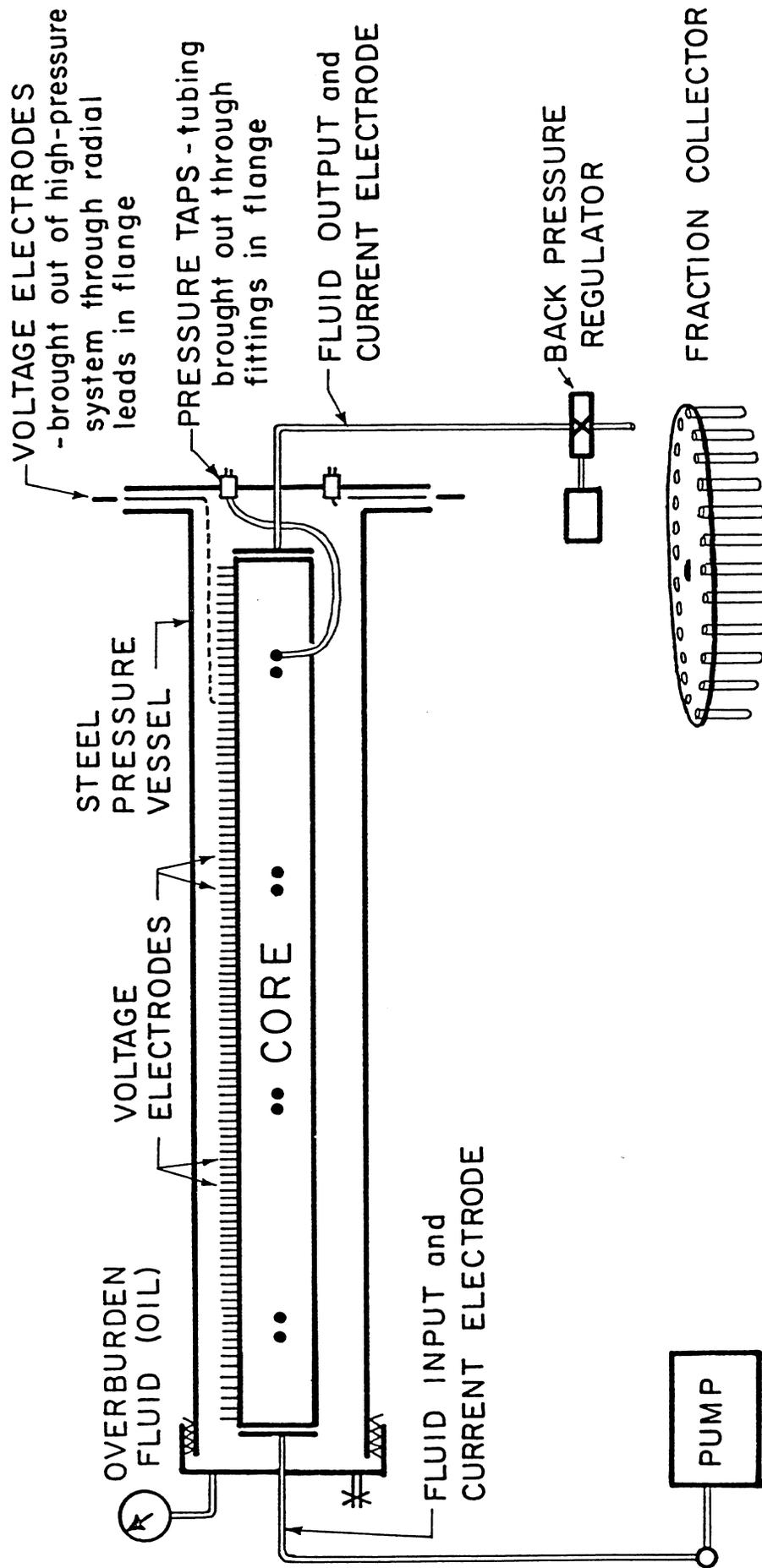


Figure 7. Instrumented core for high pressure displacements.

molecules that congregate at the water-CO₂ interfaces and stabilize the films. We presume that steady flow of foam through porous rock is accompanied by the continual motion of these lamellae through the porespace. One required property of an effective surfactant for this application is that the diffusion of surfactant molecules from within lamellae to the films should take place relatively freely. For this purpose the molecules should not be any larger than necessary, so that their diffusivity is high. Further, if they are appreciably soluble in the non-polar phase--that is, in the dense CO₂--they will not generally be available for use at the surfaces.

There are other required properties for the surfactants. Most of the requirements cannot be stated from first principles in ways that permit unequivocal selection of useful surfactants. Consequently, we are led to "screening tests" based not on general inquiry into the molecular structure of the surfactant, but rather on empirical measures of some specific properties.

Thus, for this part of the project, a screening test was developed to examine the quantity of foam-like dispersion that could be produced between isooctane and a surfactant solution of standardized concentration. Isooctane was used as an atmospheric pressure liquid that simulated the rather poor but significant solvent power of dense CO₂. A standard quantity of surfactant solution was placed in a small tube, which was then filled to overflowing with isooctane and capped so as to exclude air. Various standardized shaking procedures have been utilized, by which different surfactants could be compared under closely controlled conditions. Descriptions of the evolution of this test during Project 10689, and of the results of these "shaking tests" with a number of surfactants, are contained in the Final Report of the project [Heller & Taber, 1983]. Since that time, there has been further development of this test to provide better controlled standardization of the shaking procedure. The current form of the test is described in a report dated January 1985 from a new DOE project, "Improvement of CO₂ Flood Performance" that is partially supported by the DOE under Cooperative Agreement 21136.

Another required property of surfactants for usefulness as stabilizers of CO₂-foam would be low adsorption onto reservoir rock, and again there has seemed to be no better way to determine this suitability for the application than to measure the amount adsorbed under some standard conditions, using reservoir rock and brine.

These adsorption tests have been conducted by flowing reservoir brine through a core sample and by performing a miscible flood with a short slug of surfactant solution. This square-topped peak is introduced by operating a liquid chromatographic injection valve, putting a known amount of surfactant solution (contained in a sample loop of the valve), into the input tubing from where it is immediately carried into the core. This test makes available two different measures of adsorption, both of which can be utilized directly in design of a field operation with CO₂-foam. The test gives a measurement of the total amount of surfactant solution adsorbed on the surfaces of the rock, by comparison of the integrated quantities at the input and the output of the core. The difference can be scaled up directly to field operations to predict the quantity of surfactant that will be adsorbed by the amount of reservoir rock with which the surfactant solution will come into contact. This might be considered the total, or "irreversible" adsorption that occurs. A second measure, of the quantity that can be both adsorbed and quickly desorbed again,

is obtained by measuring the chromatographic delay in the appearance of the surfactant solution peak beyond the time of arrival which would be observed for a non-adsorbing tracer material. The amount of this delay is itself a useful quantity to measure, for the purpose of designing a field application of CO₂-foam. Details of the procedure for calculation were given in a paper presented at the Tulsa DOE/SPE Symposium on EOR in 1984 [Heller, 1984]. One further aspect of CO₂-foam mobility control research, that was investigated during the more general project, was the extent to which the mobility of CO₂ is reduced by the foam lamellae. This was measured in a steady-state experiment that simulates the flow of foam in the regions of the reservoir from which most oil has been displaced. It was envisaged that in an effective, mobility-controlled CO₂ flood, frontal instability would be greatly reduced. In this case the proven ability of CO₂ to displace most of the oil by building a Hutchinson-Braun transition zone would cause the formation of an oil bank. The bank would then grow by continual accretion of residual oil and be displaced ahead of the CO₂-foam. Large-scale finger growth would depend on the ratio between the mobilities of oil ahead of the displacement zone and of the CO₂-foam in the region far behind the front.

Following this rationale, it has seemed clear that the critical data needed to evaluate the effectiveness of CO₂-foam made with a particular surfactant would be that obtained from steady-state measurements of foam mobility. The latter is simply the ratio of the superficial flow velocity attained by CO₂-foam to the pressure gradient along the core sample. Accordingly, a large part of the CO₂-foam mobility control effort has been devoted to the development of data and experimental methods to obtain steady-state mobility measurements. In these experiments, generated CO₂-foam is forced through a core sample at a measured rate, and after a steady flow condition is attained, the pressure drop across the core is measured. Then the pump rates of CO₂ and surfactant solution are each changed by the same ratio and the experiment repeated. The reason for performing the measurement at several different total flow rates is that the mobility depends on the rate.

The experiments performed in this study have shown that foam mobility is not independent of flow rate, but rather that it increases with increasing rate. This "shear thinning" behavior is beneficial in this application. The benefit is that near the injection wellbore, where the rate is necessarily high, the CO₂-foam acts like a lower viscosity fluid--the pressure gradient is not as high as it would be for a less mobile fluid. But lowered mobility is available further out from the well, and nearer to the displacement front, in the region where such lowered mobility is needed to control the frontal instability that causes fingers. To study this behavior more closely, it is obviously important that the mobility measurements be operable at low flow rates such as those found in the majority of the reservoir area.

Performing these experiments at very low rates is especially time-consuming and difficult, however, for several reasons. Resolutions of all of the related questions was not completed in the project under review; in fact, several of these questions are still under investigation. Descriptions of the problems involved in these measurements, and of the experimental work on them, are contained in the two technical reports mentioned above [Heller & Taber, 1981; Heller & Taber, 1983] and also in reports of the current PRRC project, "Improvement of CO₂ Flood Performance."

The fourth subject studied in the Project 10689, the "Development of Mobility Control Methods to Improve Oil Recovery by CO₂," was the possibility of finding polymeric materials which could somehow be used to thicken dense CO₂. It had been realized early in the project that under the reservoir conditions in which CO₂ can displace oil efficiently, dense CO₂ can also serve as a non-polar solvent of other materials. Its solvent power depends strongly on its density. This dependence is, in fact, the reason for the increase of CO₂ flooding efficiency at higher pressures. Even at temperatures above the critical point of CO₂ at 88°F, CO₂ can be an effective solvent if it is held at sufficient pressure to keep its density in the range from 0.5 to 0.9 g/cc.

The first task of this part of the project was thus to measure the solubility of commercially available polymers in dense CO₂ and the viscosity of the solutions. This work required the development of new laboratory apparatus to enable these measurements to be made. Because of the high pressure required, and the lack of an interface by which to track fluid motion, acquisition of such data was an experimentally unusual problem that demanded the design and construction of specialized apparatus. A device was built by which to measure both solubility and, with modifications, viscosity. This apparatus is described in the second annual report [Heller & Taber, 1981] and in an SPE paper [Heller, Dandge, Card & Donaruma, 1985].

In brief, the result of this part of the project was that no commercially available polymers are suitable as direct thickeners. The ones that exhibit high enough solubility in dense CO₂ (up to 0.5% or so could be economically acceptable) are generally of low molecular weight and do not viscosify the solution nearly enough to be useful.

On the other hand, the results were also encouraging in the following way. In the course of the investigations, several specific structural attributes of hydrocarbon molecules and polymers were identified, which lead to high solubility in CO₂. It was furthermore realized that little exploration in these indicated directions had been done by polymer manufacturers and investigators. This, of course, was not surprising--most commercial polymers are designed (or selected) to meet quite different criteria than those of interest in this project. It, thus, seemed reasonable to proceed on a program of synthesis, in which new polymers would be made and tested for possible application as direct thickeners. This is the direction that has been taken in the further work that has been done, since the completion of Project 10689, in a continued search for direct thickeners.

This section has been a brief review of the work done in the more general, "Development of Methods..." project. It has been discussed here because this project, like that discussed in Section III, was an important element of the background for the Rock Creek Mobility Control Test.

V. DESIGN OF THE MOBILITY CONTROL INJECTION PROJECT

As has been pointed out in the previous section, two different methods for CO₂ mobility control were under development in the general project, '10689. Soon after the start of the project '16426, however, it became apparent that no suitable direct thickening polymer was available commercially, or could be developed in time to be of use in the Rock Creek injection project. On the other hand, there were many surfactants that were obtainable on the market and might be suitable to stabilize CO₂-foam--especially at the low temperature (75°F) of the Rock Creek field.

Laboratory Activities

It was thus decided that the work to be performed on '16426 should concentrate on the selection of an appropriate surfactant, on the testing of its needed properties for use in CO₂-foam in the Rock Creek sandstone, and on the design of the operating procedures for that use. As has been discussed previously [Heller, Lien & Kuntamukkula, 1985; Heller, 1984; Heller, Boone & Watts, 1986], the choice of surfactant was based on several testing procedures:

- 1) A "screening" test to determine the ability of the surfactant to form a relatively durable "foam-like dispersion" with isooctane (used to simulate the solvent properties of dense CO₂ in this atmospheric pressure test).
- 2) A dynamic adsorption test to determine two parameters involved in the use of the surfactant in the field. This laboratory test measures both the amount of surfactant adsorbed irreversibly on the rock, and the magnitude of the chromatographic delay to be expected.
- 3) A set of laboratory measurements of foam mobility--the ratio of foam flow rate per unit cross-sectional area, to the pressure gradient. Because the mobility varies with flow rate, a number of such measurements are required,

The above laboratory work was a necessary preparation for the field injection program, to provide reasonable assurance that the flow behavior in the field would be as expected. For this purpose, those reservoir conditions deemed most important were duplicated in the tests. Both the adsorption and the foam mobility measurements were made with core material from the Big Injun sandstone formation of the Rock Creek field, and utilized a synthetic brine for the aqueous phase. Although the natural formation water from the Big Injun had very high salt content, the field had been flushed with virtually fresh water for a considerable time. The make-up water to be used in the CO₂-foam would also be very low in total dissolved solids. A compromise composition was thus chosen for the synthetic brine used in the tests. It contained one half weight percent each of NaCl and CaCl₂.

The mobility measurements were performed at reservoir pressures and were operated under steady input conditions to simulate the flow of CO₂-foam through the already swept region of the reservoir. These measurements resulted in the choice of a low surfactant concentration, 0.05% by weight, as sufficient to cause a lowered mobility of the CO₂-foam in the sandstone of the Rock Creek field. The maintenance of this level in the face of adsorption was also

considered.

Of the surfactants screened for their ability to form a suitable foam-like dispersion, three also showed promise for use in this project by virtue of their lower adsorption on Rock Creek core material. These were Alipal CD-128 and Neodol 25-3A (which are both anionic surfactants) and Monateric ADFA (a "zwitterionic"). This is not to say that the requirements of the project could not be met by other foamer-type surfactants; even among the others tested, there might be a superior material for the purpose. Considering the a priori standards that had been set up, and the particular surfactants which had been tested by then, these three appeared to be most likely to be suitable.

In the dynamic adsorption tests, two parameters were measured. The first was the velocity ratio, r , the factor by which the flow time prior to the appearance of a surfactant peak was increased over that for a non-adsorbing tracer, and the second was M , the amount of surfactant adsorbed "irreversibly" on the rock within the time-frame of a reservoir-rate flow. These values are shown in Table 2.

Table 2

<u>Surfactant</u>	<u>Velocity Ratio, r</u>	<u>M lbs surfactant/bulk cubic feet of rock</u>
Alipal CD-128	0.91	5.6×10^{-4}
Monateric ADFA	0.91	8.7×10^{-4}
Neodol 25-3	0.95	9.8×10^{-4}

According to the measurements given above, any one of the three surfactants would be suitable for use at Rock Creek. The amount of surfactant adsorption, M , is low enough that the surfactant cost would be considerably less than that of the CO_2 . The chromatographic delay resulting even from this relatively low adsorption rate was disturbing, however. It would suggest that the surfactant would be stripped from the leading edge of a CO_2 -foam slug--from precisely where it was needed most. This would leave unprotected a growing quantity of CO_2 that could then establish fingers or channels of higher flow rate.

Surfactant Pad Design

To counter this chromatographic delay, it was proposed to use two distinct periods of surfactant injection. In the first period, surfactant solution (without any CO_2) would be injected with the purpose of supplying sufficient surfactant to the rock to satisfy its measured adsorption appetite. This "surfactant pad" would contain $M \cdot B$, the quantity of surfactant that the laboratory tests had shown could be irreversibly adsorbed on the bulk volume of rock contained in the region of the reservoir to be swept. This amount of surfactant should be contained in the volume of aqueous fluid making up the surfactant pad. The actual volume to use for carrying this amount of surfactant is not arbitrary, but should be sufficient to satisfy a second purpose. This requirement is that the pad should remain distinct during its displacement out to the limit of the designated region--that the front and rear boundaries of the pad region should remain separate. This can be estimated by

considering the speed of the different displacement fronts.

A general formula for these velocities can be obtained from material balance of fluids in the rock, and from the assumption that for the time and distance scales considered, the width of the "frontal region" is small and relatively unchanging as the displacement proceeds. We may under this condition consider two successive locations of the front, separated from each other in distance by Δx and in time by Δt . Two different expressions are derived by mass balance for the volume of fluid involved.

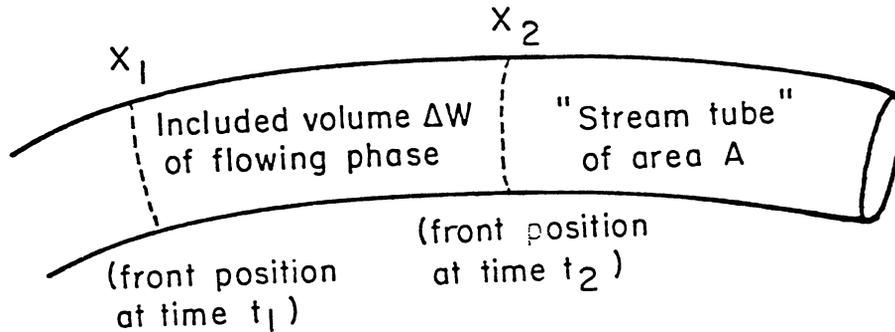


Figure 8

Two successive locations of front, at x_1 and x_2

The first of these equations, for the change in quantity of fluid ΔW contained between two successive positions of the front, is simply that this change must equal the difference between the quantity there before passage of the front and the quantity present afterwards:

$$\Delta W = \phi A \Delta x (S_{\text{before}} - S_{\text{after}}) \quad (1)$$

$$\Delta W = \phi A \Delta x \Delta S$$

An independent expression for the change in the amount of fluid in the region can be obtained by considering the flow into the region. Upstream of the frontal boundary, the fractional flow of the fluid being considered is f_u , and downstream of the boundary the fractional flow is f_d . Then in the time during which the front moves through the distance Δx , the net quantity of fluid brought into the volume must be

$$\Delta W = Q \Delta t (f_u - f_d) \quad (2)$$

$$\Delta W = Q \Delta t \Delta f$$

Equating these two expressions for ΔW yields the following equation for the velocity of the frontal region considered, V :

$$V = \frac{\Delta x}{\Delta t} = \frac{Q}{\phi A} \cdot \frac{\Delta f}{\Delta S} \quad (3)$$

This gives the velocity of a front between two fluid-filled regions in a porous rock that differ from each other in saturation of a particular phase by ΔS and in fractional flow of the same phase by Δf . Q/A is, of course, the total Darcy velocity, best obtained by evaluating Q and A at the injection or production well face where they are known, and correcting for any change of fluid density--since these are all volumetric measurements. The front in question may become broader in time, so that the velocity given by the equation must be regarded as that of some average position centered in a transition zone of increasing width. On the other hand, the front may be to some extent self-sharpening, in which case this "average position" can be equated to the position of the central isosaturation line of the front itself.

Although the equation was derived in terms of volume, it can be used for compressible fluids like CO_2 so long as proper local values of saturation and fractional flow are used in it.

The equation may also be applied to miscible displacement fronts, in which case it is almost trivial. In a miscible displacement no residual is left, and both ΔS and Δf are equal to one. V then becomes the displacement velocity of the 50% isoconcentration surface. Of course, the equation must be modified, if there is adsorption behavior, to take into account in the mass balance the quantity of a tracer adsorbed onto the porespace walls. In that case, the velocity of a miscible front marking a change in the concentration of that tracer will be reduced by a factor r . This is given by the equation:

$$r = 1(1+M'/\phi) \quad (4)$$

In this equation, M' is the derivative with respect to tracer concentration of the function $M_{(C)}$, that represents the mass of adsorbed tracer on the pore walls of a unit bulk volume of the rock.

Eq. 3 has been applied to the problem of calculating pad volume in the following way. In the field as in the laboratory, the front of the surfactant pad will move more slowly because of chromatographic adsorption. In the field it also travels through a rock that has a residual oil saturation, S_{ro} , and thus has an "effective porosity" of $\phi(1-S_{ro})$. Then the velocity of the front of the pad is:

$$V_{\text{pad front}} = \frac{Q}{\phi A} \cdot \frac{r}{1-S_{ro}} \quad (5)$$

At the rear boundary of the surfactant pad, where the oil bank is replaced by the CO_2 -foam, the velocity may be conveniently calculated by considering the changes in saturation and fractional flow of the oil. The saturation change of the oil is from $(1-S_{iw})$ in the oil bank to S_{roc} , which is close to zero, in the CO_2 -foam-swept region. The change in fractional flow on either side of the front is from unity (only oil flows in the oil bank) to zero. Thus, Eq. 3 gives

$$V_{\text{foam front}} = \frac{Q}{\phi A} \cdot \frac{1}{(1-S_{iw}-S_{roc})} \quad (6)$$

The frontal velocities of these and the other fronts, symbol definition and an idealized sketch of the succession of displacement fronts, are shown in Fig. 9. In the formulas, the symbol A represents the lateral area of the front, perpendicular to its direction of motion. Thus for instance in a radial geometry, the area A is $2\pi Rh$, and the actual front velocity decreases as $1/R$ with radial distance from the injection well.

It is also to be noted that the surfactant pad front velocity given in Eq. 5 may be less than the foam front velocity given in Eq. 6. In this case, the equations would therefore predict that the foam front would overtake the pad front after travelling some critical distance from the injection well. The distance would be further out from the injector, if a larger volume of surfactant pad fluid were injected prior to the CO_2 -foam, or if the excess of the foam front velocity over the pad front velocity were smaller. The needed volume of the surfactant pad can be approximated by the means used in the Topical Report, "Proposed Injection Design for Mobility Control of CO_2 in Rock Creek Field," [Heller, 1983]. In the case where $V_{\text{foam front}} > V_{\text{pad front}}$, one might specify that the foam front should just overtake the pad front at a "target" radial distance R_w . This distance is related to the water-filled pore volume W by $W = \pi R_w^2 \phi (1-S_{ro})h$. For the pad front to overtake the foam front at this radial distance, the travel time of the foam front from the injection well radius out to R_w should equal the travel time of the pad front over the shorter distance from its initial position after injection of the pad, out to R_w . These travel times can be calculated as the integrals $\int dR/V$ over the appropriate radial distances and with the velocities V as given in Eqs. 5 and 6. By equating the two travel times, an expression is obtained for the needed volume of pad fluid:

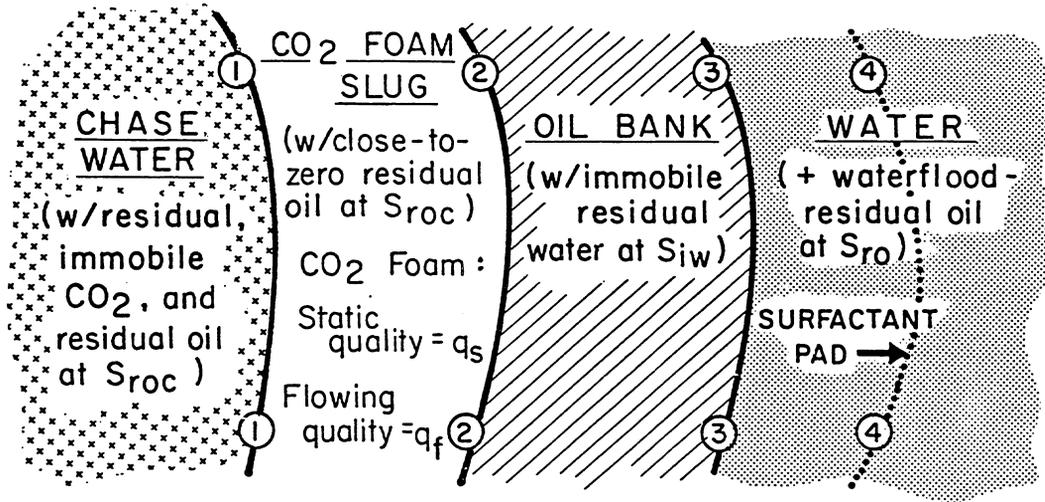
$$W_{\text{pad}} = W[1 - r(1-S_{iw}-S_{roc})/(1-S_{ro})] \quad (7)$$

As is apparent from the formula, the surfactant pad volume called for is only greater than zero if the reversible adsorption is so high that the velocity ratio r is less than a critical value r_c :

$$r_c = \frac{1 - S_{ro}}{(1-S_{iw}-S_{roc})} \quad (7a)$$

In many cases--Rock Creek included--the values of the various residual saturations are not well known. In this case r_c cannot be estimated with assurance. A minimum value for the needed surfactant pad volume can be calculated by the assumption that the waterflood residual oil is equal to the sum of the residual water saturation in the oil bank, and the residual oil left after a stable displacement by CO_2 -foam. The assumption is that $S_{ro} = S_{iw} + S_{roc}$. In this case, r_c will be unity, and the specified pad volume becomes

$$W_{\text{pad}} = W(1-r) \quad (7b)$$



$$\textcircled{1} \quad v_{\text{chase water front}} = \frac{Q}{QA} \cdot \frac{q_f}{(q_s - S_{roc})}$$

$$\textcircled{2} \quad v_{\text{foam front}} = \frac{Q}{QA} \cdot \frac{i}{1 - S_{iw} - S_{roc}}$$

$$\textcircled{3} \quad v_{\text{oil bank front}} = \frac{Q}{QA} \cdot \frac{1}{(1 - S_{iw} - S_{ro})}$$

$$\textcircled{4} \quad v_{\text{surfactant pad front}} = \frac{Q}{QA} \cdot \frac{r}{(1 - S_{ro})}$$

Fig. 9 Idealized succession of displacement fronts (with theoretical front velocities).

Even if this formula may call for a larger volume than that required by the velocity criterion, it will specify a positive minimum pad volume to contain the needed extra surfactant to satisfy the "irreversible" adsorption.

If the target pore volume W is measured in barrels, the bulk volume of rock associated with it, in cubic feet, is then

$$(5.6145) W / (\phi(1-S_{ro})) \text{ ft}^3$$

Multiplying this by the value of M lb/cu ft. obtained in the laboratory tests, the additional amount of surfactant needed for irreversible adsorption onto this much rock is then

$$(5.6145) MW / (\phi(1-S_{ro})) \text{ lb}$$

Thus the surfactant pad should contain the sum of this much surfactant in an aqueous volume given by Eq. 7b, plus that amount required to increase the concentration by the 0.05% determined by the steady state CO_2 -foam mobility test. The total concentration of surfactant in the given volume of the pad can then be calculated from:

$$C_{\text{pad}} = 0.0005 + 0.0160 M / (\phi(1-S_{ro})(1-r)) \text{ lb/lb} \quad (8)$$

(The factor .016 brings the second term in the parentheses into the same units as the first; it is the ratio of 5.6145 cu ft/bbl to 350.4, the density of water in lb/bbl).

The formula of Eq. 8 gives the concentration of surfactant in the pad, leaving the quantity to be used as a function of the volume of reservoir to be swept in the test. A second quantity that depends on the target volume selected is the volume of the CO_2 -foam "slug." How large a volume is needed to displace mobility-controlled CO_2 out to the same target volume W is determined by the mechanisms of decay of the slug, and on how active they will be in a given reservoir.

Continuing the view in which the CO_2 -foam slug is displaced stably, and so remains confined between fairly regular frontal boundaries, one can seek the rates at which the slug is eroded at both its frontward and rearward boundaries. At the leading edge, the CO_2 -foam presumably displaces the oil bank almost completely. A Hutchinson-Braun transition zone forms at this front, which expands slowly by longitudinal dispersion. In the absence of viscous instability, which is presumably suppressed by the use of CO_2 only in the lowered mobility CO_2 -foam, the front of the slug suffers only moderate degradation. At the back end, however, there is an entirely different situation. The displacement of CO_2 -foam by water is an immiscible displacement, and leaves behind a residual, which in this case is CO_2 . Consequently the CO_2 -foam slug can be expected to lose volume as it is displaced by chase water. One way to calculate the loss is given in the Topical Report [Heller, 1983] mentioned previously, by a simple balance based on the idea that after the displacement of the oil bank out to the target volume, V , there will still be a small, marginal slug of CO_2 -foam remaining between the oil bank and the chase water. If the marginal volume remaining is a fraction, g , of the original slug, if the slug itself was a fraction, s , of the target pore volume, W , and if S_{CO_2} is the residual CO_2 that remains in the

regions from which chase water has displaced "CO₂-foam," then

$$sq_s V = q_s g_s V + (1-sg) S_{CO_2} V \quad (9a)$$

Here the fractional CO₂ content of the CO₂-foam--its "static quality"--is denoted by q_s. This fraction may of course be somewhat different from the "flowing quality" q_f as noted on Fig. 9. Simplifying Eq. 9a, one obtains:

$$s = S_{CO_2} / (q_s - g(q_s - S_{CO_2})) \quad (9b)$$

This is the fractional slug size needed to provide a margin against the disappearance of the CO₂-foam by the time the target volume is reached. The static quality of the foam is also important in determining the needed slug size. In the Topical Report [Heller, 1983], it was suggested that a reasonable slug design could be based on

$$q_s = 0.8$$

$$S_{CO_2} = 0.3$$

and the safety fraction g by:

$$g = 0.1 \quad ,$$

then

$$s = 0.4$$

Cutting the margin (that is, the fraction, g) to zero would enable the slug size in the above case to be cut to 27%, but would seem risky.

The above paragraphs give the basis for the design of the mobility control trial at the Rock Creek Field. In summary, it was recommended that as a lowered-mobility displacing fluid, a "foam" consisting of dense CO₂ and an 0.05 wt percent, aqueous solution of surfactant be injected simultaneously in the attempt to sweep out more oil from a portion of this reservoir. Because of adsorption of the surfactant on the rock, a properly sized, sacrificial pre-injection of surfactant solution or "surfactant pad" was also called for. The recommended size of this pad was calculated by the techniques given above. Design criteria were also given for the volume of CO₂-foam needed in the displacement slug.

In addition to those aspects of the design discussed above, other matters more directly connected to the field operations were also to be considered. These included the planned injection facilities and location, and the means for assessment and interpretation of the results of the test.

Test Plans for the Mini-Pilot Area

Since the beginning of the project, it had been assumed that the mobility control test would be performed in the "mini-pilot" area --the triangular area defined by PI-6, PI-7 and PI-8. Roughly in the center of this area was a well, OB-1, that had been drilled for observations, but which had been converted to

use as a producer. It had, in fact, produced approximately 4000 bbl of oil during the earlier trials of CO₂ flooding in the field. Halfway between PI-7 and OB-1, another observation well had been drilled. This was OB-3, which was completed openhole for use as an observation well during the mobility control test, in the manner described below.

With the development of the overall CO₂-foam injection plans described above and in the Topical Report [Heller, 1983], detailed procedures for field operations and assessment means were in order.

In this mini-pilot area, there would only be enough CO₂ for injection into PI-7, but PI-6 and PI-8 would be kept on water injection to maintain a symmetrical pattern. The effect of injection into some of the surrounding wells, for "backup" purposes, was investigated by a computer program that simulated unit mobility ratio flow in the reservoir. The design and results of that study are shown in Appendix A. (Also contained in that Appendix are results of a similar investigation performed in the actual area chosen for the mobility control injection).

The injection plans were modified through discussions with personnel of the Morgantown Energy Technology Center of the DOE, the Pennzoil Company (at both Houston, Texas and Parkersburg, West Virginia) and the PRRC, at a meeting held in Socorro on April 8, 1983. These discussions were supplemented by telephone. These modified plans and the results of new experiments were summarized in a July 14, 1983 report to the DOE. They called for the injection of a surfactant pad consisting of 8130 barrels of 0.1% solution of Alipal CD-128 into well PI-7. This was to be followed by a slug of 32,500 reservoir barrels of CO₂-foam, consisting of 6500 bbl of 0.05% surfactant solution and 26,000 reservoir bbl of dense CO₂. At a density of 0.8 g/cc this amounts to 3660 tons of CO₂.

A method was needed for the simultaneous injection of the surfactant solution and the dense CO₂ with reasonable assurance of good mixing to prevent gravitational segregation in the wellbore, and at the same time to reduce the number of parts exposed to the corrosive action of wet CO₂. An original suggestion of a dual completion and downhole spraying mixer was dropped, at the suggestion of Paul King of Pennzoil. This new method involved the use of 2" ID fiberglass tubing to carry the mixture of CO₂ and surfactant solution downhole into the packed-off region opposite the wellbore. Turbulence in this tubing would presumably keep the surfactant solution and CO₂ well mixed.

At this time also, plans and preparations were being completed to monitor operations at the observation well, OB-3. Early in the project, a hydrological consultant, Mr. Kelly Summers, had suggested the use of a technique of hydrology for the measurement of the velocity of fluids going by a well. This technique, the "Point Dilution Method" (PDM), calls for the periodic chemical analysis to determine the rate of decline of tracer concentration caused by flow past the well. The contents of the shut-in well are kept well mixed, and the tracer concentration declines as the external flow replaces wellbore fluid with formation water. The PDM had never before been used on deep wells, but had been useful for measuring drift velocities in shallow sediments near dams. With Mr. Summers' help, plans were developed to adapt the test procedures for use in OB-3.

To these plans were added means of gathering other information at OB-3 through automatic analysis. Although the original usages of the PDM call for the continuous measurement of tracer concentration without removal of sample fluids from the well, it was apparent that small but appreciable quantities of fluid would have to be taken from the otherwise closed circulation system that had been designed to keep mixed the wellbore contents of OB-3. Mathematical analysis, given in Appendix B, show that the taking of moderate quantities of fluid from the wellbore system affects only slightly the value of a constant used in relating the primary tracer concentration decline to the drift velocity.

The detailed plans that were developed for the analytical procedures, and the automation of data acquisition equipment, were given in a report to the DOE dated March 15, 1984. Briefly, the system was contained in an insulated instrument box that would be located near the OB-3 wellhead, and be connected by high-pressure tubing to a TEE on the circulation piping from the well. Inside the data-acquisition box, a computer-controlled system would periodically open a valve in tubing leading to that line, to take small samples from the circulating well fluid. These would be analyzed by periodically calibrated specific ion electrodes, for Iodide (the primary tracer ion), and also for pH and for calcium ion. The instrument system also contained an oil-water separator and sampling outlets for the collection of cumulative samples. The March 1984 report also contained plans for much of the specialized electronic measuring circuits constructed for the system, and outlines of the computer code written to automate the taking and storage of data from the devices.

Unfortunately, most of the planning and equipment development described in the above paragraphs were bypassed by the discovery in the fall of 1984 that the mini-pilot area could not be used for the mobility control tests.

Field testing, performed in preparation for the test, revealed to Pennzoil engineers that the condition of planned main injection well PI-7 was impaired by a serious leak directly into the "Big Lime," the formation overlying the Big Injun pay sand. This previously unsuspected situation, evidently irreparable, would have frustrated the mobility control test by allowing a large and uncontrolled portion of both surfactant pad and the CO₂-foam to be injected into a "thief formation" containing no oil. The loss of PI-7 caused abandonment of the mini-pilot area as a site for the test, since the observation well OB-3 was in the wrong place to allow timely assessment of flow from either of the other two injectors in the area.

Plans for the PI-2 - OB-2 Area

As a consequence of the situation described in the previous section, a different area for the mobility control test was chosen. This is the region including the injector PI-2 and the observation well OB-2. As shown on Fig. 5, it is about 800 feet northeast of the mini-pilot area.

The well OB-2 had been drilled after the CO₂ flood to enable direct measurements of the amount of oil recovery in that project. For this purpose, pressure core had been taken to enable the oil saturation to be determined. These data are showing in Fig. 6. As can be seen, the sweep in the upper ten to twelve feet of the formation had been quite effective, with residual oil

saturations averaging about 4.5%. In the lower half of the formation, however, the oil content had not been reduced to such an extent; the core extractions there showed residual oil saturations of 20-28%. After coring, the well OB-2 had been completed with two joints of fiberglass casing through the zones containing the Big Injun. This unperforated completion, of course, had been performed to allow subsequent operations to include induction and neutron logging.

Referring to Fig. 2, it can be seen that injection well PI-2 was one of six injectors making up the original ten-acre five spots. OB-2 lies in the direction from PI-2 to the producer, LWS-4. This normal production well is about 460 feet from PI-2. Thus, water injected into PI-2 at 200 bpd and displacing uniformly the water saturation in place would not reach LWS-4 for almost 6 years. On the other hand, the time required for such a displacement front to reach OB-2 (75 feet from the injector) would be only about two months. It was thus clear from considerations of time alone that a full-scale test from this location would be impossibly long. In addition, of course, the cost of CO₂ for a full-scale test would also be excessive.

These considerations meant that test information would be available only from PI-2 (where injectivity could be measured) and from OB-2. With the latter cased and not initially in contact with the formation fluid, it would not be possible to perform a point-dilution experiment to measure drift velocity. Two options were nevertheless available for obtaining information on the progress of a test from OB-2. These were discussed in a report to the DOE dated April 10, 1984. These mutually exclusive choices were either to leave the well unperforated and to rely on repeated logging results to assess the mobility control test, or to perforate the fiberglass casing to enable the taking of fluid samples. Each choice would lead to a different set of problems and limitations; neither could offer as much information as was needed for a conclusive test.

The logging method would require the use of some log-observable tracers to enable the leading edge of the water-borne front to be detected. But logging a non-producing well would mean that the tracer would only be observable for the relatively short time during which the injected band of tracer material was passing the well. Consequently, in order not to miss the band as it went by, fairly frequent logging runs would have to be taken. How frequently these runs would be necessary would depend on the radial extent of the tracerband--the results of radial dispersion during its displacement from the injector.

The progress of dispersion during radial flow is influenced greatly by the distribution of flow heterogeneities which happen to exist in the rock in the space around the injector--a distribution which is quite unknown. To the extent that the rock can be considered to have been deposited in fairly uniform layers, however, the progress of the dispersion can be calculated numerically. There is no published literature on this subject that starts from the accepted convection-dispersion equation. A recent analysis of this system enables the computation of the profile of a radially propagated tracer band. This work is described in Appendix C.

A significant advantage of the logging method would be the added information that could be obtained about the formation. Frequent logging after the injection of the slug of radioactive tracer would enable the direct

observation of arrival times and peak widths at different levels of the hole. This data would be more descriptive of the statistical distribution of formation properties than are the results of the usual suite of laboratory permeability and porosity measurements of samples taken from core taken over the interval of interest.

In the first part of this program, observations by gamma ray logs of the passing tracer bands would give information on flow velocities and dispersion coefficients, and (by peak area) of aqueous phase saturation of different sections of the Big Injun formation. Results of a similar gamma-ray logging program to detect the passage of tracer bands at an observation well have been described by researchers at the Shell Koninklijke Laboratories [Gesink et al., 1983]. Following these, successive induction logs could give data about the changing conductivity of the formation to indicate the passing of an oil bank by the wellbore.

A major difficulty seen in this flood assessment method was that no actual fluid samples would be available, by which tracer, surfactant and oil content could be directly measured. Reliance on logging information alone, it was felt, might lead to serious and unanswerable questions about the results of the mobility control tests. Pennzoil engineers also expressed concern that the thickness of cement around the fiberglass casing of OB-2 might be enough to prevent detection of the gamma rays from the tracer.

In consequence, it was decided not to use radioactive tracers, but to investigate yet another means of assessing the results of CO₂-foam injection. This method would involve the perforation of OB-2 so that fluid samples could be taken from it. In this way, it was reasoned that the nature of the fluids could be determined directly. This data-taking activity would be performed under some constraints, however. Perhaps the most important of these was the fact that, with a surface injection pressure at PI-2 of 1000 psi, the formation fluid was at relatively high pressure even 75 feet away. Thus, the surface pressure at OB-2 was also high. Although there were no production facilities at the well, a brine production rate of as much as 10 bpd could be handled. This volume of flow could be taken through throttling valves on the well head.

Another problem area in this method concerned the method and efficiency of sampling. Initial plans (after abandonment of the "logging" method) called for perforation of the fiberglass casing in OB-2 at two levels (with two shots in the middle of the topmost 12 feet and two in the middle of the central 12 feet).

These two levels were placed in the centers of two zones in which there might be a large difference in test results. As was pointed out in Section III, the residual oil saturation was much less in the uppermost than in the central zone. If these readings could be taken as an indication of conditions existing throughout the region between OB-2 and PI-2, there would be little or no oil to be recovered from the topmost zone by the mobility control test.

The two zones were to be kept separate by a packer and sampled separately through a dual completion--the upperzone flowing through the annulus and the middle zone through the tubing. After the well was perforated and the packer and tubing had been installed, it was discovered that there was "crossflow" between the zones. It was apparently decided on the spot that this would

prevent independent samples from being taken by the planned procedure, so the tubing and packer were removed.

Consideration was given instead to the use of a wire-line fluid sampler made by the Kuster Company of Long Beach, California. With this 1 1/4" diameter device, a 500 cc sample of fluid could be taken. The opening of the sample valve was accomplished by a clock, set at the surface for a time exceeding that required for the trip to depth. With this device, it would be possible to capture clean samples of the fluid entering the wellbore from the lower perforations, if the sample bottle were opened just above them. On the other hand, if the valve were opened while the inlet was just above the upper perforations, a mixed sample from the two levels would be obtained. The mixture ratio would depend on the relative rates from the two horizons. Although several runs were made with the Kuster Sampler, no data was obtained to distinguish samples from different levels. Adding further to the difficulties of proper use of the device, it was necessary to run it into the well through a lubricator, because of the substantial surface pressure. Apparently because of these and other complexities that required professional attendance during operation of the device, it proved to be uneconomic to use the sampler on a regular basis.

Instead, a reduced goal was accepted for the sampling in which mixed samples from both zones would be taken at the surface on an every-other-day schedule. There was unfortunately no information available on the relative rates from the two zones, a situation that has contributed greatly to the uncertainty concerning the results of the test.

Surface sampling of the combined fluids would be subject to two problems connected with the flow rate. If the produced flow rate from OB-2 were kept large in order to minimize the delay between entry and recovery of the fluid sample, then two uncertainties could arise in the sampling--the distortion in the flow through the formation near the well bore, and the excessive Taylor dispersion during flow up the casing to the surface. (The latter problem, first described by G.I. Taylor [1953] concerns the large "effective dispersion coefficient" during flow in a tube or pipe. The resultant transition zone length after traversing a fixed length tube, is proportional to the square root of the flow rate). If the production rate is decreased so as to reduce these uncertainties, then the travel time to the surface is increased. The flow rate range of 7-10 bpd was adopted as a reasonable compromise. In this case, the range of travel time to the surface would be 3.1 to 4.4 days, and the Taylor dispersion zone (to which a sharp concentration change introduced at the perforations would disperse, during flow of the fluids to the surface) would vary from 1250 to 1490 feet. The corresponding dispersion in travel time is 2.3 to 2.8 days.

In the meantime, preparations were being made for the actual injection sequence. This series of events would be initiated by the injection into PI-2 of a quantity of ammonium thiocyanate to serve as a tracer. The amount to be used was estimated in the following way. If the tracer were to be mixed uniformly with all of the formation water contained in the porespace of a volume of rock 75 feet in radius and 36 feet thick, the concentration should still be high enough to distinguish from background. The volume in that amount of formation water would be $\pi(2268)^2 (1098) (0.21) (0.7) = 2.6 \times 10^9 \text{ cm}^3$. The analytical laboratory of the New Mexico Bureau of Mines and Mineral Resources

claimed that one part per million of NH_4CNS can be distinguished from background. The calculation thus leads to a requirement for 2.6 kg of the tracer to be injected into PI-2. With this final recommendation, Pennzoil engineers were ready to start the test, the events of which are chronicled in the next sections.

VI. INJECTION FACILITIES

The injection operation at PI-2 has involved the combination of a number of activities which had been performed previously at the field. By way of introduction to the chronology of these activities, a short description is given here of the facilities used for these operations.

The largest flow rate injection activity was designed to pump dense CO₂ into the six injectors of the initial five-spots shown in Fig. 2. In a now-standard operation, liquid CO₂ under several hundred psi and at a temperature somewhat below zero (Fahrenheit) was trucked to the field and unloaded into storage tanks maintained at the same conditions of temperature and pressure. From these tanks, CO₂ was taken as needed, sent through a heater to bring it most of the way to the reservoir temperature of 75°F, and pumped into high-pressure lines leading to the injectors. A bypass pressure regulation system kept the injection pressure below a set point of 1000 psi by leading enough of the output of the pump back into the input, to maintain the set point. Before reaching the injector, the dense CO₂ was routed through a turbine flowmeter to give independent readings of the amount injected. This existing system was also used for the mobility control test and generally behaved as Pennzoil engineers had designed it, with no major difficulties.

Plant facilities for water injection were also standard. Almost fresh water was used from a shallow water-bearing sand. The aqueous part of the CO₂-foam was also injected by pumps having bypass regulation--that is, the injection pressure was kept at or below a set-point pressure by means of a bypass valve that shunted some of the output from the pump to the low-pressure side when necessary. The water also was routed through a turbine meter to provide independent measurement of the flow rate and cumulative volume injected.

For the pre-foam part of the mobility control tests, the water was generally not injected alone, but with some dissolved material--either tracer or surfactant. This auxiliary injection was accomplished by use of a chemical injection pump which can take fluid at atmospheric pressure and pump it into a high-pressure line at a set rate that is ideally independent of the pressure it works against. Unlike the main water and CO₂ pumps, the chemical injection pump gave frequent trouble. It needed almost continual maintenance and eventually needed to be replaced.

During the mobility control test in the PI-2 to OB-2 region, there was a change in flow pattern from that which characterized the previous conventional CO₂ flood. In the CO₂-foam test, in contrast to the previous flooding situation, the backup water injection wells (particularly LW Shafer #11, 10 and 2, and E. Lewis #18) were not operated. Whereas the difference would have been quite noticeable at the original producer LWS-4, there would probably have been very little change of flow rate or pattern at a location as close to PI-2 as OB-2. This conclusion is reached by analogy with a similar comparison made in Appendix A.

VII. CHRONOLOGY OF MOBILITY CONTROL TEST

A. Injection

The principal events taking place during the test are given in Table 3, and are also annotated briefly below.

Although it had been estimated that less than 3 kg of ammonium thiocyanate would be sufficient as a tracer chemical, it seemed wise to increase this by a factor of five. Thus, the mobility control test was initiated on August 28, 1984, with the injection of 15 kg of NH_4CNS . It was injected over the course of two days in a total of 371 barrels of water. The salt is highly soluble in water and relatively non-toxic and non-irritating. It was dissolved in a smaller quantity of water (4 or 5 gallons would have been sufficient, given its large solubility) and pumped into the water-injection line by use of the chemical injection pump.

Injection of the thiocyanate had apparently represented quite a strain for the chemical pump. It barely managed to finish the injection before the packing failed. The first two weeks of September 1984 were marked by continual troubles with the chemical injection pump. September 10 was given as the official day for the start of injection of the surfactant pad. Difficulties with the chemical injection pump were not the only trouble experienced when the surfactant pad was started. The greatest difficulty was that the injectivity continually declined as more of the surfactant was injected. The cause of this problem is still unknown: it could not be duplicated in the laboratory, where both extracted and oil-saturated Rock Creek core material was flushed with both fresh water and with a 0.1% solution of Alipal CD-128. No significant increase in pressure drop was observed in these laboratory corefloods under any circumstances. It has been pointed out that these tests were performed with water-saturated, or water-and-oil saturated cores with no CO_2 . The presence of a high saturation of dense CO_2 in the formation may indeed have caused some in-situ foam formation or unexpected plugging reaction with the surfactant. Other speculation on possible causes for the gradual loss of injectivity in the field ranged from plugging caused by debris from the pump or injection system, to an unexpected interaction between the surfactant and some component of the injection water. The condition of the injection pump certainly was involved, as shown by the figures on Table 3. In the period from September 10 to 28, only 300 bbl of the surfactant solution were injected. After the pump was replaced on September 28, a like period (until October 18) show 2464 additional barrels of surfactant solution injected. At this point, surfactant injection was halted, and preparations were made for start of the CO_2 -foam. Fresh water injection continued until October 31, when after a short back flush period, the CO_2 -foam was started. For the CO_2 -foam, a 2" fiberglass tubing string had been installed to the bottom of PI-2. At the surface, this was joined through separate valves to the lines from the dense CO_2 and the surfactant-water supply. Injection of CO_2 -foam, with a target CO_2 water volumetric ratio of 4:1 (to give a "flowing quality" of 80% for the CO_2 -foam), was started November 6, 1984.

The injection of CO_2 -foam was also erratic, both in the volumetric ratio that could be maintained, and in the total rate possible. A temporary halt occurred on November 27, but the system was restarted December 3. From this time, the injectivity again declined gradually until on December 20, 1984, zero

TABLE 3

PI-2 Fluid Injection History*

Fluids Injected During Interval (reservoir barrel)		Date	Cumulative Injected Fluids (reservoir barrels)				Comments		
fresh water	CO ₂		Surfactant Solution pad 0.10%	Surfactant in foam 0.05%	All Aqueous	CO ₂		"CO ₂ Foam"	All Fluids
0	0	8/28/84	0	0	0	0	0	0	Start
371	0	8/30/84	0	0	371	0	0	371	Finish } (15 kg NH ₄ CNS)
1583	0	9/10/84	0	0	1954	0	0	1954	Initial start surfactant**
2166	300	9/28/84	300	0	4420	0	0	4420	Replace feed pump
0	2464	10/18/84	2764	0	6684	0	0	6684	Halt surfactant injection
1781	0	11/ 2/84	2764	0	8665	0	0	8665	Halt fresh water injection
0	0	11/ 6/84	2764	0	8665	0	0	8665	Start "CO ₂ foam" injection
0	399	11/27/84	2764	399	9064	1376	1775	10440	Temporary halt
0	0	12/ 3/84	2764	399	9064	1376	1775	10440	Re-start "CO ₂ foam"
0	371	12/20/84	2764	770	9435	2752	3522	12187	Halt CO ₂ foam, zero injectivity
0	522	1/31/85	2764	1292	9957	3865	5157	13822	Spotty injection
0	62	2/ 8/85	2764	1354	10020	3981	5335	14000	Spotty injection shut down
0	0	5/28/85	2764	1354	10020	3981	5335	14000	Start up after well work on PI-2
0	644	6/26/85	2764	1998	10664	5695	7693	16358	Temporary supply shut down
0	583	8/ 4/85	2764	2581	11247	8027	10608	19273	Target volume of CO ₂ -foam injected
~700 continuing	0	8/14/85	2764	2581	11947	8027	10608	19973	Start chase water
8500	0	10/24/85	2764	2581	20447	8027	10608	28473	Complete chase water injection

Radioactive tracer Injection Test 11/22/85 to determine injection profile - Test results indicate fluid injected into Big Injun with no casing leaks

*Data estimated from Pennzoil's graphical injection record.

**Initial start of surfactant pad injection was erratic because of deteriorating chemical pump.

injectivity was reported at PI-2. At that time, a total of 3522 bbl of CO₂-foam had been injected.

From this time until February 8, 1985, additional spotty injection of CO₂ and 0.05% surfactant solution was accomplished. In the second week of February, injection into PI-2 was increased, and by February 19, it was reported that the well was taking 200 bpd at a plant pressure of only 800 psi. Water production into the common production heater-treater soon indicated that a leak had developed from PI-2, probably into the Big Line. Subsequent further testing in March 1985 revealed that the leak had become quite serious. Fluorescein-dyed water injected into PI-2 showed up in less than a day's time at the heater-treater, apparently having broken through from PI-2 to LWS-1. Using a new gamma-ray diagnostic tool, Pennzoil engineers early in April ascertained that the leak was behind the pipe, 13 feet above the top perforation in PI-2.

In order to resume meaningful injection of CO₂-foam in the well, arrangements were made for a service company to perform a cement squeeze operation through the top three perforations of PI-2. This work was done in the second week of April, and during the next two weeks the excess cement was drilled out (by cable tool) and the sand bailed from the bottom of the well. Because injectivity of the cleaned well was still low, it was re-perforated and cleaned with acid. After this treatment, PI-2 was able to take 80 bpd (at a plant pressure of 800 psi) with no flow evident into LWS-1. Thus, by the end of May, the injection well was once again ready to receive CO₂-foam.

From the re-start on May 28, 1985, injection went well, at rates of about 80 bpd of CO₂-foam, except for a temporary shut-down from loss of CO₂ supply. The target injection of 10,608 barrels of CO₂-foam was reached on August 4, 1985, and injection was switched to "chase water."

B. Sampling

As was pointed out in an earlier section, assessment of project results was to be performed on the twin bases of the injection performance of PI-7 and the analysis of samples taken from the 7 to 10 bpd production at OB-2. The latter subject is dealt with here and in Table 4.

Briefly, and disappointingly, all of the results of the analyses for ammonium thiocyanate tracer were unequivocally negative. Most of the tests for surfactant also indicated that the concentration was less than procedure's minimum detectable level. There were some indications in the later samples that surfactant was barely above that level, but those results were made uncertain by chemical interference from crude oil components.

Starting from early after the start of the mobility control test, samples of the water produced from OB-2 were taken at intervals of two days, with every other one of these being sent to New Mexico Tech. These samples were analyzed both for NH₄CNS and for surfactant. The work was done at the New Mexico Bureau of Mines and Mineral Resources laboratory at New Mexico Tech, by standard procedures.

For thiocyanate, this involved adding an excess of Fe⁺⁺⁺ ion and measuring optical absorption in the blue region of the spectrum. The normal detection

Table 4

Rock Creek Water Samples Test Results^{1,2}

<u>Date Sampled</u>	<u>Thiocyanate</u>	<u>Surfactant (Alipal)</u>	<u>Date Sampled</u>	<u>Thiocyanate</u>	<u>Surfactant (Alipal)</u>
8-20-84	ND*	ND*	11-27-84	ND*	ND*
8-27-84	↓	↓	11-29-84	↓	↓
9-26-84	↓	↓	12-01-84	↓	↓
9-07-84	↓	↓	12-03-84	↓	↓
9-13-84	↓	↓	12-05-84	↓	↓
9-18-84	↓	↓	12-07-84	↓	↓
9-20-84	↓	↓	12-09-84	↓	↓
9-22-84	↓	↓	12-11-84	↓	↓
9-24-84	↓	↓	12-13-84	↓	↓
9-28-84	↓	↓	12-15-84	↓	↓
9-30-84	↓	↓	12-17-84	↓	↓
10-02-84	↓	↓	12-19-84	↓	↓
10-04-84	↓	↓	12-21-84	↓	↓
10-06-84	↓	↓	2-22-85**	↓	↓
10-08-84	↓	↓	2-24-85**	↓	↓
10-10-84	↓	↓	12-23-84	↓	↓
10-12-84	↓	↓	2-26-85**	↓	↓
10-14-84	↓	↓	12-25-84	↓	↓
10-16-84	↓	↓	2-20-85	↓	↓
10-18-84	↓	↓	2-23-85	↓	↓
10-20-84	↓	↓	3-08-85	↓	↓
10-22-84	↓	↓	3-11-85	↓	↓
10-24-84	↓	↓	3-12-85	↓	↓
10-26-84	↓	↓	3-15-85	↓	↓
10-28-84	↓	↓	3-19-85	↓	↓
10-30-84	↓	↓	3-25-85	↓	↓
11-01-84	↓	↓	3-26-85	↓	↓
11-03-84	↓	↓	4-01-85	↓	↓
11-05-84	↓	↓	5-16-85	↓	***
11-07-84	↓	↓	5-19-85	↓	***
11-09-84	↓	↓	5-20-85	↓	***
11-11-84	↓	↓	5-29-85	↓	***
11-13-84	↓	↓	6-10-85	↓	***
11-15-84	↓	↓	6-13-85	↓	***
11-17-84	↓	↓	8-24-85	↓	↓
11-19-84	↓	↓	8-26-85	↓	↓
11-21-84	↓	↓	8-27-85	↓	↓
11-23-84	↓	↓	8-28-85	↓	↓
11-25-84	↓	↓	8-29-85	↓	↓

¹All samples are from OB-2, unless noted otherwise.

²Samples are listed in the order they were received by PRRC.

*Not detectable.

**Shafer-I samples.

***Detectable levels of surfactant.

limit is 1 ppm; but after the negative results referred to above, most of the samples obtained during the period when tracer was expected were evaporatively concentrated by factors of 5 to 10, and the concentrate retested. Although this gave an effective detection limit of 0.2 ppm, no positive indications of the tracer were obtained. The concentration of thiocyanate in these samples was then less than would have been observed in a uniform solution of 15 kg in 400,000 bbl.

The test for surfactant is standard for any anionic surfactant, with specific dyes used for particular surfactants. Methylene blue is used with Alipal CD-128. The test involves first the reaction of the unknown with methylene blue in aqueous solution, and then the extraction by chloroform. After shaking the two phases, retention of blue color by the aqueous phase is taken as a qualitative indication of Alipal in the water. The cationic surfactant, Hyamine, is then added in a titration procedure until enough blue has been transferred into the chloroform, so that the two phases are equally colored.

Most of the samples showed no detectable surfactant concentration. Attachment I to this section describes the test procedure, and points out that the verified detection limit is about 19 ppm. Attachment I also contains a memo from Susan Weber, that points out the details of the uncertainty that exists concerning the surfactant content of two groups of samples which also contained crude oil contamination. One of these groups consisted of the OB-2 samples taken after April 1985, and the other consisted of three samples taken from the common heater-treater that takes the produced fluids from LWS-1 and LWS-4. These three samples were taken in late December 1984, when it was observed that fluid was being produced into that tank. Because it was at first thought that LWS-4 was the source of this fluid, the samples (as well as initial references to them) were mislabeled. Later it was realized from independent evidence (the low salinity of the produced water) that this fluid was being produced by LWS-1, and this then was recognized as an indication of the developing leak between PI-2 and the Big Lime that was referred to above.

Samples from OB-2 were unfortunately not continuous as had been planned, but were taken regularly on the original schedule up to December 21, 1985. With the decline to zero of CO₂-foam injection on December 20, it was decided to log OB-2, and the work was done on December 28, 1984. Unfortunately, the lubricator in place on the well was designed for small diameter tools (like the Kuster fluid sampler that had earlier been used), not the larger tools designed for logging. When the small diameter lubricator was removed, the consequent waterspout from OB-2 apparently allowed the loss of about 175 bbl of water (the estimate is made from the observed dimensions of the spout--4" diameter, 50 feet high and of '4 or 5 minutes' duration). When the well stabilized, the logging tool found the liquid level at 1415 feet from the bottom of the hole. Subsequent to this incident, liquid did not flow from OB-2 until the middle of February, and then was intermittent with gas (presumably CO₂).

New Mexico
PETROLEUM RECOVERY RESEARCH CENTER
A Division of New Mexico Tech
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Date: August 23, 1985.

To: J.P. Heller
From: S.J. Weber 
Subject: Update on Rock Creek OB-2 water sample analyses

This is to provide a report on results of laboratory analyses of Rock Creek water samples sent by Pennzoil to the PRRC in recent months. The samples were tested for the presence of thiocyanate and surfactant. To date, no thiocyanate has been found in any of the samples. The lab report states that thiocyanate levels were below the detectable limits.

The method for quantitative analysis for Alipal is described in a Shell Development Company reprint, identified as Attachment A. The calibration curve developed by the Bureau chemists (who performed the analyses) is located in Attachment B.

There has been strong indication that surfactant is present in the OB-2 samples dated May 16, 1985 thru June 13, 1985. Interpretation of these results may require a review of the method used. The procedure, as noted in Attachment A, is a two-phase titration. In the above samples, color developed in the test solutions, which is the qualitative, positive response for surfactant. However, to determine quantitatively the concentration of surfactant, a titration endpoint must be found. Normally, when the color develops in the test, and no interferences are present, an endpoint at which the color moves from one phase to the other can be observed, and a surfactant concentration can be calculated. Although these particular samples did develop the characteristic blue color, the color never shifted phases even after the addition of vast excess of titrant.

A possible explanation for this effect is that crude oil, which is visibly present in some of the samples, may be interfering with the endpoint reaction. It is believed that the crude oil may exist in an emulsion with the surfactant, and this complex in turn may bind the dye in a stable association, thus preventing the dye from migrating into the other phase. This possibility was investigated and is reported in Attachment C. However, a rough estimate of a minimal surfactant concentration was made by observing a subtle color shift during the titration, although the complete endpoint reaction never occurred. These results are as follows:

OBII Water Samples

<u>Sample date</u>	<u>Concentration % actives</u>
3-8-85	not detectable (ND)
↓	↓
4-1-85	ND
5-16-85	$2.7 \times 10^{-3}\%$
5-19-85	same
5-20-85	same
5-29-85	same
6-10-85	same
6-13-85	same

Shell Method Series

470/71



Determination of

ANIONIC SURFACTANTS IN SULFATED DETERGENTS

SCOPE

1. This method¹ describes a procedure for the determination of equivalents of sulfates of detergent alcohols and of ethoxylated detergent alcohols.

METHOD SUMMARY

2. The monoalkyl sulfates or monoalkyl ethoxysulfates are allowed to react with an aqueous solution of methylene blue and the resultant colored methylene blue sulfate is extracted with chloroform. The water-chloroform mixture is titrated with a standard aqueous solution of sodium lauryl sulfate (Hyamine) until the blue color is equally distributed between the two phases. The amount of Hyamine consumed is taken as a measure of the sulfate content.

REAGENTS

3. Unless otherwise indicated, it is intended that all reagents conform to the specifications established by the Committee on Analytical Reagents of the American Chemical Society, where such specifications are available.

(a) Chloroform.

(b) Sodium Dodecyl Sulfate² (sodium lauryl sulfate) crystals.

(c) Hyamine Solution approximately 0.003 M. To prepare, dissolve 1.4 g of Hyamine 1622³ in distilled water and dilute to 1 liter. Standardize by titration against 10.0 ± 0.1 ml of a 0.003 M aqueous solution of sodium dodecyl sulfate.

(d) Methylene Blue Indicator Solution. To prepare, dissolve 50g of anhydrous sodium sulfate in 120 ml of 2 N sulfuric acid solution. Add 6.0 ml of a 0.5%w aqueous solution of methylene blue chloride. Dilute this mixture with sufficient water to give a volume of 1 liter.

(e) Sodium Sulfate, anhydrous.

PROCEDURE

4. (a) Prepare an approximately 0.003 M solution of the sample
 1 Approved by the Shell Testing Services Coordinating Committee for use in North American laboratories. Issued in 1971.
 2 Available from Eastman Kodak Company, Eastman Organic Chemicals, 343 State Street, Rochester, New York 14650; Catalog No. 5967.
 3 Hyamine 1622 powder available from Rohm and Haas Company, Independence Mall West, Philadelphia, Pennsylvania 19105.

in water. Use isopropyl alcohol to assist in the solution of the sample but do not use more than 20%v.

(b) Pipet 10 ± 0.1 ml of this solution into a glass-stoppered mixing cylinder. Add 25 ml of methylene blue indicator solution and 15 ml of chloroform. Add from a buret an amount of the Hyamine solution approximately equal to one half of the expected titration (see Note 1). Replace the stopper and shake the mixture. Continue adding Hyamine solution in small quantities and shaking after each addition, until the color of the two phases becomes identical when viewed against a white background with the main source of light behind the operator.

Note 1. Stable emulsions are sometimes formed during the early part of the titration. This difficulty is not encountered when reagent sufficient to react with at least half of the sulfate present is added before the reaction mixture is shaken.

CALCULATION

5. (a) Calculate the sulfate ester content of the sample by means of the following equation.

$$\text{Sulfate Ester Content, \%w} = MVE/10W$$

where:

M = molar concentration of the Hyamine solution.
 E = equivalent weight of the sulfate ester,
 V = volume, milliliters, of standard Hyamine solution required to titrate a 10-ml portion of the sulfonate solution, and
 W = weight of sample, grams, in a 10-ml portion of the sulfate solution.

PRECISION

6. The following data should be used for judging the acceptability of results (95% probability) according to the SMS statistical concept of precision⁴.

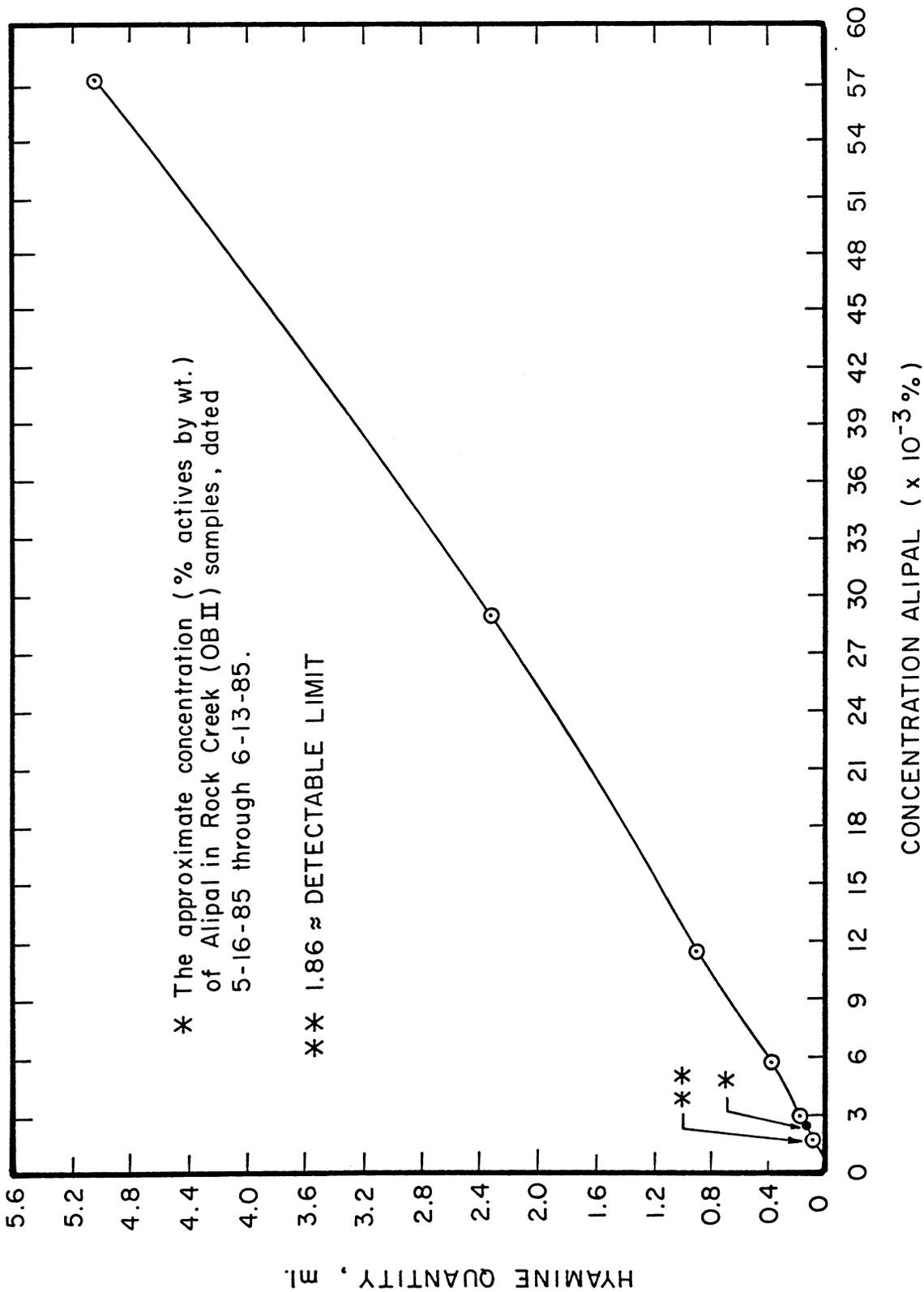
(a) Duplicate results by the same operator should be considered suspect if they differ by more than the following amounts:

REPEATABILITY

0.1 ml of titrant

(b) Sufficient data are not available for making an estimate of the Reproducibility⁴ of this method.

⁴ In accordance with the definitions given in Appendix I to the General Introduction of the Shell Method Series.



Alipal standards were formulated in Rock Creek brine samples collected previous to surfactant injection in August, 1984.

To test this possibility, samples were formulated as follows:

A 1% NaCl/CaCl₂ brine was used to make up surfactant solution of .05% Alipal.

- A. Surfactant solution shaken with Rock Creek crude oil, then filtered to give clear solution.
- B. Surfactant solution unaltered.
- C. Brine (with no surfactant) shaken with Rock Creek crude oil, then filtered.

The above three solutions were tested for surfactant, and the results are as follows:

Both the solutions containing Alipal developed color in the aqueous phase, which is the qualitative, positive response for surfactant. However, only B. reacted with the Hyamine (cationic surfactant) titer to give the expected endpoint. This endpoint is the amount (mls) of Hyamine added to drive the blue dye into the cloroform phase.

In sample A., which had been exposed to crude oil, vast excess of Hyamine titer was added, but the blue never moved into the cloroform phase. It appears that something from the crude oil is interfering with the movement of the dye from the aqueous to the cloroform phase. Sample C., which contains only brine and crude oil, showed no color development in the aqueous phase, indicating no surfactant present.

VIII. RESULTS AND DISCUSSION

The mobility control test at the Rock Creek field did not produce any appreciable volume of oil. Such production from the uppermost zone of the Big Injun would have been remarkable because of the very low residual saturation (about 4.5%) oil that had been left by the previous conventional CO₂ flood. There had been some expectation, however, that an oil bank might be generated in the central horizon to which OB-2 was exposed. If such an oil bank was formed, it was not propagated the 75 feet from the injector PI-2.

Unfortunately, it was not possible to determine the ratio of flow in the two horizons of the Big Injun sand at which the fiberglass casing of OB-2 was perforated. Analysis of core from the two zones (performed after they were extracted to measure the oil remaining after the first CO₂ flood) had shown no striking permeability difference between the two layers, but the relatively large difference in residual oil had led to some expectation that flow could be somewhat greater in the uppermost zone. It remains possible that the difference in flow rates was so large that some sort of small-diameter oil bank exists in the central and lower zones of the sand, surrounding PI-2, but not reaching as far as OB-2.

A second possibility is that the mobility control efforts were not effective, and that for some reason the co-injected surfactant was not able to prevent the CO₂ from forming instability fingers in the formation. This does not seem entirely reasonable, based on the fact that CO₂ had not broken through a separate phase to OB-2 until after December 21 (and probably not until the pressure was lowered by the December 28 incident). It is true that an increasing amount of CO₂ was entering OB-2 up to that time, but in quantity insufficient to interfere with the flow of liquid samples. During the "second period" of sample-taking, this was not the case, with liquid "slugging" out of the well for shorter times, and between longer interludes of gas production. This is evidence that the CO₂ introduced as CO₂-foam did not form fingers. The behavior of OB-2 during the second period of the test is what could be expected after CO₂ actually reached the well.

A third possibility is that the major part of the fluid injected into PI-2 did not flow outward uniformly, but took some other route determined by an unexpected permeability heterogeneity. This possibility is supported by the complete disappearance of the ammonium thiocyanate tracer. (Perhaps "disappearance" is not the proper word--after all, it was looked for in only one place). Unless the thiocyanate was adsorbed on the rock--and it was chosen because this salt has no record of adsorption in other formations--the tracer tests show either that none of the injected tracer reached OB-2 or that the tracer band passed by unobserved in one or more of the intervals between samplings. This could have happened during or just after the interruption sample collection following December 21.

Co-existing with the above possible reasons why no oil bank was observed at OB-2 is the fact that injectivity of CO₂-foam was only about 40% of the well established value of water injectivity into PI-2. Such an observation (especially if, as here, it is sustained long enough for the injected fluid to be displaced outward more than ten or twenty feet from the well) can be taken as reliable evidence of reduced mobility. This apparent viscosity of CO₂-foam in Rock Creek sandstone is approximately the same as was observed in the

laboratory, as was expected. The reduced mobility apparently was not coupled with an increased mobilization of oil, however. On November 22, 1985, PI-2 was re-entered for radioactive tracer injection, with the observations that no leaks behind casing were detected, that fluid did enter the Big Injun sand, and that the calculated saturations indicated no more oil had been displaced from the lower zones of the formation.

Field tests carried out in the oilfield are often subject to uncertainties in their interpretation, and this test is far from an exception. Many of the uncertainties in this case, as in other tests, have resulted from operational decisions which were made out of necessity, for operational, budgetary or safety reasons. Although no oil was produced, the test cannot be called a failure. Despite the uncertainty about what the results mean, it can be said, that:

1. It is possible to inject dense CO₂ and surfactant solution simultaneously, without insurmountable problems of corrosion or of control of the ratio.
2. The injection of "CO₂-foam" was effective to some extent in retarding the flow of free CO₂ to a producing observation well 75 feet away from the injector.
3. A number of operational and scientific problems related to this method of mobility control of CO₂ floods have been examined, discussed and exposed to industry scrutiny. The results of this exposure may hopefully prove valuable in future attempts at mobility control of CO₂ floods.

IX. ACKNOWLEDGEMENTS

As has been pointed out in the SPE papers reporting this mobility control test, the work was a cooperative effort by personnel from three groups with somewhat different motivations. Any success is due to this cooperation, and the principal investigator wishes to express his sincere appreciation for it. These thanks are due both to the engineering staff of Pennzoil Exploration and Production Company, especially Dan Boone, Paul King, John Blomberg, Jim Crews and Larry Guckert, and to personnel of the Morgantown Energy Technology Center, especially Royal J. Watts, Albert Yost III and Vic Owens.

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APPENDIX A - FLOODING PATTERNS

Introduction

For several reasons, it is impossible to predict precisely the velocity and the composition profiles during flow and displacement in an oil reservoir. Clearly, a major difficulty is the incompleteness of available information about the horizons in question. Detailed information that would be required for a complete description of flow would include the porosity and permeability distributions referred to in three-dimensional coordinates. A suitable and accurate boundary approximation would also need to be made of a surface surrounding the region beyond which no significant flow occurs. (In many cases a completely closed boundary might not be a viable assumption--for example, when the oil-bearing sand is in contact with an extensive aquifer). Along with all of this data about the physical features of the porous rock, detailed information about the saturation distribution and the wettability would obviously be relevant to the flow behavior. But even all of this would not be sufficient to enable the details of a flood to be predicted exactly.

In addition, a perfect computational method would be needed; one that was adapted to utilize all of the important information and to calculate flow velocity values at all points within the reservoir boundaries. The changing saturation distribution could then be obtained in a secondary calculation in which the motion of fluids along the streamlines is calculated by integration.

In the common case where the mobilities of fluids in the rock are not constant, but are influenced by the state of saturation, the two computations referred to above cannot be done separately. In this usual situation, the equations for the saturation and velocity distributions are coupled. This causes the fluid velocities in the reservoir to change as the displacement proceeds, so that the "perfect computation method" would also have to take into account this continuously changing situation in the computation of successive "flood fronts."

The purpose of the above is to make clear that approximations are required in reservoir simulation. The requirement comes first of all from the needs to reduce the generalized mathematical description to a numerical procedure in which only a finite number of algebraic equations need to be solved. Secondly, some of the mathematical representations of the physical processes of flow are uncertain, or are too complicated to solve in detail. These circumstances dictate that, even if much more data about the reservoir were known, it might not be possible to utilize the added information. Thus, these difficulties make it easier for us to accept our ignorance of the exact details of the variation of reservoir properties, and to pursue and assume various simplified alternatives to be used in the place of precise reservoir and process descriptions.

The particular alternatives chosen--that is, the specific assumptions made about the reservoir and the flow processes within it--are of great importance not only in the performance of the reservoir engineering calculation, but also in the interpretation and evaluation of their results. Thus, information on the nature of these assumptions is a necessary part of the report of reservoir calculation or simulation.

The work reported in this Appendix is the calculation of flow velocities within the Rock Creek reservoir and of the resulting flood patterns. The calculations are simplified by several major assumptions. While these flaws prevent the results from being considered as firm predictions of reservoir behavior, the calculated results are still useful to a lesser extent. One way the calculations can be utilized is by comparison with known reservoir behavior, in order to assess the extent to which the assumptions may reflect reality.

Nature of the Approximations

The assumptions made in the calculation method used in this Appendix were:

1) That the section of reservoir considered is uniform in thickness, in permeability and in porosity. This assumption could be relaxed if enough information were available. It is felt, though, that in the apparent absence of field-wide trends, significant improvement in this simplified reservoir description would require a much greater volume of data sufficient to show some of the field-wide variability of these quantities.

2) That the fluid mobility is a constant throughout the regions considered, independent of the fluid saturating the rock. This assumption is that the velocities are constant in time, and that the flow equation is not coupled with the fluid distribution equation. There can be no frontal instability in this circumstance, and no formation of viscous fingers. Whereas the utilization of this assumption prevents the calculation method from seeing a major aspect of the reservoir flow, it leads to a tremendous simplification of the calculation procedure. It is worthwhile to use this calculation with the hope of later making an approximate modification of the results to take into account the influence of a mobility ratio different than one.

3) That the displacement of one fluid by another in the rock is complete, with no residual saturation of the initial fluid. How this assumption affects the results is different for miscible and for immiscible displacements. If the displacement front is between miscible fluids, then the assumption departs from the truth only in that a dispersed zone exists between the displacing and displaced fluids. If reservoir distances are large compared to the dispersion zone, and if the moving displacement front obtained in the calculation is understood to represent the "50% isoconcentration surface," then the assumption is not very objectionable. On the other hand, if the displaced and displacing fluids are immiscible, there will not only be a Buckley-Leverett type transition zone between the fluids, but there will be a residual saturation left behind the front. This latter situation would also require that some modification be made of the results before their use. The presence of a residual phase demands first of all the recognition of an increased displacement velocity. Instead of a value of U/ϕ , the displacement front will move at the rate $Uf/(\phi\Delta S)$. Here, U is the Darcy or superficial velocity at which the displacement fluid moves through a given region of the reservoir, f is the fractional flow of the displacement fluid, ϕ is the porosity of the rock, and ΔS is the change in saturation of the displacement fluid from one side to the other of the frontal region. This modification can be made in the consideration of the results of these calculations.

4) That the fluids are incompressible. This assumption is never strictly true, and a proper course of interpretation is to consider what is the magnitude of the error caused by using it. The major influence of the non-zero compressibilities of water, oil and rock is encountered as a transient pressure change immediately after a well is shut in, or when other rapid changes in flow rates occur. Contrasting to this, when changes are slow (or if the reservoir is in a steady-state situation), the main effect of compressibility is that the flow velocity will differ from one part of the flow field to another, depending on the absolute pressure. The fractional change of velocity (from that calculated under the assumption that the compressibility is zero) is equal to the fractional change of density of the fluid from that at the reference value. Even for gases or supercritical fluids near their critical points, this effect can often be approximated as a post-calculation correction.

This description of the assumptions made is given to indicate the conditions under which results of the calculation can be accepted, and in what way they must be modified to correspond more closely with actual reservoir flow.

Program of Calculation

The listing of the main PASCAL program for the calculation is given in Fig. A-1. It consists of a number of Procedures (the Pascal nomenclature for subroutines) that perform the following tasks (given the layout of wells and the rates into or out of each):

a) The Darcy fluid velocity is calculated at points of interest, as the product of the assumed fluid mobility by the space derivative (the gradient) of the pressure. This space derivative, in two dimensions, is simply the vector sum:

$$\sum_{i=1}^{N_{\text{wells}}} \frac{Q_i}{|\vec{r}_i - \vec{r}|^2} (\vec{r}_i - \vec{r})$$

Here Q_i is the strength of the i^{th} source or sink (in bpd, positive for production wells and negative for injectors), and \vec{r}_i and \vec{r} are two position vectors. The first of these is the position of the i^{th} well, and the second is the position of the point at which velocity is to be calculated.

The values of the displacement velocity are used to calculate new positions for points on a presumed displacement front. Starting out with 120 points on a presumed initial circle around an injection well, each of the points is moved by a distance equal to the product of its calculated velocity and a small time increment. After each such step the velocities are recalculated at the new point positions and the procedure is repeated. The magnitude of the time increment is kept very small, so as to keep insignificantly small the error made by assuming the velocity to be constant along the increment of path length.

RCNU: RCDISNU . TEXT

listed on 1/18/86. Program writes disk file with successive fronts

```

{ SPAR2:NR2LST}
PROGRAM DISPLCE; {in a two dimensional, unit mobility situation, to calc-
ulate successive displacement boundaries.}
{11/20/82 REWRITTEN FOR ROCK CREEK MINIPILOT}
{4/1/83 Modified to print, & allow up to 6 producers, including OB 1}
{4/13/83 FIXED on breakthrough lines}
{5/14/83 re-FIXED on breakthru for more than one producer}
{1/6/86 fixed to close file of fronts after each front written, and
to re-open it for next front.}
CONST NP = 120; PI = 3.1415927; CFPB=5.6145; {cu ft per bbl}
POROS=0.21; RCO=1.840127; {cc.days/bbl.sec}
TYPE WEL = RECORD
      LOC : ARRAY[1..2] OF REAL;
      STREN : REAL;
      END;
      POS = ARRAY[1..2] OF REAL;
      RING = ARRAY[1..NP] OF POS;
      FRONT = RECORD
            ST : REAL;
            FR : RING;
            AR : REAL;
            END;
      HIST = FILE OF FRONT;

VAR W : ARRAY[1..26] OF WEL;
    TW : WEL;
    TRI, TRJ, TRK : RING;
    FRA, FRB : FRONT;
    FRIST : HIST;
    P, PA, PB, PC, PS, PN : POS;
    WID : ARRAY[1..26] OF STRING[6];
    A, XS, YS, X, Y, U, DEL, DE, QS, MAXDR, DR, RB, DT, R, TH, RHO, DMIN, PCOR,
    VX, PCO, FKSPEC, RZ, ALZ, AL, DA, VOL, VIL, MU, H, K, R3, CUT, VC : REAL;
    I, J, KK, L, N, NS, M, MTS, DESIPOINT, NE, II, JJ, NTR, NSFC, ISINK, NSINK,
    RNUM, TN, TNP, LSK : INTEGER;
    V, TYM, TT : ARRAY[1..50] OF REAL;
    PPS : ARRAY[1..6] OF POS;
    ISNK : ARRAY[1..6] OF INTEGER;
    CH : CHAR;
    SKRIP : STRING; {name for file of fronts}
    PR : TEXT;

{ $I PRINDATE }

{marker AA}

FUNCTION R24(VAR P1,P2 :POS):REAL; {SQUARE of distance betw P1 & P2}
BEGIN
  R24:=SQR(P1[1]-P2[1]) + SQR(P1[2]-P2[2]);
END;

FUNCTION R12(VAR P1,P2 :POS):REAL; {distance between points 1 & 2}
BEGIN
  R12:=SQRT(R24(P1,P2));
END;

```

RCNU:RCDISNU.TEXT

```

(marker A)
FUNCTION ANGL(VAR PA,PB :POS):REAL; {angle of vector from pt A to pt B}
CONST DRAD = 57.29577951 ;VAR X,Y,A : REAL;
BEGIN
  X:= PB[1]-PA[1];
  Y:= PB[2]-PA[2];
  IF (X=0) THEN IF (Y=0) THEN ANGL:=0
                ELSE IF (Y>0) THEN ANGL:=90
                ELSE ANGL:=270
  ELSE {if X<>0}
  BEGIN
    A:=DRAD*ARCTAN(Y/X);
    IF (A<0) THEN A:=A+180;
    IF (X>0) THEN IF (Y>=0) THEN ANGL:=A      {1st quadrant}
                  ELSE ANGL:=180+A {4th  " }
                  ELSE IF (Y>0) THEN ANGL:=A  {2nd  " }
                  ELSE ANGL:=180+A; {3rd  " }
  END;
END; { ANGL comes out in DEGREES }

```

(marker B)

```

PROCEDURE ENTRWELLS; {modified for ROCK CREEK wells; locations listed here}
  { OB2 and OB3 added 12/29/85, and scale revised as in MAKEFILN 1/11/86 }
CONST SC=380.50; {feet in field per inch on map}
VAR X,Y,Q,S,RW,D,RSB : REAL;
    II,IK : INTEGER;
    THISWELL : WEL;
    XX,YY : ARRAY[1..26] OF REAL;

```

PROCEDURE WELLOCA;

```

BEGIN
  WID[1]:=' OB 1'; XX[1]:=0; YY[1]:=0;
  WID[2]:=' PI 6'; XX[2]:=-0.20; YY[2]:=-0.58;
  WID[3]:=' PI 7'; XX[3]:=-0.41; YY[3]:= 0.44;
  WID[4]:=' PI 8'; XX[4]:= 0.54; YY[4]:= 0.25;
  WID[5]:=' PI 1'; XX[5]:=-0.95; YY[5]:= 0.84;
  WID[6]:=' PI 2'; XX[6]:= 0.44; YY[6]:= 1.60;
  WID[7]:=' PI 3'; XX[7]:= 1.74; YY[7]:= 2.25;
  WID[8]:=' PI 4'; XX[8]:= 2.59; YY[8]:= 0.92;
  WID[9]:=' PI 5'; XX[9]:= 1.20; YY[9]:= 0.19;
  WID[10]:=' RCE 1'; XX[10]:= 0.48; YY[10]:=-1.09;
  WID[11]:=' RCE 4'; XX[11]:= 1.72; YY[11]:=-0.13;
  WID[12]:=' EL 17'; XX[12]:=-0.97; YY[12]:=-0.77;
  WID[13]:=' EL 18'; XX[13]:=-1.45; YY[13]:= 1.15;
  WID[14]:=' EL 27'; XX[14]:=-1.31; YY[14]:= 0.14;
  WID[15]:=' EL 28'; XX[15]:=-1.12; YY[15]:= 1.96;
  WID[16]:=' EL 29'; XX[16]:=-0.33; YY[16]:=-1.52;
  WID[17]:=' JHL 1'; XX[17]:= 2.28; YY[17]:= 0.38;
  WID[18]:=' JHL 5'; XX[18]:= 3.42; YY[18]:= 1.16;
END;

```

PROCEDURE WELOC2;

```

BEGIN
  WID[19]:=' LWS 2'; XX[19]:=-0.26; YY[19]:= 2.10;
  WID[20]:=' LWS 8'; XX[20]:= 3.01; YY[20]:= 2.01;
  WID[21]:=' LWS 10'; XX[21]:= 0.81; YY[21]:= 2.65;
  WID[22]:=' LWS 11'; XX[22]:= 2.06; YY[22]:= 2.92;
  WID[23]:=' LWS 1'; XX[23]:= 0.17; YY[23]:= 0.56;

```

```

RCNU:RCDISNU.TEXT

      WID[24]:= 'LWS 4'; XX[24]:= 1.45; YY[24]:= 1.24;
      WID[25]:= ' OB 2'; XX[25]:= 0.62; YY[25]:= 1.53;
      WID[26]:= ' OB 3'; XX[26]:= -0.20; YY[26]:= 0.22;
      END;

PROCEDURE QUESVAR (QS:STRING;VAR QX:REAL);
  VAR CHQ : CHAR;
      QY : REAL;
  BEGIN
    WRITE('DEFAULT VALUE: ',QS);
    REPEAT
      WRITE(' IS THAT OK ? ');
      READLN(CHQ);
      IF (CHQ<>'Y') AND (CHQ<>'y') THEN
        BEGIN
          WRITE ('ENTER A NEW VALUE ');
          READLN(QY);
          QX:=QY;
        END;
      UNTIL (CHQ='Y') OR (CHQ='y');
      WRITELN(QX);
    END;

PROCEDURE READEM;
  BEGIN
    S:=0;
    ISINK:=0;
    FOR II:=1 TO 26 DO
      BEGIN
        XX[II]:=SC*XX[II];
        YY[II]:=SC*YY[II];
        WRITE(' ',II:2,' ',WID[II],XX[II]:10:0,YY[II]:8:0,' ');
        READLN(Q);
        P[1]:=XX[II];
        P[2]:=YY[II];
        THISWELL.LOC:=P;
        THISWELL.STREN:=Q;
        S:=S+Q;
        W[II]:=THISWELL;
        IF (W[II].STREN>0) THEN { for positive Q}
          BEGIN
            ISINK:=ISINK+1;
            PPS[ISINK]:=W[II].LOC;
            ISNK[ISINK]:=II;
          END;
        NSINK:=ISINK; {is number of production wells}
      END;
    WRITELN('THAT'S ALL. THE SUM OF WELLRATES IS ',S:5:1,'bpd');
  END;

  BEGIN {procedure ENTRWELLS}
    WELLOCA;
    WELOC2;
    H:=24.7;
    QUESVAR('H=24.7 feet',H);
    MU:=1.1;
    QUESVAR('MU=1.1 cp.',MU);
    K:=0.01;
    QUESVAR('K=0.010 md.',K);
  END;

```

RCNU:RCDISNU.TEXT

```

WRITELN('for WELL LOCATIONS (X,Y), enter RATES (bpd,+ for prod'n)');
WRITELN(' I WELL ID   X(ft)   Y(ft)   Q(bpd)');
READLN;
WRITE('WHICH WELL TO SPECIFY PRESSURE, AND BHP THERE? ENTER NSPC,BHP ');
READLN(NSPC,PKSPEC);
  REPEAT
    WRITELN;
    WRITE(' DESIGNATE WELL # FROM WHICH TO TRACE FRONT ');
    READLN(NS);
    IF (WINSJ.STREN>0) THEN WRITELN('THAT'S A PRODUCER. TRY AGAIN');
    UNTIL (WINSJ.STREN<0);
  PS:=WINSJ.LOC; {position of well sourcing traces}
  XS:=PS[1]; YS:=PS[2];
  QS:=WINSJ.STREN;
  WRITELN('The producers are:');
  WRITELN(' NAME           X           Y           RSB');
  RB:=1000;
  FOR ISINK:=1 TO NSINK DO
    BEGIN
      IK:=ISNK[ISINK]; {IK is the wellnumber of the ISINKth prod'r}
      RSB:=R12(PS,PPS[ISINK]);
      WRITELN(WID[IK],XX[IK]:10:3,YY[IK]:8:3,RSB:11:3);
      IF (RB>RSB) THEN
        BEGIN
          RB:=RSB;
          LSK:=ISINK;
        END;
    END; {RB is now the distance between source and the closest sink well,
        and LSK is the producer # of that well}
  MAXDR:=0.08*RB;
  TH:=ANGL(PS,PPS[LSK]);
  WRITELN('MAXDR= ',MAXDR:7:4,', and THETA= ',TH:6:2,
    ' towards closest producer');
  DESIPOINT:=(ROUND(TH*NP/360)) MOD NP;
  IF (DESIPOINT=0) THEN DESIPOINT:= NP;
  WRITE(' DIRECTION NUMBER IS ',DESIPOINT,', IS THAT OK? ');
  READLN(CH);
  IF (CH<>'Y') THEN BEGIN
    WRITE('ENTER DESIPOINT ');
    READLN(DESIPOINT);
  END;
END; {procedure ENTRWELLS}

      {marker C}

PROCEDURE CORRPRS; {to compute PCOR, from one specified well pressure}
CONST  RW=0.25; {assumed well radius}
VAR    PCDM,RR      : REAL;
        PI,PSP      : POS;
        I           : INTEGER;

BEGIN {PCOR is a correction pressure,to be added to calcd value}
  PSP:=W[NSPC].LOC; {NSPC is the number of the well at which ..}
  RR:=0; {..Pressure is specified to be PKSPEC. }
  FOR I:=1 TO 26 DO
    IF (I<>NSPC) THEN
      BEGIN
        PI:=W[I].LOC;
        RR:=RR+W[I].STREN * LN(R12(PSP,PI)/RW);
      END;

```

```
RCNU:RCDISNU.TEXT
```

```
PCOR:=PKSPEC - PCO*MU*RR/(H*K);
END;
```

```
FUNCTION PHI(VAR F1:POS):REAL;
      { to calculate potentials in unbounded, uniform field}
CONST { for PHI to come out in psi,if MU in cp,K in darcy}
      RW=0.25; {and H and R12 in feet. Well radii assumed 0.25 ft}

VAR RR : REAL;
      I : INTEGER;
      PE : POS;
BEGIN
  RR:=0;
  FOR I:=1 TO 26 DO
    BEGIN
      PE:=W[I].LOC;
      RR:=RR+W[I].STREN*LN(R12(PE,F1)/RW);
    END;
  PHI:=PCOR+PCO*MU*RR/(H*K);
END; {function PHI}
```

```
PROCEDURE SHOWPOT(VAR II,JJ:INTEGER); {lists potential at
      19 pts along line between wells II and JJ}
```

```
BEGIN
  IF (II<=26) AND (JJ<=26) THEN
    BEGIN
      PA:=W[II].LOC;
      PB:=W[JJ].LOC;
    END;
    FOR I:=1 TO 19 DO
      BEGIN
        FOR J:=1 TO 2 DO
          PC[J]:=PA[J] + I*(PB[J]-PA[J])/20;
          U:=PHI(PC);
          WRITELN ('at I=',I:2,' PHI= ',U:9:4);
        END;
      END;
    END; {procedure SHOWPOT}
```

```
      {marker D}
```

```
PROCEDURE SETTYMS; { sets times at which fronts will be calcd}
      {max time is 3550 days}
```

```
VAR I : INTEGER;
      CH : CHAR;
BEGIN
  TT[I]:=0.04; {is ignored anyway}
  WRITE('Is this for Near or Far well study? (N/F) ');
  READLN(CH);
  IF (CH='N') OR (CH='n')
    THEN BEGIN
      FOR I:=2 TO 11 DO TT[I]:=(I-1)*20;
      FOR I:=12 TO 19 DO TT[I]:=200+(I-11)*50;
      FOR I:=20 TO 50 DO TT[I]:=600+(I-19)*100;
      {Above is schedule designed for looking at front near
      injector, such as in region between PI-2 and OB-2}
    END
  END
```

RCNU:RCDISNU.TEXT

```

ELSE BEGIN
  FOR I:=2 TO 5 DO
    TTII1:=(I-1)*50;
  FOR I:=6 TO 14 DO
    TTII1:= 200+(I-5)*150;
  FOR I:=15 TO 50 DO
    TTII1:= 1550+(I-14)*50;
  END;
END; {proc SETTYMS}

PROCEDURE MOVE (TN:INTEGER;DELT:REAL);
  {moves points of TNth trace MTS times,each time for a
   time interval DELT (in days), to take it to (TN+1)th trace.}
TYPE VEL = ARRAY[1..2] OF REAL;
VAR UU
  P1,P2,P3,P4
  DLR,TAR,DSK,D1,D2,DU,G
  I,J,KK,IT
  BROKE
  : VEL;
  : POS;
  : REAL;
  : INTEGER;
  : BOGLEAN;

PROCEDURE VELOC(VAR P1: POS);{to calculate flow velocity in unit
  mobility ratio situations, with potential calculated
  from 26 injection and production wells.}

VAR Q,RS,QVC
  : REAL;

BEGIN
  FOR J:=1 TO 2 DO UUIJ:=0;
  FOR KK:= 1 TO 26 DO
    BEGIN
      Q:=WIKK].STREN;
      IF (Q<>0) THEN
        BEGIN
          P2:=WIKK].LOC;
          RS:=R24(P1,P2);
          QVC:=Q*VC/RS;
          FOR J:= 1 TO 2 DO {Q for injector is minus}
            UUIJ:=UUIJ-QVC*(P1[J]-P2[J]);
          END; {UU should now be in feet per day}
        END; {loop on wells; velocity of lth point on trace is UU}
    END; {procedure VELOC}

BEGIN {procedure MOVE}
  TNP:=TN+1;
  TAR:=TTITNP]; {target time}
  TYMITNP]:=TYMITN];
  IF (DELT<0) THEN WRITELN('DELT IS NEGATIVE!');
  G:=0.04;
  IT:=1;
  REPEAT
    DU:=0;
    FOR I:=1 TO NP DO {loop over points on front}
      BEGIN
        P1:=TRII];
        BROKE:=FALSE; {assume no breakthroughs }
        FOR ISINK:=1 TO NSINK DO
          IF (P1=FPS[ISINK]) THEN
            BEGIN
              BROKE:=TRUE; {FPS are pos. of producers}
              P3:=P1;
            END;
          END;
        END;
      END;
    END;
  UNTIL DU>0;
  END;

```

RCNU:RCDISNU.TEXT

```

                                END; {if already broken thru on dir'n I}
IF (NOT BROKE) THEN
  BEGIN
    VELOC(P1);
    DLR:=DELT*SQRT(SQR(UU[1])+SQR(UU[2])); {max distance pt could go}
    FOR ISINK:=1 TO NSINK DO {over production wells}
      BEGIN
        DSK:=R12(P1,PPS[ISINK]); {distance from frontpt to prod'n well}
        IF (DLR>DSK) THEN
          BEGIN
            P3:=PPS[ISINK];
            BROKE:=TRUE;
            END; {if dir'n I broke thru in this iteration}
          END;
        IF (NOT BROKE) THEN
          FOR J:=1 TO 2 DO P3[J]:=P1[J]+DELT*UU[J];
            {calculate new frontpt}
          END;
        TRJ[I]:=P3;
        END; {loop on points I of trace; new points TRJ have been calc'd}
        FOR I:=1 TO NP DO TRI[I]:=TRJ[I];
        P4:=TRI[DESIPPOINT];
        IT:=IT+1;
        TYM[TNP]:=TYM[TNP]+DELT;
        IF (IT MOD 10 =0) THEN
          BEGIN
            WRITE(CHR(7));
            L:=IT DIV 10;
            END; {IF IT MOD 10 =0 , loop to update DELT}
            IF (TAR-TYM[TNP] <= DELT)
              THEN DELT:=TAR-TYM[TNP]; {override on DELT calc'n to end on TT[I+1]}
            UNTIL (TYM[TNP]>TAR); {end of loop that gets new front}
            IF (L MOD 10 <>0) AND (TNP < 8) THEN WRITELN;
            MTS:=IT;
            V[TNP]:=-TYM[TNP]*QS; {this should be the volume injected, in bbl}
            END; {PROCEDURE MOVE}

```

{marker E}

```

PROCEDURE VOLSUM; {this is volume in swept-out area around well NS}
VAR PF,PK : FOS; {mod. to new formula 4/2/83}
    XF,YF,XK,YK : REAL;

```

```

BEGIN
  VOL:=0;
  PF:=TRJ[NP];
  XF:=PF[1];
  YF:=PF[2];
  FOR KK:=1 TO NP DO
    BEGIN
      PK:=TRJ[KK];
      XK:=PK[1];
      YK:=PK[2];
      VOL:=VOL+((XF-XS)*(YK-YS)-(XK-XS)*(YF-YS)); {not in bbl}
      XF:=XK;
      YF:=YK;
    END;
  VOL:=VOL*H*POROS/(2*CFPB); {is now in bbl}
  END; { procedure VOLSUM }

```

RCNU:RCDISNU.TEXT

```

PROCEDURE DISCRIBBL (VAR TRK : RING);
  BEGIN
    RESET(FRIST,SKRIP);
    SEEK(FRIST,RCNUM);
    FRA.ST:=VIL; {is FI(tracenum)}
    FRA.FR:=TRK; {is array of points on trace}
    FRA.AR:=VOL; {is NP(tracenum)}
    FRIST^:=FRA;
    PUT(FRIST);
    CLOSE(FRIST);
  END; {procedure DISCRIBBL}

```

```

PROCEDURE POTLIST; {asks if a list of pressures is wanted}
  BEGIN
    REPEAT
      WRITE('WANT A LIST OF POTENTIALS ? ');
      READLN(CH);
      IF (CH = 'Y') OR (CH = 'y') THEN
        BEGIN
          WRITE('ALONG LINE FROM WELL II TO JJ : ENTER II,JJ ');
          READLN(II,JJ);
          SHOWPOT(II,JJ);
        END;
      UNTIL (CH='N');
  END; {procedure POTLIST}
      {marker F}

```

```

PROCEDURE PRNHEAD;
  BEGIN
    WRITE('ENTER NAME FOR HIST FILE ');
    READLN(SKIP);
    WRITELN(PR,' FRONT HISTORY FILE NAME IS ',SKIP);
    WRITELN(PR,'H= ',H:4:1,'ft, MU= ',MU:3:1,'cp, K= ',K:5:3,'md');
    WRITELN(PR,' non shut-in wells are listed below');
    WRITELN(PR,' I WELL ID RATE(+ for prod'n)');
    FOR II:=1 TO 26 DO
      IF (W[II].STREN <> 0) THEN
        WRITELN(PR,II:5,WID[II]:10,W[II].STREN:14:2);
    WRITELN(PR);
    WRITELN(PR,' front traced from well ',WIDENS);
    WRITE(PR,
      ' Traceno Time(days) R at Theta FI NP ');
    WRITELN(PR,'MTS Np/Fi');
    TN:=1;
    VX:=PI*DEL*DEL*H*POROS/CFPB; {volume behind starting front, in bbl}
    TH:=3*(DESIFPOINT-1);
    V[1]:=VX;
    TYM[1]:=-VX/QS; {time from zero to have reached "starting front"}
    WRITELN('Traceno TIME R at THETA FI NP');
    WRITELN(' 1',TYM[1]:8:2,DEL:7:2,TH:8:1,VX:11:2,VX:10:2);
    WRITELN(PR,TN:5,TYM[1]:13:2,DEL:10:2,TH:12:1,VX:11:2,VX:10:2);
  END;

```

```

PROCEDURE INITIZE;
  BEGIN
    PRINDATE('Output from RCDISNU, ');
    PCO:=RCD*14.696/(2*PI*30.48); {coefficient to put P in psi}

```

RCNU:RCDISNU.TEXT

```

XS:=0; YS:=0;
FOR I:=1 TO 50 DO V[I]:=0;
WRITE('ENTER NUMBER OF TRACES TO CONSTRUCT,NTR= ');
READLN(NTR);
WRITELN
  ('Time increment between iterations is 1/N of inter front time. ');
N:=50;
WRITE('DEFAULT value of N is 50. is that ok? (Y/N) ');
READLN(CH);
IF (CH<>'Y') AND (CH<>'y') THEN BEGIN
      WRITE('Enter new value N= ');
      READLN(N);
      END;
WRITELN(PR,
  ' OUTPUT from RCDISNU run, constructing ',NTR,' front traces');
ENTRWELLS; {takes data on rates to or from each well, etc.}
CORRPRS; {computes constant pressure correction, from PKSPEC at NSPC}
POTLIST; {repeatedly offers to print pressures along an interwell line}
WRITE('ENTER STARTING RADIUS OF CIRCULAR FRONT AROUND WELL ',NS,' ');
READLN(DEL);
FOR I:=1 TO NP DO
  BEGIN
    A:=2*PI*(I-1)/NP;
    P[1]:=XS+DEL*COS(A);
    P[2]:=YS+DEL*SIN(A);
    TRI[I]:=P;
  END;
PRNHEAD;
END; {INITIZE}

      {marker G }

BEGIN {MAIN}
REWRITE(PR,'#6:');
INITIZE;
SETTYMS;
DE:=MAXDR/25;
DT:=(TT[2]-TT[1])/N;
VC:=CFPB/(2*PI*H*POROS);
REWRITE (FRIST,SKRIP);
VIL:=V[1]; {volume injected,barrels}
VOL:=V[1]; {volume displaced,barrels}
CLOSE(FRIST,LOCK);
RCNUM:=0;
DISCRIBBL(TRI);
FOR I:=1 TO (NTR-1) DO
  BEGIN {computing traces}
    WRITE(CHR(7));
    TN:=I;
    J:=I+1;
    RCNUM:=RCNUM+1;
    MOVE(I,DT);
    VIL:=V[1]; {volume injected,barrels}
    P:=TRJ[DESIFPOINT];
    RZ:=R12(P,PS);
    TH:=ANGL(PS,P);
    VOLSUM;
    CUT:=VOL/V[J]; {cumulative produced/injected fluid}
    WRITE(J:5,TYM[1]:8:2,RZ:7:2,TH:8:1,V[J]:11:2,VOL:10:2);
    WRITELN(' MTS= ',MTS);

    WRITELN(PR,J:5,
      TYM[1]:13:2,RZ:10:2,TH:12:1,V[J]:11:2,VOL:10:2,MTS:5,CUT:9:5);
    DISCRIBBL(TRJ);
    IF I<50 THEN DT:=(TT[I+2]-TT[I+1])/N; {DT is used as DELT in MOVE}
    TRI:=TRJ;
  END;
CLOSE(FRIST,LOCK);
WRITELN('DONE');
CLOSE(PR,NORMAL);
END.

```

The program also contains procedures to obtain initialization data from the operator, to facilitate the calculations, and to print out and also to write the results onto disk for later use. These disk files contain information on the positions of the 120 points on successive displacement fronts, from a chosen injector. In usual computer runs, somewhere between 25 and 50 such fronts are calculated and recorded--these more than span the distance between the injector and the nearest producers, and shows the typical cusp-shape in the neighborhood of the producing wells both before and after "breakthrough." The distance between recorded displacement fronts is, of course, many times larger than the incremental distance steps between those that are calculated in the fundamental algorithm.

Several other programs have been designed to examine these recorded displacement front patterns. Perhaps the most useful are two mapping programs. The first of these draws a map on which are shown both the wells and a chosen group of the successive fronts. These successive fronts have also been called "isochrons"--lines along which the flow time from the injector are constant. A similar program draws a map in which the streamlines are connected, so as to indicate the direction and magnitude of the flow (by the degree to which the streamlines are crowded together). On both types of maps a short label can optionally be printed to identify each of the wells. These two programs are listed in Figs. A-2 and A-3.

Disk files of successive fronts have been recorded from many computer runs that were performed to study how displacement patterns in the Rock Creek field would be affected by changes in the injection rates of water into surrounding backup wells, or by the production from a more distant well. During examination of these displacement patterns, it should be remembered that they are subject to the inaccuracies referred to previously, and can only be taken as probable indications of average fluid behavior, subject to the assumptions described.

Application to Rock Creek

Despite the caveats given above, some useful results are obtained by a consideration of Figs. A4 through A7. These maps represent only a portion of the field shown in Fig. 5 of the main text in this report. These maps represent respectively the displacement fronts and the streamlines in the mini-pilot area in two different circumstances. In A4 and A5, injection is only into wells PI-6, PI-7 and PI-8, and production is from OB-1. The relative number of streamlines entering OB-1 is fixed by the assumed well rates, which are 12 bpd produced from OB-1, and 75 bpd injected into each of the three injectors. Over much of the region outside of the mini-pilot area, the streamlines from the injectors are nearly radial, and the successive fronts are nearly circular. It may be noted that the programs were not run out far enough to show breakthrough from PI-6, or the entry of all of the streamlines from PI-7 and PI-8 which were heading for the producer OB-1. The destinations and approximate continuations of the streamlines shown in Fig. A5 should be clear, however. It is apparent that much of the fluid displaced from the region around the injectors is pushed out of the mini-pilot area. Note that the well labels have been omitted from Fig. A-5 to avoid obscuring any of the streamlines.

RCNU:FRNTGRF.TEXT

listed on 1/18/86. This program draws up to ten of the fronts filed by RCDISNU.

```

PROGRAM FRNTGRAF;
USES MPSTGRAF;
CONST RPD=0.0174533;
TYPE POS = ARRAY[1..2] OF REAL;
STRNG = STRING[6];
STRN = STRING[13];
WELL = RECORD
    WELLID : STRNG;
    WELLOC : POS;
END;
RING = ARRAY[1..120] OF POS;
FRONT = RECORD
    ST : REAL;
    FR : RING;
    AR : REAL;
END;

VAR XC, YC, X, Y, FPIS, MXLN : REAL;
    FPI, XF, YF, XS, YS : REAL;
    II, IOR, IFR, ITR, I, L, LASTFR, WELLCOUNT : INTEGER;
    JF : ARRAY[1..12] OF INTEGER;
    P, PT : POS;
    THISWL : WELL;
    FOFW : FILE OF WELL;
    FOFFR : FILE OF FRONT;
    FOF : FRONT;
    TRK : RING;
    XX, YY : ARRAY[1..26] OF REAL;
    WID : ARRAY[1..26] OF STRNG;
    FRF, WLF : STRN;
    FRFA : ARRAY[1..10] OF STRN;
    CH : CHAR;
    LABLOK, WANTLBL, CNCTOK, NEWFYL : BOOLEAN;

<$I PRINDATE>

PROCEDURE QUESTA;
BEGIN
    WRITELN('This draws fronts from injector');
    WRITE('ENTER NAME OF FILE OF WELLS ');
    READLN(WLF);
    RESET(FOFW, WLF);
    WRITE('IS A LIST OF LOCATIONS WANTED ON SCREEN ? (Y,N) ');
    READLN(CH);
    IF (CH='Y') THEN
        WRITELN(' I WELL ID X ft Y ft');
    FOR II:=1 TO 26 DO
        BEGIN
            THISWL:=FOFW^;
            WID[II]:=THISWL.WELLID;
            P:=THISWL.WELLOC;
            XX[II]:=P[1];
            YY[II]:=P[2];
            IF (CH='Y') THEN
                BEGIN

```

RCNU:FRNTGRF.TEXT

```

        IF (II=18) THEN READLN;
        WRITELN(II:3,WID[III]:9,XX[III]:10:2,YY[III]:10:2);
    END;
    GET(FOFW);
    END;
    CLOSE(FOFW);
    WRITE('CENTER OF MAP AT WHAT COORDINATES (in ft)? ');
    READLN(XC,YC);
    WRITE('WHAT SCALE - HOW MANY FEET TO ONE INCH ON PAPER? ');
    READLN(FPI);
    WRITE('Do you want well labels printed on map? (Y/N) ');
    READLN(CH);
    IF (CH<>'Y') AND (CH<>'y') THEN WANTLBL:=FALSE ELSE WANTLBL:=TRUE;
    WRITE
        ('Enter longest straight line (inches) to be allowed in any front ');
    READLN(MXLN);
    MXLN:=SQRT(MXLN);
    END; {proc QUESTA}

PROCEDURE QUESTB;
BEGIN
    WRITELN('ENTER FILEID WHERE DESIRED FRONTS ARE STORED ');
    READLN(FRF);
    I:=1;
    IF WELLCOUNT=1 THEN
        BEGIN
            WRITELN('WHICH FRONT NUMBERS (max of 10) ARE TO BE MAPPED ? ');
            REPEAT
                WRITE('JF[I],I,']= ');
                READLN(JF[I]);
                IF JF[I]<>0 THEN IFR:=I;
                I:=I+1;
            UNTIL ((I=11) OR (JF[I-1]=0));
        END;
    END; {procedure QUESTB}

PROCEDURE QS; {initializes and asks questions for sizes}
BEGIN
    WRITE('ENTER WIDTH in inches ');
    READLN(WIDTH);
    WRITE('ENTER HEIGHT in inches ');
    READLN(HEIGHT);
    END; {procedure QS}

FUNCTION R24(VAR PA,PB :POS):REAL; {square of distance [PA,PB]}
BEGIN
    R24:=SQRT(PA[1]-PB[1]) + SQRT(PA[2]-PB[2]);
    END;

PROCEDURE INIT;
BEGIN
    WELLCOUNT:=1;
    FRINDATE('Output from FRNTGRF, ');
    QS;
    QUESTA;
    FPIS:=SQRT(FPI);

```

```

RCNU:FRNTGRF.TEXT

MAPINIT(WIDTH,HEIGHT,-1);
FOR II:=1 TO 26 DO
  BEGIN
    X:=(XX[II]-XC)/FPI;
    Y:=(YY[II]-YC)/FPI;
    IF (II=1) THEN PUNKT(X,Y,4)
      ELSE PUNKT(X,Y,2);
    IF WANTLBL THEN
      BEGIN
        X:=X+0.05;
        LABLOK:=(X<(WIDTH/2-0.65)) AND (X>-WIDTH/2)
          AND (Y<HEIGHT/2+0.2) AND (Y>-HEIGHT/2);
        IF LABLOK THEN LABYL(X,Y,WID[II]); {checks range of label coords}
      END;
    WRITE(' ');
    END; {prints well symbols and labels}
  WRITELN;
  END; {procedure INIT}

PROCEDURE CKFILE;
BEGIN
  {$I-}
  REPEAT
    RESET(FOFFR,FRF);
    IOR:=IORESULT;
    IF (IOR<>0)
      THEN
        BEGIN
          WRITELN('IOR= ',IOR,' ! ENTER A GOOD FILENAME FOR F of FRONTS ');
          READLN(FRF);
        END;
  UNTIL (IOR=0);
  {$I+}
  FRFAC[WELLCOUNT]:=FRF;
  WRITELN('ok');
END; {proc CKFILE}

PROCEDURE PRNTLB;
BEGIN
  REWRITE(PR,'#6:');
  L:=24;
  FOR ITR:=1 TO LASTFR DO IF JF[ITR]<10 THEN L:=L+2 ELSE L:=L+3;
  L:=39-(L DIV 2);
  IF L<0 THEN L:=0;
  FOR I:=1 TO L DO WRITE(PR,' ');
  WRITE(PR,'SCALE ',FPI:4:0,'ft/in. Fronts ');
  FOR ITR:=1 TO LASTFR DO WRITE(PR,JF[ITR],',');
  WRITELN(PR);
  L:=39-(12+13*WELLCOUNT) DIV 2;
  IF L>0 THEN
    BEGIN
      FOR I:=1 TO L DO WRITE(PR,' ');
      IF WELLCOUNT =1 THEN WRITE(PR,' From File ')
        ELSE WRITE(PR,' From Files ');
      FOR I:=1 TO WELLCOUNT DO BEGIN
        WRITE(PR,FRFAC[I]);
        IF I <> WELLCOUNT THEN WRITE(PR,',');
      END;
    END;
  END
  ELSE

```

RCNU:FRNTGRF.TEXT

```

      BEGIN
        WRITELN(PR,' ':27,'From Files ',FRFAC1]);
        FOR I:=2 TO WELLCOUNT DO WRITELN(PR,' ':38,FRFAC1]);
      END;
    WRITELN(PR);
    WRITELN(PR);
    CLOSE(PR,NORMAL);
  END;

BEGIN (MAIN)
  INIT;
  NEWFYL:=TRUE;
  WHILE NEWFYL=TRUE DO
    BEGIN
      QUESTB;
      CKFILE;
      FOR ITR:=1 TO IFR DO
        BEGIN
          SEEK(FOFFR,(JF[I TR]-1));
          GET(FOFFR);
          IF (EOF(FOFFR)=TRUE)
            THEN
              BEGIN
                WRITELN('File does not contain a front ',ITR);
                LASTFR:=ITR-1;
              END
            ELSE
              BEGIN
                LASTFR:=ITR;
                FOF:=FOFFR^;
                TRK:=FOF.FR;
                PT:=TRK[120];
                XS:=(PT[1]-XC)/FPI;
                YS:=(PT[2]-YC)/FPI;
                FOR I:=1 TO 120 DO
                  BEGIN
                    CNCTOK:=TRUE;
                    P:=TRK[I];
                    XF:=(P[1]-XC)/FPI;
                    YF:=(P[2]-YC)/FPI;
                    CNCTOK:=(R24(P,PT)/FPI)<MXLN) AND (XF<>XS) AND (YF<>YS);
                    IF CNCTOK THEN
                      CNNECT(XS,YS,XF,YF);
                    PT:=P;
                    XS:=XF;
                    YS:=YF;
                    IF CNCTOK THEN WRITE(' ')
                      ELSE WRITE(' ');
                    IF (I=60) THEN WRITELN;
                  END;
                WRITELN;
              END; {ELSE}
            WRITELN(' ');
          END;
        WELLCOUNT:=WELLCOUNT+1;
        CLOSE(FOFFR);
        WRITE('Ready for a new file? ');
        READLN(CH);
        IF (CH<>'y') AND (CH<>'Y') THEN NEWFYL:=FALSE;
      END;
      WELLCOUNT:=WELLCOUNT-1;
      WRITELN('WELLCOUNT= ',WELLCOUNT);
      MAFFIT;
      WRITELN('MAFFIT COMPLETED');
      PRNTLB;
    END.

```

RCNU: STRMLYNS. TEXT

listed on 1/18/86. This program draws streamlines from files by RCDISNU.

```
PROGRAM STRMLYNS;
  {To draw streamlines from one or more wells on same map}
  USES MPSGRAF;
  CONST RP3D=0.0523599;
  TYPE POS = ARRAY[1..2] OF REAL;
  STRNG = STRING[6];
  STRN = STRING[13];
  WELL = RECORD
    WELLID : STRNG;
    WELLOC : POS;
  END;
  RING = ARRAY[1..120] OF POS;
  FRONT = RECORD
    ST : REAL;
    FR : RING;
    AR : REAL;
  END;

  VAR XC, YC, X, Y, TH, XS,
      YS, XN, YN, FPI, IR : REAL;
      I, J, F, UL, XT, SL, ITR, WELLCOUNT, II, IOR : INTEGER;
      XA, YA : ARRAY[1..120] OF REAL;
      P, PT, PI : POS;
      THISWL : WELL;
      FOFW : FILE OF WELL;
      FOFFR : FILE OF FRONT;
      FOF : FRONT;
      TRK : RING;
      XX, YY : ARRAY[1..26] OF REAL;
      WID : ARRAY[1..26] OF STRING;
      FRF, WLF : STRING;
      FRFA : ARRAY[1..10] OF STRN;
      CH : CHAR;
      LABLOK, WANTLEL, CNCTOK, NEWFYL : BOOLEAN;
```

```
{ $I PRINDATE }
```

```
PROCEDURE QUESTA;
  BEGIN
    WRITELN('This draws streamlines from injector');
    WRITE('ENTER NAME OF FILE OF WELLS ');
    READLN(WLF);
    RESET(FOFW, WLF);
    WRITE('IS A LIST OF LOCATIONS WANTED? (Y,N) ');
    READLN(CH);
    IF (CH='Y') THEN
      WRITELN(' I WELL ID X ft Y ft');
      FOR II:=1 TO 26 DO
        BEGIN
          THISWL:=FOFW^;
          WID[II]:=THISWL.WELLID;
          P:=THISWL.WELLOC;
          XX[II]:=P[1];
          YY[II]:=P[2];
          IF (CH='Y') THEN
```

```
RCNU:STRMLYNS.TEXT
```

```

      BEGIN
        IF (II=18) THEN READLN;
        WRITELN(II:3,WID[III]:9,XX[III]:10:2,YY[III]:10:2);
      END;
      GET(FOFW);
      END;
      CLOSE(FOFW);
      WRITE('CENTER OF MAP AT WHAT COORDINATES (in ft)? ');
      READLN(XC,YC);
      WRITE('WHAT SCALE - HOW MANY FEET TO ONE INCH ON PAPER? ');
      READLN(FPI);
      WRITE('Do you want well labels printed on map? (Y/N) ');
      READLN(CH);
      IF (CH='Y') OR (CH='y') THEN WANTLBL:=TRUE ELSE WANTLBL:=FALSE;
      END; {proc QUESTA}

```

```

PROCEDURE QUESTB;
BEGIN
  WRITELN('ENTER FILEID WHERE FRONTS ARE STORED ');
  READLN(FRF);
  SL:=LENGTH(FRF);
  IF WELLCOUNT=1 THEN
    BEGIN
      WRITE('Want to draw only (1/F) of the streamlines? ENTER F= ');
      READLN(F);
    END;
  END; {procedure QUESTB}

```

```

PROCEDURE QS; {initializes and asks questions for sizes}
BEGIN
  WRITE('ENTER WIDTH in inches ');
  READLN(WIDTH);
  WRITE('ENTER HEIGHT in inches ');
  READLN(HEIGHT);
  END; {procedure QS}

```

```

PROCEDURE INIT;
BEGIN
  WELLCOUNT:=1;
  PRINDATE('Output of STRMLYNS');
  QS;
  QUESTA;
  MAPINIT(WIDTH,HEIGHT,-1);
  IR:=0.125; {radius of blank circle around injector}
  FOR II:=1 TO 26 DO
    BEGIN
      X:=(XX[III]-XC)/FPI;
      Y:=(YY[III]-YC)/FPI;
      IF (II=1) THEN PUNKT(X,Y,4)
        ELSE PUNKT(X,Y,2);
      IF WANTLBL THEN
        BEGIN
          X:=X+0.05;
          LABLOK:=(X<(WIDTH/2-0.65)) AND (X)>-WIDTH/2)
            AND (Y<HEIGHT/2+0.2) AND (Y)>-HEIGHT/2);
          IF LABLOK THEN LABYL(X,Y,WID[III]); {checks range of label coords}
        END;
    END;

```

```

RCNU:STRMLYNS.TEXT

    WRITE(' ');
    END; {prints well symbols and labels}
    WRITELN;
    END; {procedure INIT}

PROCEDURE CKFILE;
BEGIN
    { $I- }
    REPEAT
        RESET (FOFFR,FRF);
        IOR:=IORESULT;
        IF (IOR<>0)
            THEN
                BEGIN
                    WRITELN(' IOR= ',IOR,' ! ENTER A GOOD FILENAME FOR FOFFR ');
                    READLN(FRF);
                END;
    UNTIL (IOR=0);
    { $I+ }
    FRFA[WELLCOUNT]:=FRF;
    WRITELN(' ok ');
    END; {proc CKFILE}

FUNCTION R24(VAR PA,PB:POS):REAL; {square of distance [PA,PB]}
BEGIN
    R24:=SQR(PA[1]-PB[1]) + SQR(PA[2]-PB[2]);
    END;

PROCEDURE PRNTLB;
BEGIN
    REWRITE(PR, '#6:');
    FOR J:=1 TO 17 DO WRITE(PR, ' ');
    WRITELN(PR,
    ' SCALE ',FPI:4:0,' ft/in. Map Center at (',XC:4:0,',',YC:4:0,') ft. ');
    XT:=28;
    IF XT>0 THEN
        BEGIN
            FOR J:=1 TO WELLCOUNT DO XT:=XT-(LENGTH(FRFA[J])) DIV 2;
            FOR J:=1 TO XT DO WRITE (PR, ' ');
            WRITE(PR, 'Streamlines from Files ');
            FOR J:=1 TO WELLCOUNT-1 DO WRITE(PR,FRFA[J],', ');
            WRITE(PR,FRFA[WELLCOUNT]);
        END
        ELSE
        BEGIN
            XT:=23;
            WRITELN(PR, ' ':XT, 'Streamlines from Files ',FRFA[1]);
            FOR J:=2 TO WELLCOUNT DO WRITELN(PR, ' ':XT+23,FRFA[J]);
        END;
    WRITELN(PR);
    CLOSE(PR,NORMAL);
    END;

BEGIN (MAIN)
    INIT;
    NEWFYL:=TRUE;
    WHILE NEWFYL = TRUE DO
        BEGIN

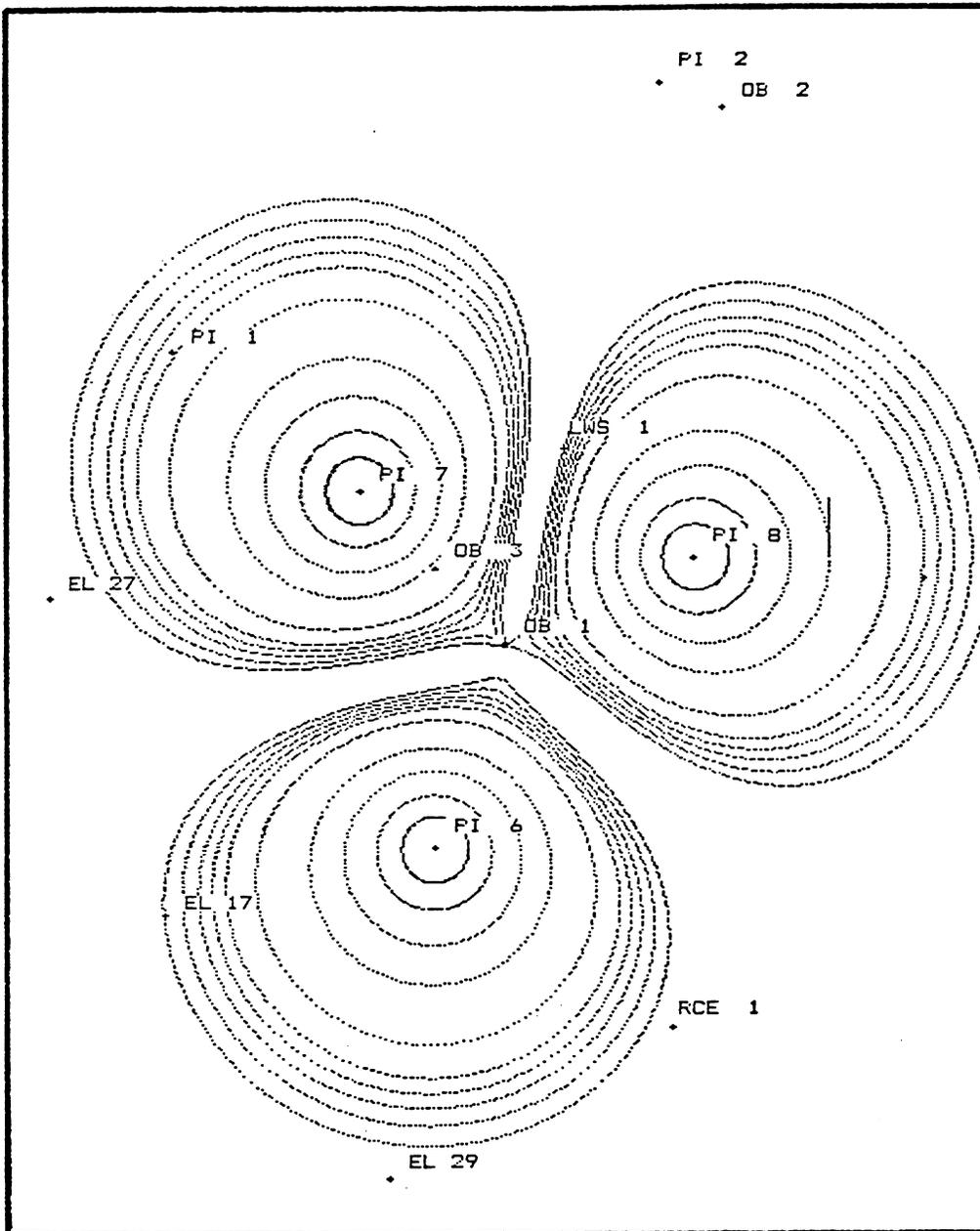
```

RCNU:STRMLYNS.TEXT

```

ITR:=1;
CNCTOK:=FALSE;
QUESTB;
CKFILE;
REPEAT
  SEEK(FOFFR, ITR-1);
  GET(FOFFR);
  UL:=120 DIV F;
  IF (EOF(FOFFR)<>TRUE)
    THEN
      BEGIN
        FOF:=FOFFR^;
        TRK:=FOF.FR;
        IF (ITR=1) THEN
          BEGIN
            P:=TRK[30];
            PT:=TRK[90];
            XS:=(P[1]+PT[1])/2;
            YS:=(P[2]+PT[2])/2;
            PI[1]:=XS;
            PI[2]:=YS;
            FOR I:=1 TO UL DO
              BEGIN
                J:=F*I;
                TH:=RP3D*J;
                XA[J]:=(XS-XC)/FPI+IR*COS(TH);
                YA[J]:=(YS-YC)/FPI+IR*SIN(TH);
              END;
            END; <if this was first trace>
            IF NOT CNCTOK
              THEN BEGIN
                P:=TRK[1];
                CNCTOK:=(R24(P,PI)>IR*FPI);
              END
            ELSE
              FOR I:=1 TO UL DO
                BEGIN
                  J:=F*I;
                  P:=TRK[J];
                  XN:=(P[1]-XC)/FPI;
                  YN:=(P[2]-YC)/FPI;
                  IF ((XN<>XA[J]) AND (YN<>YA[J])) THEN
                    CNNECT(XA[J],YA[J],XN,YN);
                  XA[J]:=XN;
                  YA[J]:=YN;
                END;
              ITR:=ITR+1;
              WRITE(' ');
            END {THEN}
          ELSE
            WRITELN('DONE');
          UNTIL (EOF(FOFFR)=TRUE);
          WELLCOUNT:=WELLCOUNT+1;
          CLOSE(FOFFR);
          WRITE('Want to look at another file on same map? ');
          READLN(CH);
          IF (CH<>'Y') AND (CH<>'y') THEN NEWFYL:=FALSE;
        END;
        WELLCOUNT:=WELLCOUNT-1;
        WRITELN('WELLCOUNT= ',WELLCOUNT);
      MAPPIT;
      FRNTLB;
    END.

```

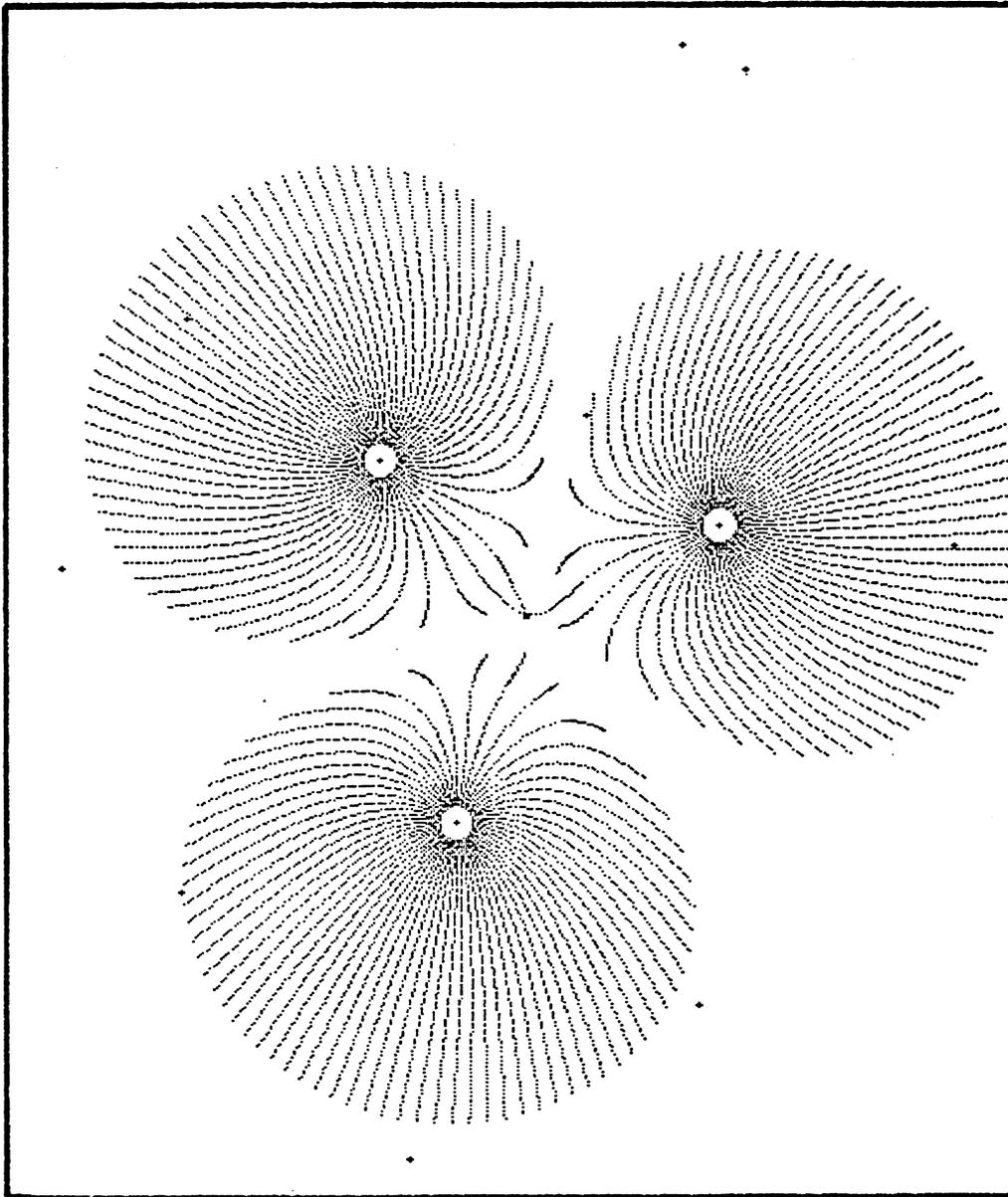


Scale: 193 ft/in

Fig. A4. Successive fronts from pattern injectors
(No injection into surrounding auxiliary wells)

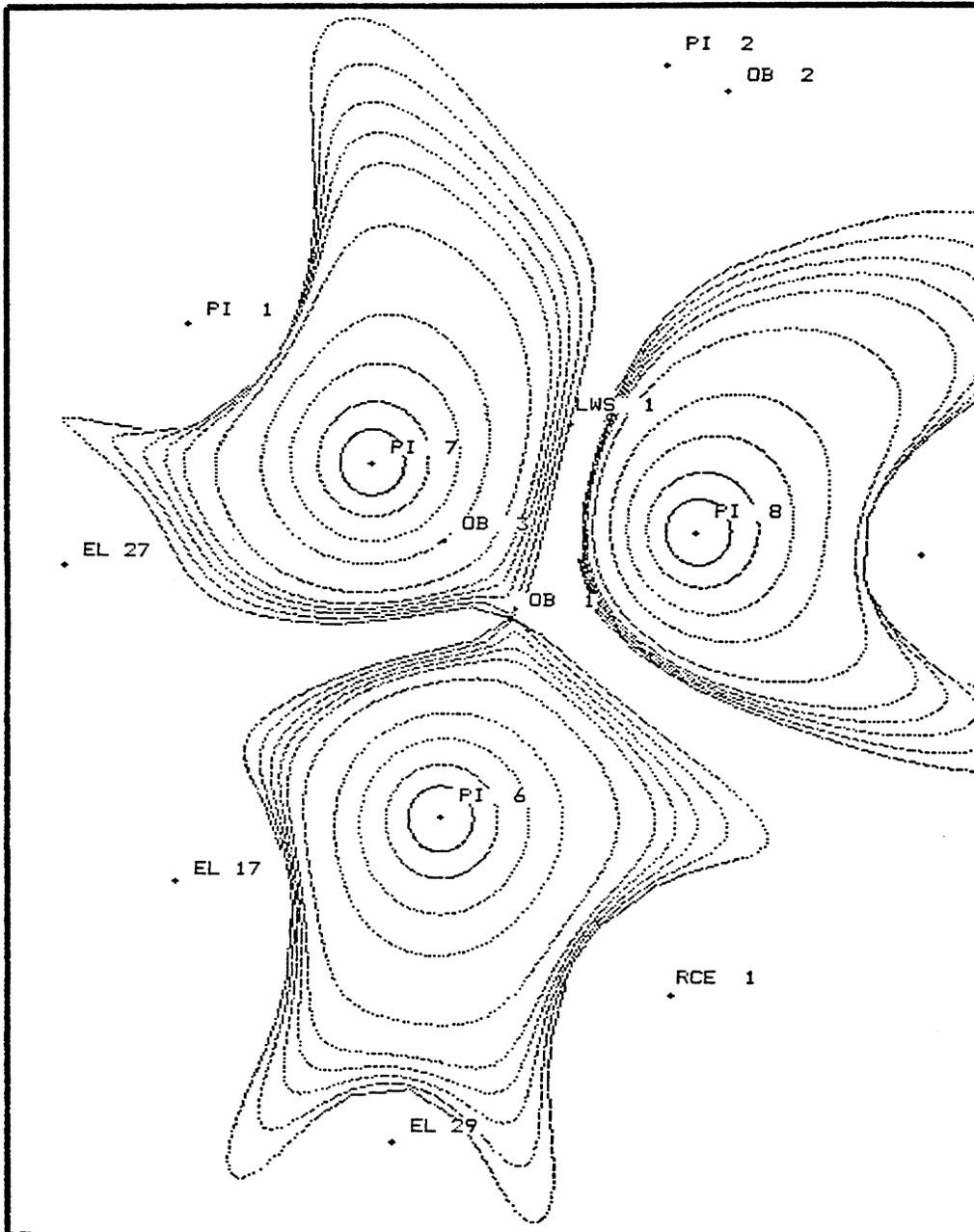
Fronts shown at 3750 bbl into each of PI-6, 7, 8

- 11250 bbl
- 26250 bbl
- 48750 bbl
- 93750 bbl
- 123750 bbl
- 138750 bbl
- 153750 bbl
- 172500 bbl
- 195000 bbl



Scale: 193 ft/in

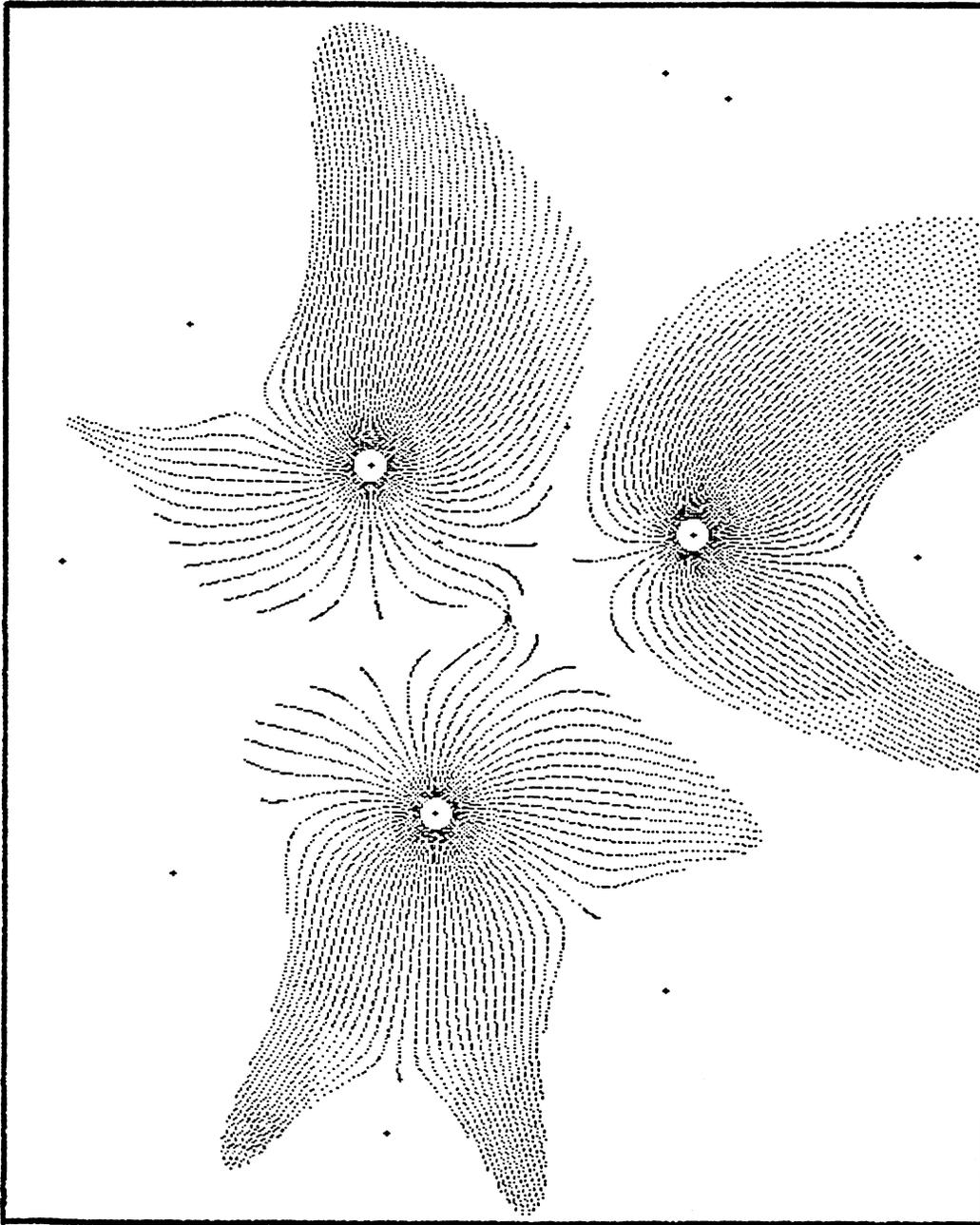
Fig. A5. Streamline pattern in mini-pilot area
(auxiliary injectors not operating).



Scale: 193 ft/in

Fig. A6. Successive fronts (from pattern injectors)
(injection also into auxiliary injection wells)

Fronts shown at 3750 bbl into each of PI-6, 7, 8
 11250 bbl
 26250 bbl
 48750 bbl
 93750 bbl
 123750 bbl
 138750 bbl
 153750 bbl
 172500 bbl
 195000 bbl



Scale: 193 ft/in

Fig. A7. Streamline pattern in mini-pilot area (auxiliary injectors operating).

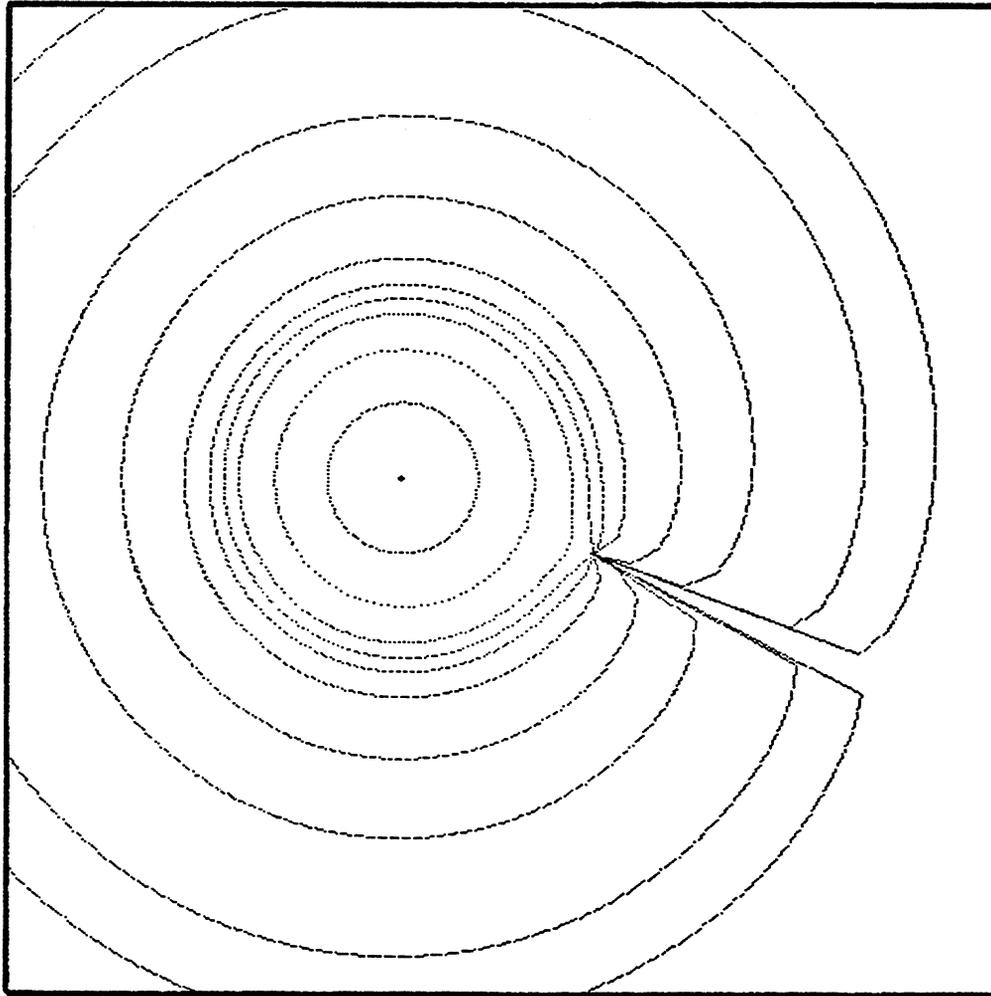
In Figs. A6 and A7, the same well rates in the producer OB-1 and in the primary injectors PI-6, 7 and 8 are maintained. In addition, however, fluid is injected into wells PI-1 at 100 bpd, PI-2 at 90 bpd, PI-5 at 70 bpd, RCE-1 at 100 bpd, EL-29 at 60 into EL-17 at 100 bpd and into EL-27 at 70 bpd. Because in this case too the programs were not run quite far enough, the reader's imagination is solicited to extend the streamlines shown in Fig. A7. In this figure, as in the previous case, well labels are omitted for the same reason. It is apparent that the additional injection into the "back-up wells" has greatly affected the distribution of flow in the overall area. At the same time, the distribution within the triangular mini-pilot area, itself, was not changed significantly. This disappointing situation follows mostly from the relative proximity of three primary producers. To increase markedly the sweep of an area like the mini-pilot area of Rock Creek, the injection into backup wells would need to be combined with an increase in the relative production rate of the center well.

A second application of the simplified flood pattern simulator described in this Appendix is concerned with the alternate area chosen for the mobility control test--the region between PI-2 and OB-2. It is the purpose of these calculations, displayed in Figs. A8 through A11, to show the influence of simultaneous production from a more distant well. Here also the scale of the maps is enlarged to show the critical area between the injector and the producer. The distant producer LWS-4 (which is the center of the northeast five-spot) is not shown, being far off these maps. From Fig. 5 of the main text, it can be seen that OB-2 is about 75 feet from PI-2, and LWS-4 is about 385 feet further from it in the same direction. For these maps, all of the well labels have been omitted to avoid obscuring any of the fronts or streamlines. It is believed, though, that the reader will be able to recognize PI-2 and OB-2 after only a short period of panic. The production from OB-2 is only one tenth of the injection into PI-2, and the third well involved is the original producer of the pattern, LWS-4. Here again the effect of proximity is almost overwhelming. Initial data for the calculations leading to Figures A8 and A9 included the well rates of -100 bpd for PI-2, +10 for OB-2 and 0.0 for LWS-4. As can be seen, twelve of the 120 streamlines that issue from PI-2 are terminated in OB-2. Cusping of the fronts is observable from front #6, after the injection of 10,000 barrels.

A comparison can be made between these results and those shown in Figs. A10 and A11, where the well rates for the same three wells were -100, +10 and +100. In this case, even with the relatively large additional production from LWS-4, flow in the smaller region directly between PI-2 and OB-2 is not much affected. In particular, it can be seen from a comparison of the figures that the cusping of the fronts in this case is only slightly more advanced in front #6 after the same quantity of injection. This pattern similarity does not extend, however, to the flow at greater distances from the PI-2-OB-2 interwell region where the influence of the production from LWS-4 is quite important.

Conclusion

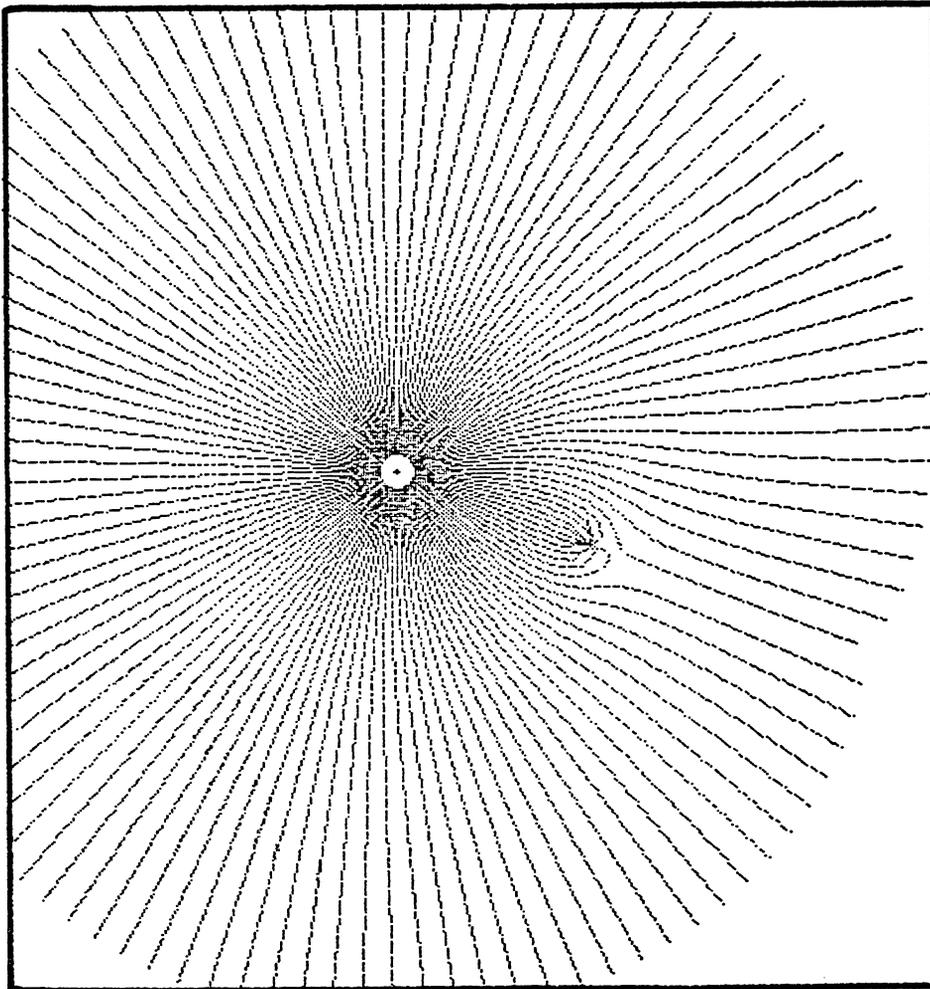
It is necessary to reiterate, as discussed earlier, that the flow descriptions given by this calculation program do not take into account a number of important aspects of the actual reservoir problems. Nevertheless they do give an accurate indication of displacement in the simplified reservoir situation, and when taken together with our firm knowledge of the direction of



Scale: 68 ft/in

Fig. A8. Displacement fronts from PI-2
(production from OB-2 only; LWS-4 turned off)

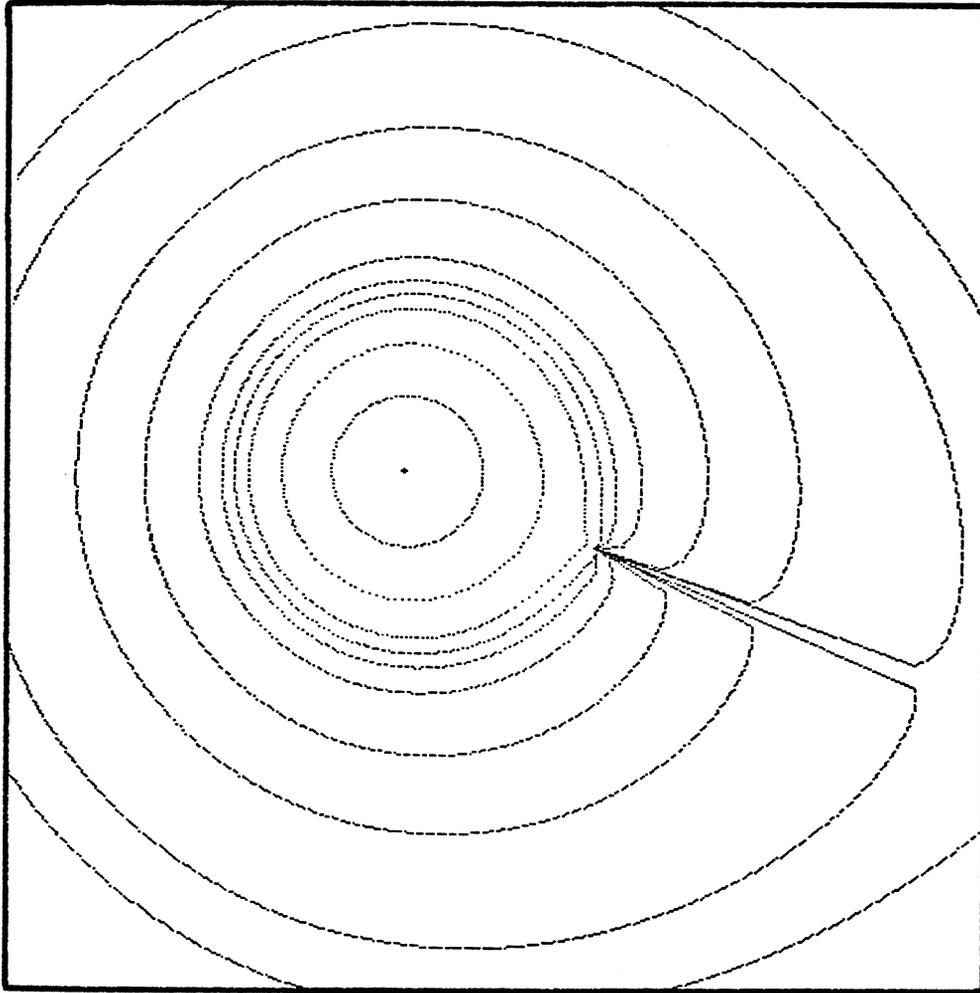
Fronts shown at	2000 bbl injected	
	6000 bbl	"
	10000 bbl	"
	12000 bbl	"
	14000 bbl	"
	18000 bbl	"
	30000 bbl	"
	50000 bbl	"
	90000 bbl	"
	120000 bbl	"



Scale: 68 ft/in

Fig. A9. Streamlines from PI-2

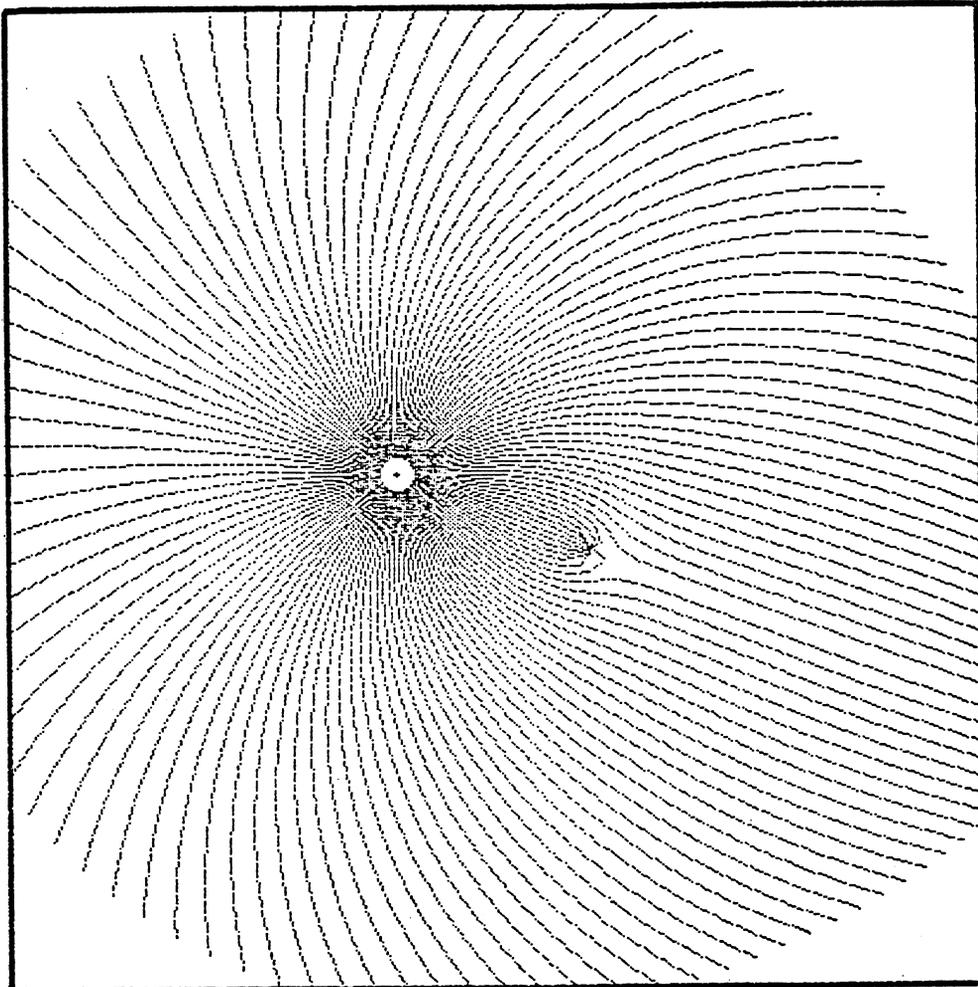
Well rates:	PI-2	-100 BPD
	OB-2	+ 10 BPD
	LWS-4	0



Scale: 68 ft/in

Fig. A10. Displacement fronts from PI-2
(production from LWS-4--out of frame to right)

Fronts shown at	2000 bbl	injected
	6000 bbl	"
	10000 bbl	"
	12000 bbl	"
	14000 bbl	"
	18000 bbl	"
	30000 bbl	"
	50000 bbl	"
	90000 bbl	"
	120000 bbl	"



Scale: 68 ft/in

Figure All. Streamlines from Injector PI-2

Well rates:	PI-2	-100 BPD
	OB-2	+ 10 BPD
	LWS-4	+100 BPD

error, constitute reliable, semi-quantitative pictures of the average displacement behavior in the real rock. In particular, the comparisons given can be taken as good indications of the influence of changes in well rates on the displacement patterns.

APPENDIX B - THE POINT DILUTION METHOD

Introduction

The Point Dilution Method (PDM) is an experimental procedure of hydrology by which to measure the magnitude of the horizontal fluid velocity field in the permeable formation surrounding a well. This is done by introducing into the wellbore a tracer that is initially absent from the formation water, and allowing the natural motion of that water to dilute the wellbore contents gradually by its horizontal flow through the well. If during this dilution process the water in the wellbore is continually stirred, then the decrease of tracer concentration will be exponential in time, with the decay constant directly proportional to the horizontal velocity field. In normal use of the method, tracer concentration in the well fluid has been measured without removing any fluid from the well--the PDM is a "zero-net-flow" method, in which as much formation fluid flows into one "side" of the well as out of the other. A schematic representation of this situation is shown in Fig. B1.

This situation was also a goal of an adaptation of the PDM to oilfield use that was recently considered and prepared for in the West Virginia mobility control test. The major innovations required in that design are described below.

Deep Well Design of the Point Dilution Method

For measurement of "drift" velocity of fluid in the formation part of the wellbore, an openhole completion is required. Further, it would be necessary to isolate the horizon in question by packers, and to allow fluid communication with a stirring pump at the surface by a dual completion. Details of a closed circulating system to accomplish this were worked out with the aid of the Pennzoil Exploration and Production Company and are shown in Fig. B2. This system was intended to be used in well OB-3, situated about half-way between PI-7, the mini-pilot injector into which it had been decided to introduce "CO₂-foam," and the producer OB-1. The Darcy velocity of fluid past OB-3 would be measured by monitoring the rate of decrease of iodide ion in the water introduced into the wellbore and continually circulated during the experiment. The effectiveness of CO₂ foam as a mobility control additive was to be assessed by observing the time variation of the velocity as the CO₂-foam displacement front approached. It was reasoned that the presence of major instability fingers on the injection front would cause relatively rapid changes in the measured velocity past the observation well as the front approached.

In addition to the velocity measurements to be accomplished by monitoring I⁻ concentration, a data-acquisition system prepared for use at OB-3 contained two other chemical analysis electrodes--one to measure pH and the other to measure calcium ion concentration. Because the surface pressure of the wellbore fluid was anticipated to be about 500 psi, it was necessary to remove samples of the fluid periodically so that the electrochemical measurements could be performed at atmospheric pressure. Arrangements were also made to save these samples for possible later analysis. In addition, when the samples were first taken, they were held for a short time to achieve gravitational separation. Any oil produced with the sample could then be kept out of the electrode system and conducted to separate storage.

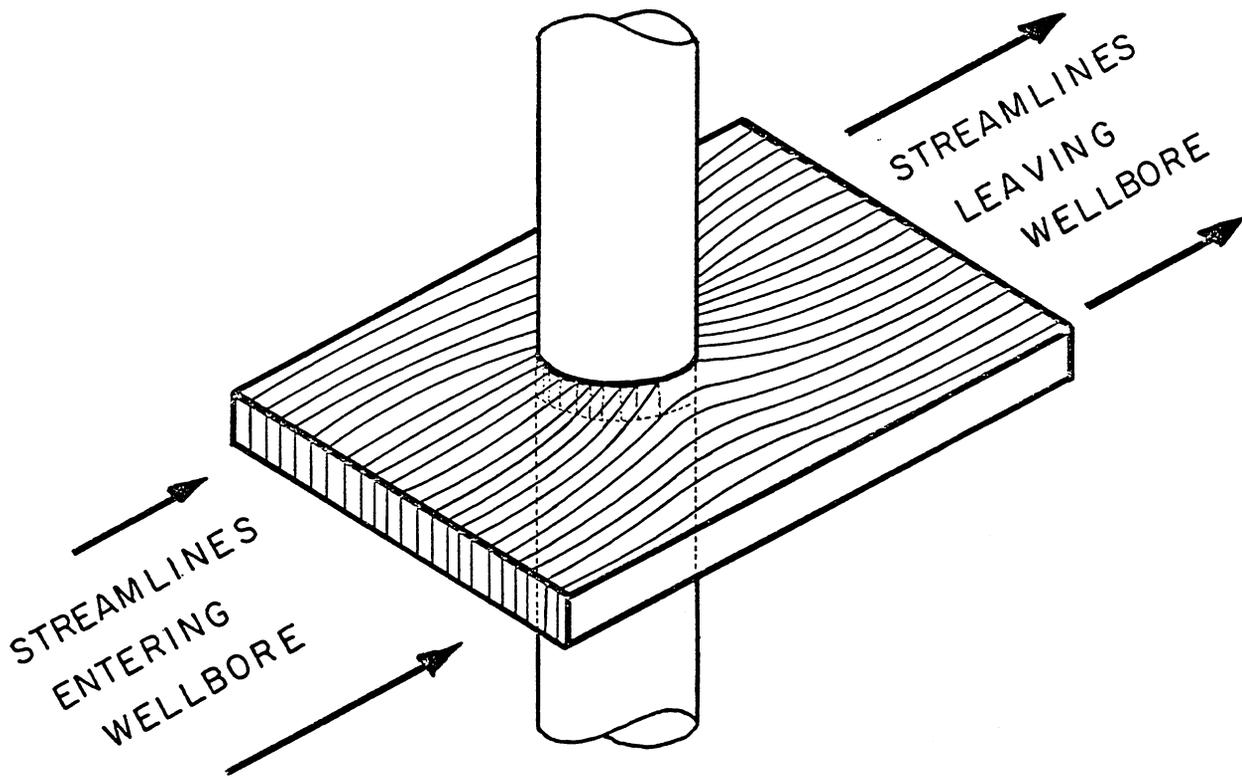


Fig. B1. Drift velocity past shut-in well.

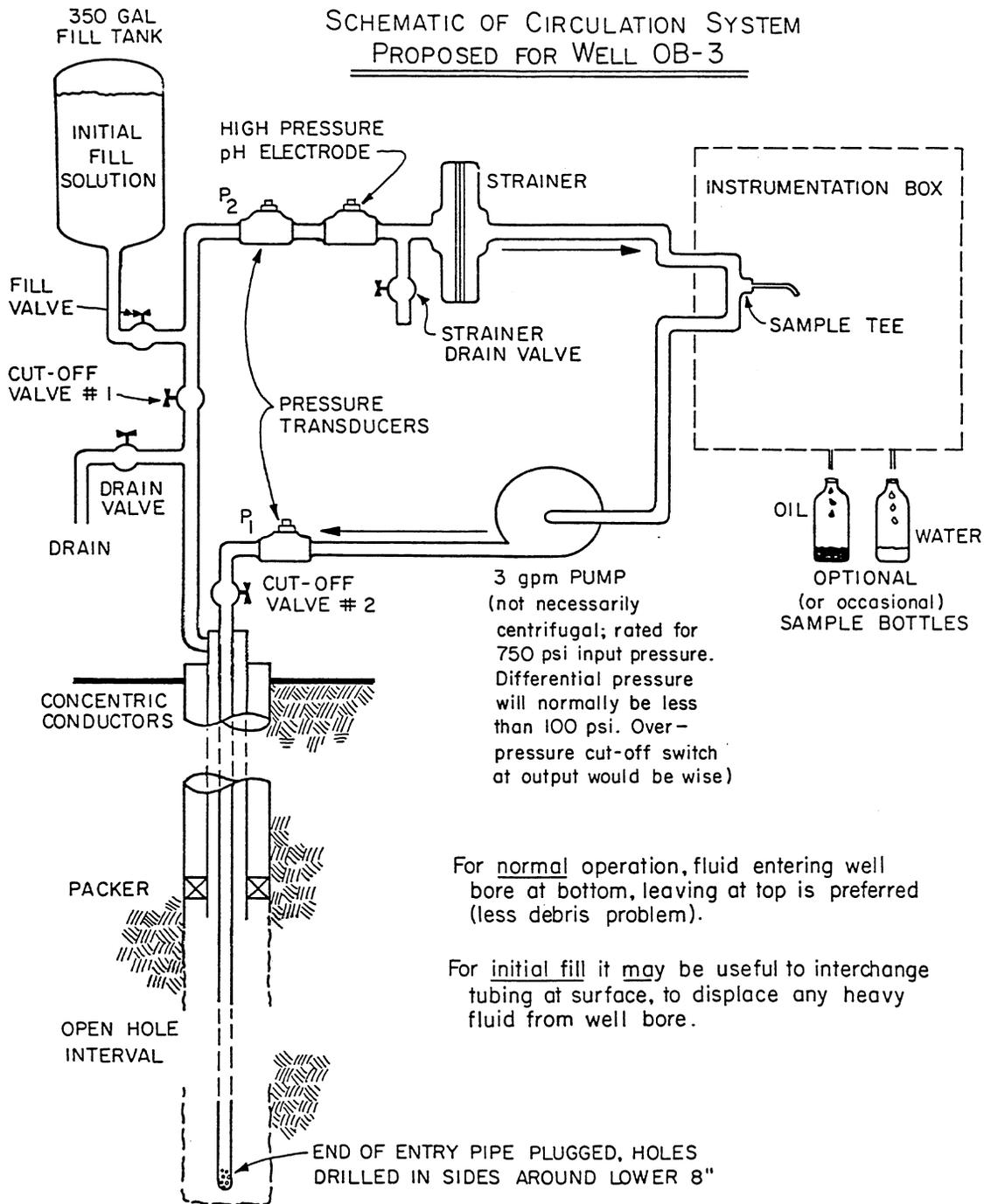


Fig. B2. Proposed circulating system for Point Dilution Method experiment at well OB-3.

The entire system was designed for automatic operation and data collection. A small computer, programmed in BASIC, contained the instructions by which the sample valves were periodically opened and closed. After appropriate delays in this regular sequence of events, the electrode systems were operated and then calibrated by the alternate use of standard solutions that could also be flushed through the flow system containing the electrodes. In addition, data was periodically stored on a magnetic cassette which would require replacement only once a week. The readings of several pressure and temperature sensors were included in the tape record to enable proper operation of the system to be independently assessed. The entire analytical apparatus together with its associated control equipment were contained inside a thermally insulated box to be installed next to the surface circulating pump alongside OB-3.

Descriptions of this system in greater detail are given in the quarterly report dated March 15, 1984 [Heller, 1984].

Because of the need for withdrawal of fluid samples, the question arose of the sensitivity of the Point Dilution Method to such flow. This called for some mathematical clarification of the basis of PDM and the value of the coefficients to be used. These are discussed in the following sections.

The Concentration Decay Time

The rate of decrease of an initial tracer concentration in the wellbore depends on the drift velocity of fluids in the formation and on various geometric parameters of the well. Between the packers that define the height over which it is exposed to the formation, the well receives flow on one side of the wellbore face and allows it back out into the formation through the opposite side.

If the total quantity of tracer in the wellbore and associated volume is represented by M , then its concentration in the wellbore fluid is

$$C = M / (\pi a^2 h + V_a) \quad (B1)$$

Here a is the radius of the well and h is the height between packers. V_a is the associated volume (completion tubing to the surface, and the pump and circulating system).

No new tracer enters the wellbore on the input side of the well, but tracer does leave on the other side. The rate of decrease is the product of the average concentration of tracer inside the wellbore and the rate of flow out of the well on the "downstream" side. That flow, while it is obviously proportional to the Darcy drift velocity as it would be measured far from the well, will be greater than the product of the velocity with $2ah$ (the sideways area presented by the well) because of distortion in the flow field caused by the presence of the well. For now, the proportionality constant can be denoted as α . Then the rate of loss of tracer out of the side of the well is

$$dM/dt = - C 2ah\alpha U_{inf} \quad (B2)$$

where U_{inf} is the drift velocity. The rate of change of concentration in the whole volume is then given by

$$dC/dt = - (2ah\alpha U_{inf}) C / (\pi a^2 h + V_a) \quad (B3)$$

This differential equation defines an exponential decrease of the tracer concentration, with the characteristic decay time of

$$T_c = (\pi a^2 h + V_a) / (2ah\alpha U_{inf}) \quad (B4)$$

It turns out that when the rate of fluid withdrawal is zero, and if the wellbore is completely exposed to the formation along its length, h , and the rock immediately surrounding the wellbore is undamaged (i.e., no skin effect), then the value of the constant is 2.0. To evaluate α in the more general case when the fluid withdrawal rate is not zero, one must examine the pressure field in the rock surrounding the well. This is done in the next section.

The Pressure Field

The horizontal flow of fluids in the rock is described here by use of Darcy's equation in the differential form, in which the vector velocity field is given as

$$\vec{U} = - \frac{k}{\mu} \vec{\nabla} p \quad (B5)$$

Here k is the permeability of the porous medium and μ , the fluid viscosity. The gradient of the pressure is represented by $\vec{\nabla} p$.

The most important aspects of the present problem of fluid motion around a well are described by this familiar equation, along with the assumption of reservoir homogeneity. The geometry can be simplified by considering only two space coordinates, the cylindrical variables (r, ϕ) with the origin centered in the wellbore. If the mobility $\lambda = k/\mu$ is independent of the coordinates and if the fluid is considered to be incompressible, differentiation of Darcy's equation leads to Laplace's equation in the pressure, which for the above cylindrical coordinates is

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} + \frac{1}{r^2} \frac{\partial^2 P}{\partial \phi^2} = 0 \quad (B6)$$

Reasonable and appropriate solutions of Eq. (B6) in this case are generated by presuming that $P(r, \phi)$ is a separable function of the coordinates, that is, that P is a sum of products of the form:

$$P_{i(r, \phi)} = R_{i(r)} \phi_{i(\phi)} \quad (B7)$$

Taking the indicated derivatives and substituting into Eq. (B6) gives

$$\phi_i R''_i + \phi_i R'_i / r + \phi''_i R_i / r^2 = 0$$

where the primes designate differentiation. If we now divide by $(\phi_i R_i / r^2)$, we obtain the separated equation

$$r^2 R''_i / R_i + r R'_i / R_i = - \phi''_i / \phi_i$$

In this equation, the lefthand side is a function of the radial coordinate r alone, while the righthand side depends only on the angle ϕ . Each side must then be equal to the same constant that can be designated m_i^2 . (For m_i real, this expresses a definite choice for the sign of the constant. This choice is necessary if the functions of angle $\phi_i(\phi)$ are to be periodic, as they must be for $P_i(r, \phi)$ to be single-valued. In addition, periodicity also requires that the m_i be equal to the successive integers). We then have the two ordinary differential equations

$$\phi''_i + m_i^2 \phi_i = 0 \tag{B9a}$$

and

$$R''_i + R'_i / r - m_i^2 R_i / r^2 = 0 \tag{B9b}$$

The general solutions of Eq. B9a are

$$\phi_i(\phi) = A_i \cos m_i \phi + B_i \sin m_i \phi$$

for $|m_i| > 0$, and (B10a)

$$\phi_o(\phi) = A_o \phi + B_o$$

for $m_i = m_o = 0$

The general solution for Eq. B9b are

$$R_i(r) = C_i r^i + D_i r^{-i}, \text{ for } |m_i| > 0$$

and

$$R_o(r) = C_o + D_o \ln r, \text{ for } m_o = 0 \tag{B10b}$$

The general solution to Laplace's equation, i.e., for Eq. (B6), in this geometry is then the sum of products of the expressions in Eqs.(B10a) and (B10b). From this large collection of possible solutions, those are to be

selected which also satisfy the boundary conditions imposed by the physical situation.

For instance, the well-known case in which the external pressure field is symmetrical about the well and is driven by a production rate Q corresponds to the solution with:

$$\begin{aligned} m &= 0, \\ A_0 &= 0, \\ B_0 D_0 &= + \frac{\mu Q}{2\pi h k}, \end{aligned} \tag{B11a}$$

and

$$B_0 C_0 = 0, \tag{B11b}$$

where h is the thickness of the pay zone. The constant $B_0 D_0$ may also contain a numerical factor so that with the use of preferred units for μ , k , h , and Q , it will be in convenient units of pressure. A non-zero value of $B_0 C_0$ may be similarly needed, if a pressure known at some other part of the flow system than the wellbore is to be utilized. In equation form, then, the symmetrical pressure field around the well due to a production rate Q is the familiar expression:

$$P(r) = + \frac{\mu Q}{2\pi h k} \ln r \tag{B12}$$

The Zero Net Flow Solution

In the circumstances under which the usual point dilution experiment is performed, there is no net production from the well, and the observation of concentration change is made in order to evaluate the external flow field past the well. If the source of this flow is a very distant well or set of wells, it is reasonable to consider that in the absence of the observation well, the streamlines would be parallel, representing a constant Darcy flow velocity of \vec{U}_{inf} in this region. The placement of the observation well would modify this pattern in the following way. Those streamlines of the external flow which are headed toward the well will be deflected into it, so they intersect its boundary at right angles. This is because horizontal flow through the wellbore is accompanied there by zero Darcy pressure gradient, and so the cylindrical perimeter of the wellbore must be an isopotential surface. Similarly, nearby streamlines will be bent towards and then away from the well as they pass by it. At larger distances from wellbore, the amount of deflection will be less; further than three or four well-diameters, one would expect the flow velocity to be very little different from the constant \vec{U}_{inf} .

This Zero Net Flow solution is obtained from the general solutions of Eqs. (B10a) and (B10b) by setting non-zero values only for A_1 , B_1 , C_1 and D_1 , with $m_1 = 1$. We then obtain a pressure distribution which is symmetrical about a line through the origin. This line will be the Y axis, if $A_1 = 0$ as well. The

solution for zero net flow is then given by

$$P_{z(r,\phi)} = B_1(C_1 r + D_1/r) \sin \phi \quad (B13)$$

In order for the far-field velocity to be \vec{U}_{inf} , and for Darcy's equation to be satisfied at large distances where the term in $1/r$ becomes small, we must have

$$B_1 C_1 = \frac{\mu}{k} U_\infty \quad (B14)$$

Here, the scalar constant U_∞ is the magnitude of the constant vector \vec{U}_{inf} :

$$U_\infty = |\vec{U}_{inf}| \quad (B15)$$

It is convenient to put the zero of pressure, for this solution, at the wellbore surface, i.e., at $r = a$, where a is the radius of the well. This specification fixes the value of the constant D_1 . Setting $P(a,\phi) = 0$ in Eq. (B13) we obtain $D_1 = -C_1 a^2$, and thus, using Eq. (B14), the equation for the pressure variation in this Zero Net Flow case becomes:

$$P_{z(r,\phi)} = \frac{\mu}{k} U_\infty \left(r - \frac{a^2}{r} \right) \sin \phi \quad (B16)$$

The Combined Solution

Both of the solutions given, Eqs. (B12) and (B16) satisfy Laplace's equation for the spatial variation in pressure, and so does their sum. In fact, the addition of Eqs. (B12) to (B16) does not alter the needed boundary conditions, since the velocity far away from the well, obtained by taking the gradient of the pressure, will still approach U_∞ as r increases, and the potential will be zero at the wellbore. This combined solution is

$$P_c(r,\phi) = \frac{\mu U_\infty}{k} \left[\left(r - \frac{a^2}{r} \right) \sin \phi + a \tilde{A} \ln r \right] \quad (B17)$$

where

$$\tilde{A} = \frac{Q}{2\pi h a U_\infty}$$

It is convenient for computation to write this equation in rectangular coordinates. Using the usual transformation equations:

$$r = (x^2 + y^2)^{1/2} \quad (\text{B18})$$

$$\phi = \tan^{-1}(y/x),$$

then

$$P_{c(x,y)} = + \frac{\mu U_{\infty}}{k} \left[y \left(1 - \frac{a^2}{(x^2+y^2)} \right) + \frac{a\tilde{A}}{2} \cdot \ln (x^2 + y^2) \right] \quad (\text{B19})$$

The dimensionless parameter \tilde{A} is the ratio of the average flow velocity into the well at its perimeter to the parallel velocity at great distances, U_{∞} . When this ratio is zero, we have the Zero Net Flow solution. When U_{∞} is zero, the solution becomes that for the pressure distribution when there is no "drift velocity," but production rate Q . Subject to the various assumptions cited, the equation gives the exact pressure distribution for any value of \tilde{A} .

Velocity Computation and Display

While Eq. (B17) or (B19) give the pressures around the well, it is both instructive and helpful in obtaining a graphical representation to calculate the Darcy velocities resulting from that pressure distribution. In component form of Eq. (B1), these are:

$$(v_x, v_y) = - \frac{k}{\mu} \left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y} \right) \quad (\text{B20a})$$

or

$$(v_r, v_{\phi}) = - \frac{k}{\mu} \left(\frac{\partial P}{\partial r}, \frac{1}{r} \frac{\partial P}{\partial \phi} \right) \quad (\text{B20b})$$

From Eq. (B20a) and by differentiation of Eq. (B19), one obtains

$$v_x = - U_{\infty} \left[\frac{2xya^2}{(x^2+y^2)^2} + \frac{ax\tilde{A}}{x^2+y^2} \right] \quad (\text{B21a})$$

and

$$v_y = - U_{\infty} \left[\frac{2y^2a^2}{(x^2+y^2)^2} + \frac{x^2 + y^2 + ay\tilde{A} - a^2}{x^2 + y^2} \right] \quad (\text{B21b})$$

Eqs. B21a and B21b are used to calculate the fluid velocities at different locations, so that the streamline paths can be plotted. Some of these, for various values of the dimensionless ratio \tilde{A} , are given as Figs. B3 to B8. Note that Fig. B3, the Zero Net Flow case, is that for which $\tilde{A} = 0$. The PASCAL

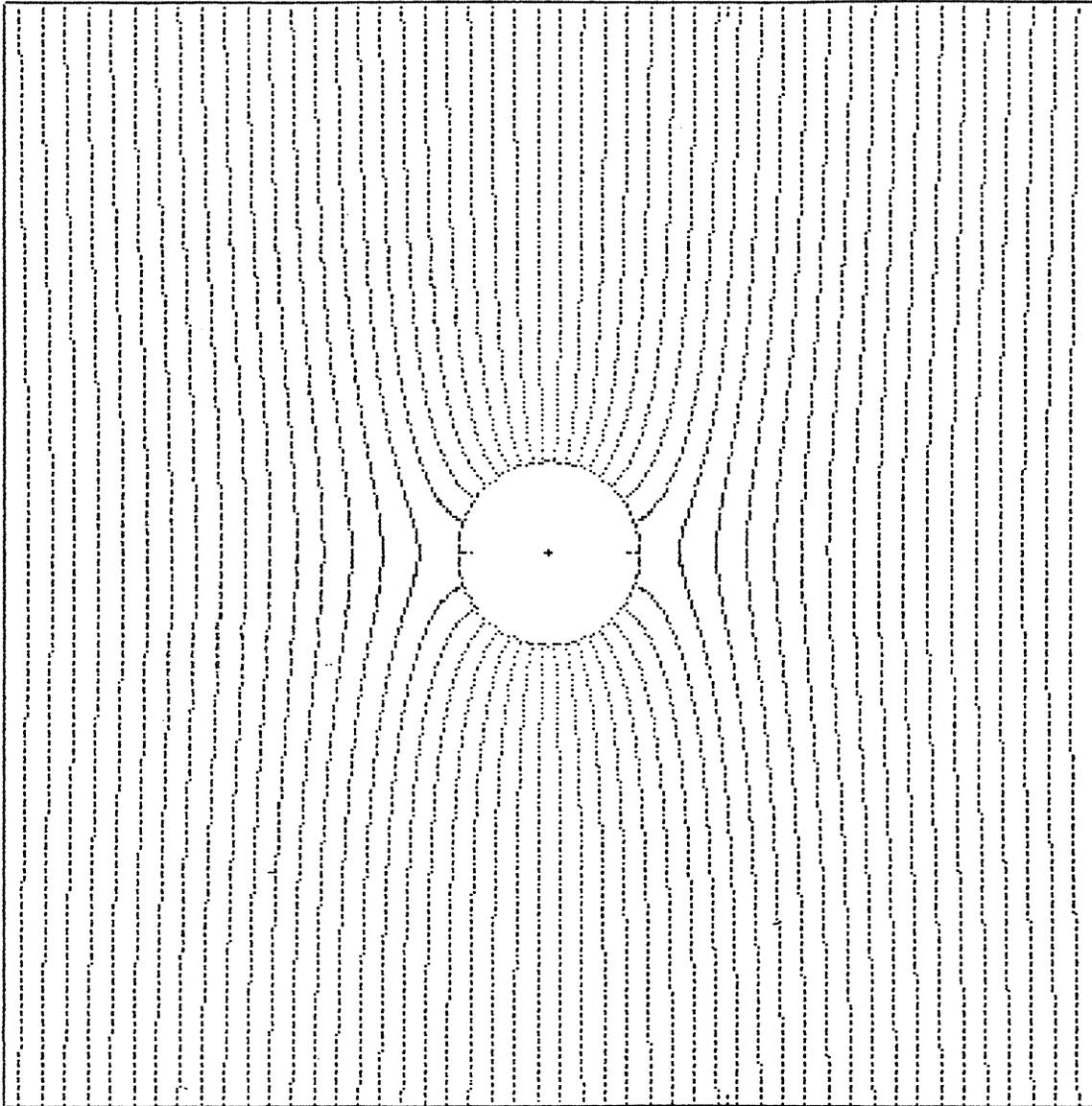


Fig. B3. Streamlines around well - Point Dilution Method.
 $\tilde{\Lambda} = 0.0$

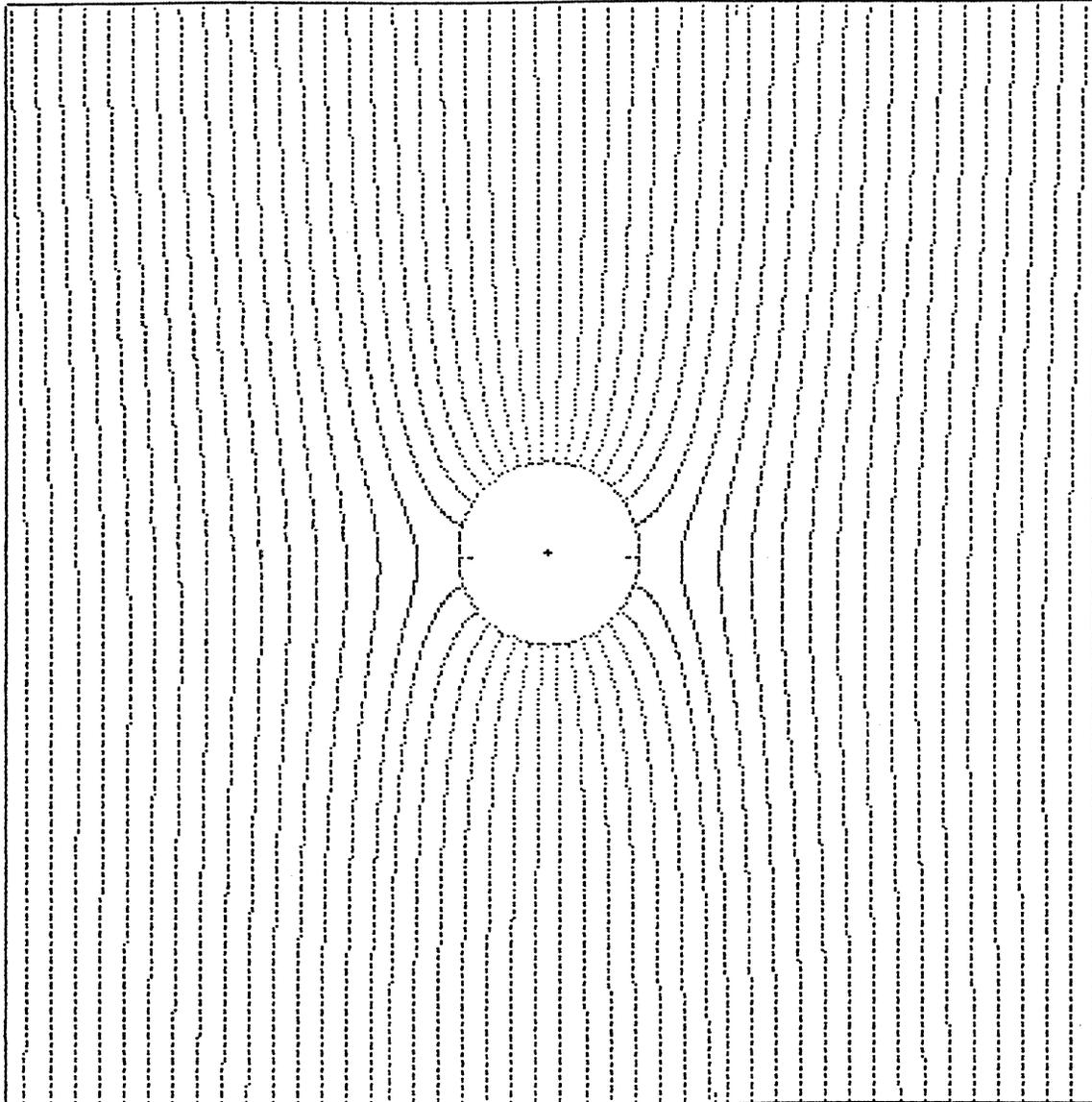


Fig. B4. Streamlines around well - Point Dilution Method.

$$\tilde{A} = 0.10$$

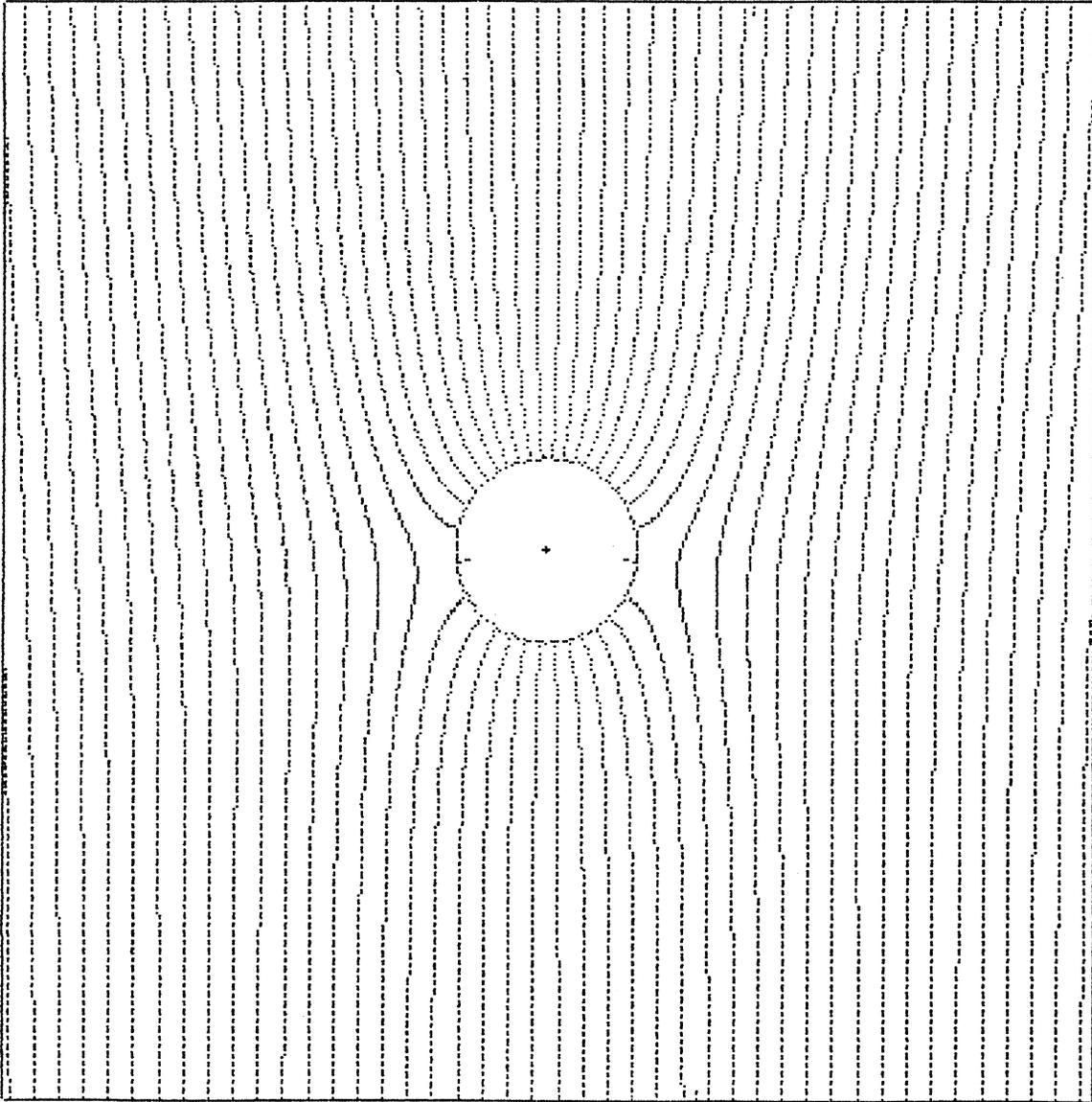


Fig. B5. Streamlines around well - Point Dilution Method.

$$\tilde{A} = 0.25$$

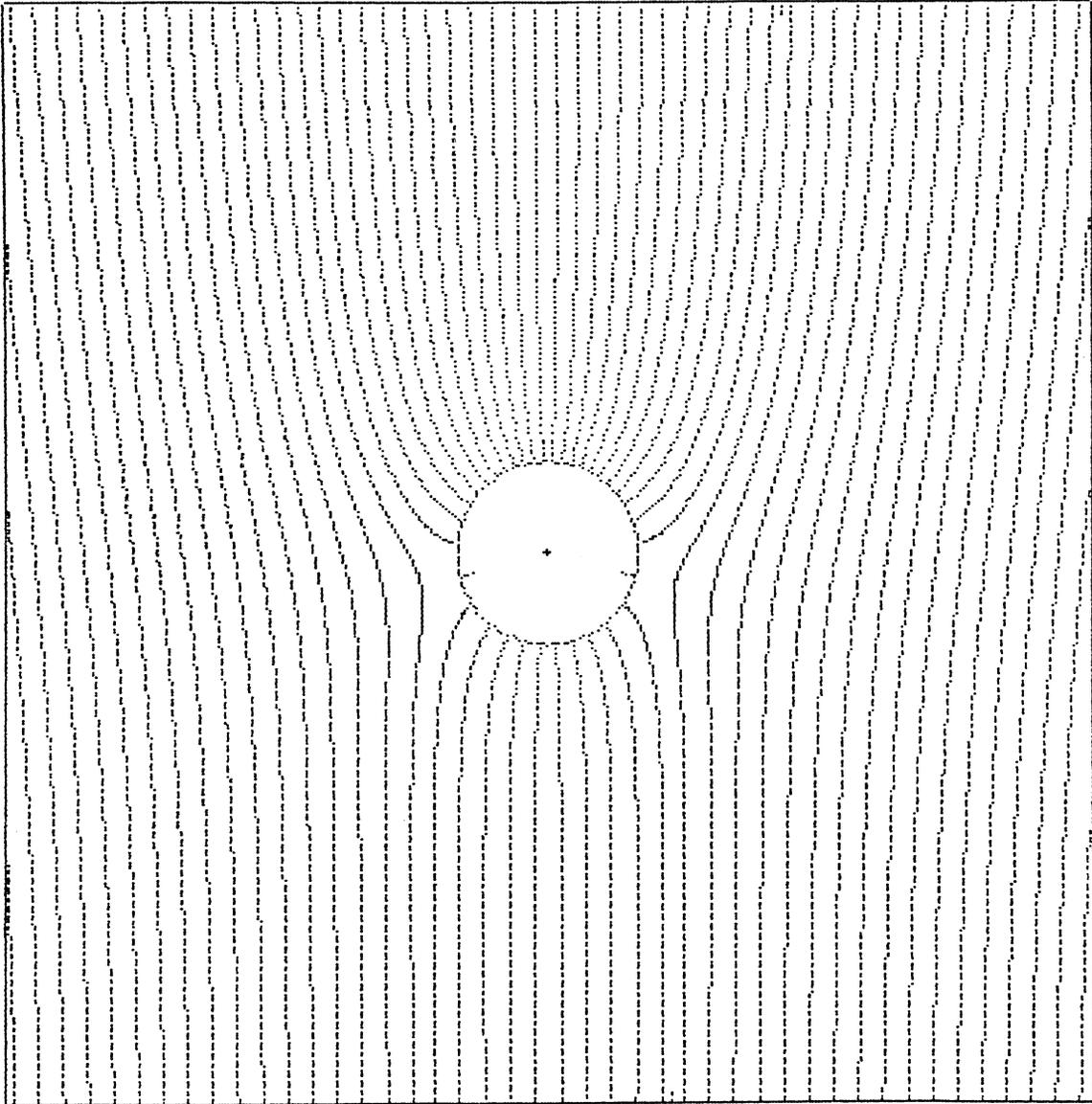


Fig. B6. Streamlines around well - Point Dilution Method.
 $\tilde{A} = 0.50$

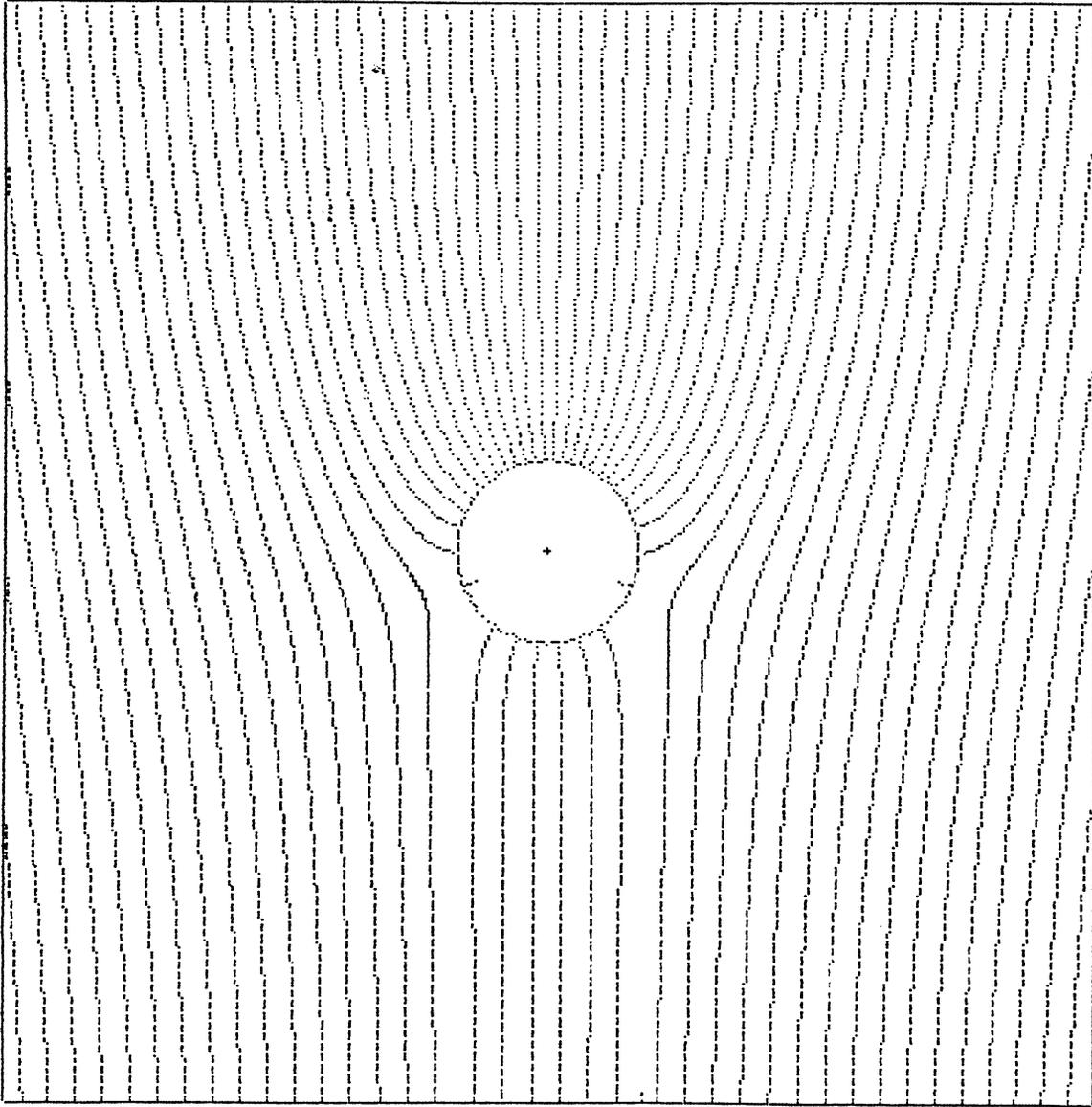


Fig. B7. Streamlines around well - Point Dilution Method.

$$\tilde{A} = 0.75$$

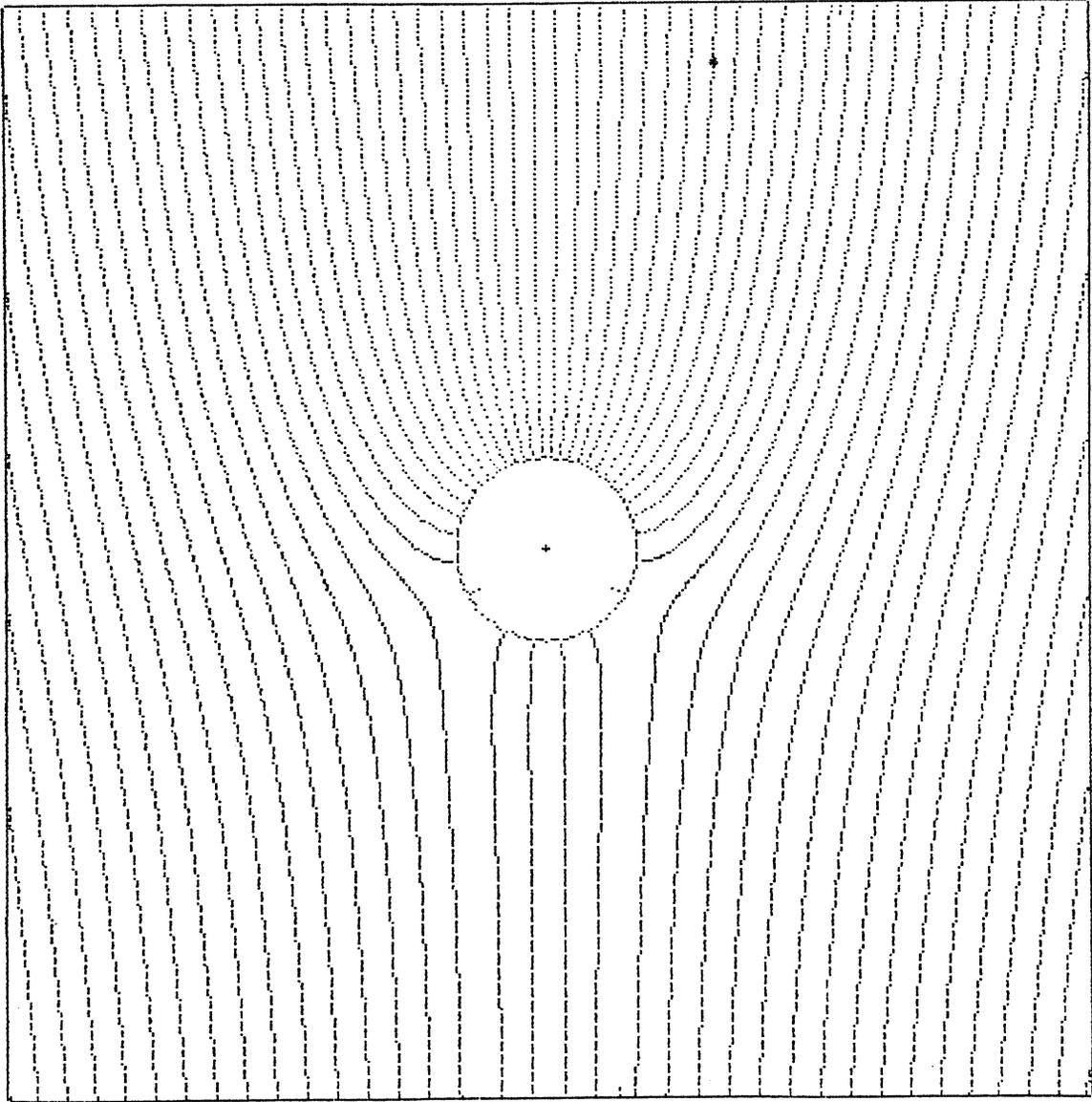


Fig. B8. Streamlines around well - Point Dilution Method.
 $\tilde{A} = 1.00$

program that produced these plots is listed in Fig. B10.

Flux Leaving the Wellbore - The Value of α

The derivatives with respect to the radial coordinate, evaluated at the wellbore, will enable the flow into and out of the well to be computed. From Eqs. (B17b) and (B13), one can write:

$$v_{r(r,\phi)} = -U_{\infty} \left[\left(1 + \frac{a^2}{r^2} \right) \sin \phi + \frac{a\tilde{A}}{r} \right] \quad (\text{B22a})$$

from which

$$v_{r(a,\phi)} = -U_{\infty} (2 \sin \phi + \tilde{A}) \quad (\text{B22b})$$

This quantity is the outward velocity of fluid flow from the wellbore. (The azimuthal component of fluid velocity at $r = a$ is zero, since the wellbore is an isopotential.) We note that a sign change occurs at the angle,¹ ϕ_0 , where

$$\phi_0 = \sin^{-1} \left(-\frac{\tilde{A}}{2} \right) \quad (\text{B23})$$

For angles between ϕ_0 and 90° (and for the mirrored interval to the left of the Y axis), the value of $v_{r(a,\phi)}$ is less than zero--that is, the streamlines enter the well. For angles between -90° and ϕ_0 (and again for the mirror image of this interval on the left of the Y axis), the streamlines leave the well. If $\tilde{A} = 0$, then these neutral angles are at 0° and 180° , and the area of the well through which fluid enters is equal to the area through which it leaves--and the net fluid entry is zero. As \tilde{A} increases, the neutral angles move downward towards the lowest point of the wellbore at 270° . The two angles meet and reach 270° when $\tilde{A} = 2$. For $\tilde{A} > 2$ (i.e., for the net production rate $Q > 4\pi haU_{\infty}$), there are no streamlines leaving the well, although the density with which streamlines enter (that is, the Darcy velocity) is considerably greater at $\phi = 90^\circ$ than at $\phi = 270^\circ$.

The total flow out of the wellbore would be obtained by multiplying $2h$ into the integral (over the perimeter interval $-\pi/2$ to $\pi/2$) of the radial velocity given in Eq. (B22b). This product is just equal to the negative of Q , the production rate. To calculate the coefficient α , though, we are interested only in the mixed wellbore fluid leaving the wellbore, which is

$$X = 2ha \int_{-\pi/2}^{\phi_0} v_{r(a,\phi)} d\phi$$

¹The angle is measured counter-clockwise from the positive x-axis in these equations.

$$\begin{aligned}
X &= 2haU_{\infty} \int_{-\pi/2}^{\phi_0} (2\sin\phi + \tilde{A}) d\phi \\
&= 2haU_{\infty} \left(2\sqrt{1-\tilde{A}^2/4} - \tilde{A}\cos^{-1}(\tilde{A}/2) \right)
\end{aligned} \tag{B24}$$

But this is only part of the fluid leaving the wellbore system--the remainder is Q itself, that leaves the wellbore system at the surface. So the total rate at which tracer is lost from the fluid is

$$-\frac{dM}{dt} = C(X + Q) \tag{B25}$$

Putting Q in terms of \tilde{A} , this becomes

$$-\frac{dM}{dt} = 2ahCU_{\infty} [2\sqrt{1-\tilde{A}^2/4} + \tilde{A}(\pi - \cos^{-1}(\tilde{A}/2))] \tag{B26}$$

The change of tracer concentration is obtained by dividing this by the total fluid volume, so

$$\frac{dC}{dt} = -\frac{2ahCU_{\infty} [2\sqrt{1-\tilde{A}^2/4} + \tilde{A}(\pi - \cos^{-1}(\tilde{A}/2))]}{\pi a^2 h + V_A} \tag{B27}$$

By comparison with Eq. (B4) we see that the parameter α is

$$\alpha = [2\sqrt{1-\tilde{A}^2/4} + \tilde{A}(\pi - \cos^{-1}(\tilde{A}/2))] \tag{B28}$$

This may also be written

$$\alpha = 2\sqrt{1-\frac{\tilde{A}^2}{4}} + \tilde{A}\cos^{-1} \frac{\tilde{A}}{2} \tag{B29}$$

The value of this coefficient varies almost linearly with \tilde{A} , as can be seen in Fig. B9. So long as the dimensionless production parameter \tilde{A} is held to a small value (say 0.1 or less), then the required correction will be small.

In this case a recommended procedure to obtain the drift velocity U_{∞} would be as follows. From the observed slope of the concentration decay on a semi-log plot, the measured withdrawal rate Q, the geometric description of the well system and an initially assumed value of 2.0 for α , a first trial value of U_{∞} can be calculated. From this, a corrected value of \tilde{A} and therefore, of α , can be obtained. When \tilde{A} is small, this procedure will converge rapidly to give a

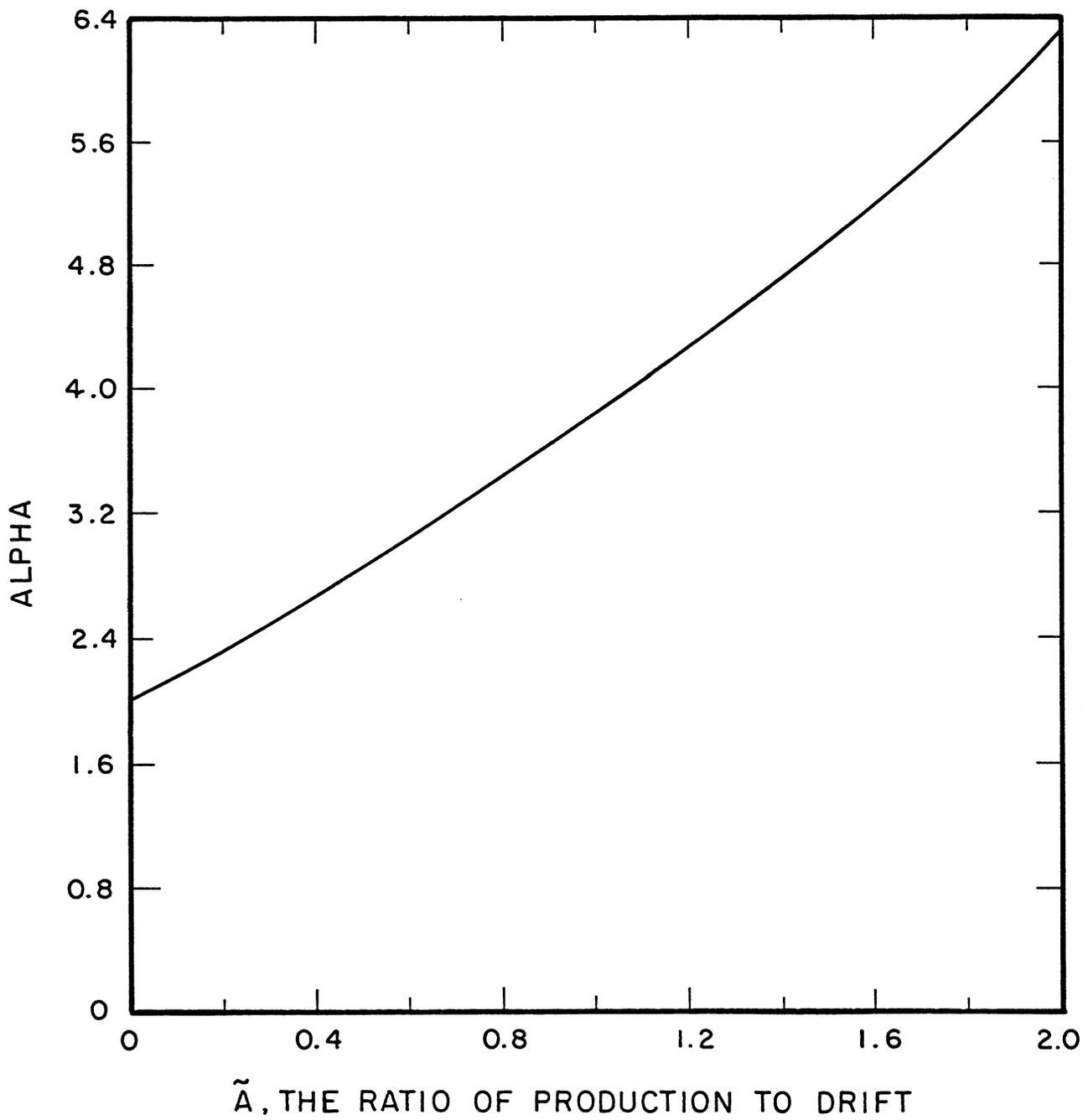


Fig. B9. Variation of parameter α with respect to \tilde{A} .

corrected value of the drift velocity U_{∞} .

Conclusion

The Point Dilution Method is a straightforward way of measuring drift velocity in saturated porous formations. It has been used principally by hydrologists and civil engineers in determining drift of ground water in shallow sands in critical locations, such as near dams and in river or lake drainage areas. In this Appendix, it has been shown that PDM can also be utilized in deeper formations, including oil sands, and that moderate rates of production from the sand do not interfere with the interpretation.

It is not suggested that this is the best way of measuring drift velocity, or even the most economical. One competitive method that might be further developed is the analysis of "single well tracer" tests, a pumpin-pumpout procedure in which the time variation of tracer concentration in the returned fluid contains information on the integrated drift of the injected pattern. Both of these methods are unable to distinguish the direction of the drift velocity--a difficulty which could be serious in both reservoir and aquifer description.

H3RC: WELLSTRM. TEXT

listed on 1/19/86.

```
PROGRAM WELLSTREAM; {Maps streamlines around wellbore of radius RA,
when flow velocity far from well (ie, "drift velocity") is VINP, which
is parallel to the Y axis on the map. The production rate from the
well is not necessarily zero, and is allowed for in terms of a
dimensionless parameter, AT. AT=Q/(2*PI*h*a*VINP) }
{4/28/84 VERSION, WITH (optional) PRINTOUTS OF FLUX ALONG BOUNDARIES}
```

```
USES MPSSGRAF; {MAP and GRAPH UNIT}
CONST PI=3.141592654; NB=50;
TYPE ARY = ARRAY[1..NB] OF REAL;
VAR AT, {dimensionless parameter - ratio of production to distant V}
RA, {well radius in inches on graph}
WX, {width of graph, inches}
HY, {height of graph, inches}
XOL,YOL,XNU,YNU,PHIZ,PHIB,THIB,RAT,XCS,RAI,XBZ,XTZ,TNFC,
NETFLIN,TFC,DFLX,DLT,DT,RAS,VX,VY,RSQ,R4,RSQO : REAL;
J,K,MA,NSTR,MNST,NOX,NTB,NBB,NBBB : INTEGER;
FTB,FBB : ARY;
CH,CHP : CHAR;
```

```
FUNCTION ARTAN(X:REAL):REAL;
BEGIN
  IF X>=0 THEN ARTAN:=ARCTAN(X)
  ELSE ARTAN:=- (ARTAN(-X));
END;
```

```
FUNCTION ARCOS(X:REAL):REAL;
BEGIN
  IF ABS(X)>1
  THEN WRITELN('ARCOS ERROR, X= ',X)
  ELSE IF X>0 THEN ARCOS:=ARCTAN(SQRT(1/(X*X)-1))
  ELSE IF X<0 THEN ARCOS:=-ARCTAN(SQRT(1/(X*X)-1))
  ELSE ARCOS:=PI/2;
END;
```

```
PROCEDURE R2AND4(U,V:REAL);
BEGIN
  RSQ:=U*U+V*V;
  R4:=RSQ*RSQ;
END;
```

```
PROCEDURE NABERP(XA,YA:ARY;XS:REAL;VAR YS:REAL;
NP:INTEGER;VAR IP:INTEGER);
{Quadratic interpolation subroutine, to compute a point YS an
an input point XS, given two arrays XA and YA, each NP long.}
{Modelled after old Fortran SR of same name, written
by G. W. Nabor of Mobil sometime in 1960s.}
{ ** NOTE THAT TYPE ARY MUST BE DECLARED IN MAIN! ** }
{NP is number of points in arrays XA and YA.
XS is input variable, YS and IP are outputs}
VAR X1,X2,X3,X4,Y1,Y2,Y3,Y4,C,A,B : REAL;
IPP,J,M : INTEGER;
```

H3RC:WELLSTRM.TEXT

```

PROCEDURE WHERE (XS:REAL;VAR IQ:INTEGER);
  {output is IP; XS lies between XA[IP] and XA[IP+1].
   if on output IP is 0 or NP, XS lies outside range!}
  VAR K      : INTEGER;
  BEGIN
    IQ:=0;
    FOR K:=1 TO NP DO
      IF (XS>XA[K]) THEN IQ:=K;
    END; {procedre WHERE}

```

```

PROCEDURE MIDL;
  VAR SM,DF,PT,W,SL,Q1,Q4,A1,A4  : REAL;
  BEGIN
    IF (IP<>IPF) THEN
      BEGIN
        SM:=X2+X3;
        DF:=X2-X3;
        PT:=X2*X3;
        W:=(Y2*X3-Y3*X2)/DF;
        SL:=(Y2-Y3)/DF;
        Q1:=X1*(X1-SM)+PT;
        Q4:=X4*(X4-SM)+PT;
        A1:=X1*SL-W;
        A4:=X4*SL-W;
        C:=((Y1-A1)*Q1+(Y4-A4)*Q4)/(Q1*Q1+Q4*Q4);
        A:=C*PT-W;
        B:=SL-C*SM;
      END;
      YS:=A+XS*(B+C*XS);
      IPP:=IP;
    END; {procdure MIDL}

```

```

PROCEDURE ENDS;
  TYPE ARY2S = ARRAY[1..3,1..3] OF REAL;
  VAR AA,BB,CC,EE,FF,DE,DEN  : REAL;
      AAR,BAR,CAR,DAR       : ARY2S;
      I                      : INTEGER;

```

```

FUNCTION DET (A:ARY2S):REAL;{calculates determinant of a 3x3 matrix}
  VAR SUM : REAL;
  BEGIN
    SUM:=A[1,1]*(A[2,2]*A[3,3]-A[3,2]*A[2,3])
        -A[1,2]*(A[2,1]*A[3,3]-A[3,1]*A[2,3])
        +A[1,3]*(A[2,1]*A[3,2]-A[3,1]*A[2,2]);
    DET:=SUM;
  END; {function DET}

```

```

BEGIN {procedure ENDS}
  IF (IP<>IPF) THEN
    BEGIN
      FOR I:=1 TO 3 DO DAR[I,1]:=1;
      DAR[1,2]:=X1;
      DAR[2,2]:=X2;
      DAR[3,2]:=X3;
      DAR[1,3]:=SQR(X1);
      DAR[2,3]:=SQR(X2);
      DAR[3,3]:=SQR(X3);
      AAR:=DAR;
    END;

```

H3RC:WELLSTRM.TEXT

```

      AAR[1,1]:=Y1;
      AAR[2,1]:=Y2;
      AAR[3,1]:=Y3;
      BAR:=DAR;
      BAR[1,2]:=Y1;
      BAR[2,2]:=Y2;
      BAR[3,2]:=Y3;
      CAR:=DAR;
      CAR[1,3]:=Y1;
      CAR[2,3]:=Y2;
      CAR[3,3]:=Y3;
      DEN:=DET(DAR);
      AA:=DET(AAR)/DEN;
      BB:=DET(BAR)/DEN;
      CC:=DET(CAR)/DEN;
      DE:=X2-X1;
      EE:=(Y1*X2-Y2*X1)/DE;
      FF:=(Y2-Y1)/DE;
    END;
    YS:=(AA+EE + XS*(BB+FF+XS*CC))/2;
    IFP:=IP;
  END; {procedure ENDS}

BEGIN {procedure NABERF}
  YS:=0;
  FOR J:=1 TO NP DO IF (XS=XAJJ) THEN YS:=YAJJ;
  IF (YS=0)
  THEN
    BEGIN
      IP:=-1;
      WHERE(XS,IP);
      {WRITE(' IP= ',IP,' '); }
      IF (IP=0) OR (IP=NP)
      THEN WRITELN('INPUT FLUX IS OUT OF RANGE!')
      ELSE
        BEGIN
          IF (IP=1) OR (IP=NP-1)
          THEN
            IF (IP=1)
            THEN
              BEGIN
                X1:=XAJ1; Y1:=YAJ1;
                X2:=XAJ2; Y2:=YAJ2;
                X3:=XAJ3; Y3:=YAJ3;
              ENDS;
            END
            ELSE
              BEGIN
                X1:=XAJNP; Y1:=YAJNP;
                X2:=XAJNP-1; Y2:=YAJNP-1;
                X3:=XAJNP-2; Y3:=YAJNP-2;
              ENDS;
            END
          ELSE
            BEGIN
              M:=IP-1;
              X1:=XAJM; Y1:=YAJM;
              X2:=XAJM+1; Y2:=YAJM+1;
              X3:=XAJM+2; Y3:=YAJM+2;
            END
          END
        END
      END
    END
  END
END {procedure ENDS}

```

H3RC:WELLSTRM.TEXT

```

        X4:=XA[M+3]; Y4:=YA[M+3];
        MIDL;
        END;
    END;
END; {procedure NABERP}

```

PROCEDURE INITASK;

```

VAR PHIZD : REAL; {PHIZ in degrees}
BEGIN
    WRITE('ENTER RADIUS OF CENTRAL CIRCLE, in INCHES. RA= ');
    READLN(RA);
    WRITE('ENTER value of dimensionless production ratio. AT= ');
    READLN(AT);
    IF AT<2 THEN PHIZ:=PI-ARCOS(AT/2) { PHIZ is measured downwards}
    ELSE PHIZ:=PI; { from north pole.}
    PHIZD:=180*PHIZ/PI; {for AT>=0, 90<=PHIZD<=180 degrees}
    WRITELN('PHIZ= ',PHIZD:6:2,'degrees');
    IF AT>=2 THEN
        WRITELN('FOR AT>2 THERE ARE NO OUTGOING STREAMLINES');
        WRITELN('Enter number of streamlines to be drawn entering ');
        WRITE('the well between phi = 0 and phi=PHIZ,on one side. NSTR= ');
        READLN(NSTR);
        NNST:=0;{temporarily: if AT<2, this'll be # of streamlines
        leaving well}
        NBBB:=0;
        TFC:=PHIZ*AT+2*SIN(PHIZ); {is total flux entering well between
        north pole and PHIZ down along one side.}
        DFLX:=TFC/NSTR; {is flux change between adjacent streamlines}
        WRITELN('Flux change between adjacent streamlines is ',DFLX:5:4);
        TNFC:=0; {default value of exiting flux, if AT>=2}
        NETFLIN:=TFC-TNFC;
        RAT:=RA*AT;
        RAS:=RA*RA;
        RAI:=RA+0.025;
        HY:=6;
        WX:=6;
        WRITE('Are WX=6 and HY=6 inches ok? (Y/N) ');
        READLN(CH);
        IF (CH<>'Y') AND (CH<>'y') THEN
            BEGIN
                WRITE(' Enter new value for WX = ');
                READLN(WX);
                WRITE(' Enter new value for HY = ');
                READLN(HY);
            END;
        MAFINIT(WX,HY,-1);
        {opens printer file,& initializes MAP mode of MFSGRAF}
    END;

```

PROCEDURE CIRC(XC,YC,RAD:REAL);

```

{computes points to draw well outline & mark center}
VAR DTH,TH,XS,YS,XF,YF : REAL;
    I : INTEGER;
BEGIN
    DTH:=PI/18;
    XS:=XC+RAD;
    YS:=YC;
    FOR I:=1 TO 36 DO

```

H3RC:WELLSTRM.TEXT

```

      BEGIN
        TH:=I*DTH;
        XF:=XC+ RAD*COS(TH);
        YF:=YC+ RAD*SIN(TH);
        CNNECT(XS,YS,XF,YF); {MPSGRAF's procedure to draw straight line}
        XS:=XF;
        YS:=YF;
      END;
      PUNKT(XC,YC,2);
    END;

  PROCEDURE TIC; {puts inward tics in perimeter at +&- PHIZ}
    VAR XS,YS,XF,YF : REAL;
    BEGIN
      XS:=RA*SIN(PHIZ);
      XF:=0.85*XS;
      YS:=RA*COS(PHIZ);
      YF:=0.85*YS;
      CNNECT(XS,YS,XF,YF);
      IF XS<>0 THEN CNNECT(-XS,YS,-XF,YF);
    END;

  PROCEDURE MOVE; {makes next dot in streamline, and in mirror image}
    BEGIN
      CNNECT(XOL,YOL,XNU,YNU); {MPSGRAF's procedure to draw straight line}
      CNNECT(-XOL,YOL,-XNU,YNU);
      XOL:=XNU;
      YOL:=YNU;
    END;

  FUNCTION FLUX(XN,XC,YB:REAL):REAL; {gives the integral of vertical flux
    upwards through a horizontal line at YB, between XN and XC}
    VAR XS1,XS2 : REAL;
    BEGIN
      XS1:=XN*XN+YB*YB;
      XS2:=XC*XC+YB*YB;
      FLUX:=(XN-XC)/RA + AT*(ARTAN(XN/YB)-ARTAN(XC/YB)) +
        RA*(XN/XS1-XC/XS2);
    END;

  PROCEDURE PARTA; {to get production streamlines entering wellbore
    between north pole and PHIZ, and mirrored}
    {Moves backwards on streamline, from well perimeter outward.}
    VAR PHA,PHAA,FLA,XX,FLX : ARY;
        PH,PHOUT,FLIN,PHIA,XT : REAL;
        I,IP : INTEGER;

  FUNCTION FL(PH:REAL):REAL; {integrated flux from pole to PH}
    BEGIN
      FL:=2*SIN(PH)+AT*PH;
    END;

  PROCEDURE AFINDM; {to fill a table of proper places on well
    perimeter to start streamlines around circle}
    BEGIN
      FOR I:=1 TO NB DO
        BEGIN
          PH:=(I-1)*PHIZ/NA; {NA is NB-1}

```

H3RC:WELLSTRM.TEXT

```

      PHA[II]:=PH;
      FLA[II]:=FL(PH);
      FTB[II]:=0;
      FBB[II]:=0;
      XT:=(I-1)*WX/(2*NA);
      XX[II]:=XT;
      FLX[II]:=FLUX(XT,0,HY/2);
    END; {filling arrays for NABERP to work from}
    NABERP(FLX,XX,TFC,XTZ,NB,IP);
    IF IP=NB THEN XTZ:=WX/2;
    FOR I:=1 TO NSTR DO
      BEGIN
        FLIN:=(I-0.5)*DFLX;{is total flux between pole and PHA[II]}
        NABERP(FLA,PHAA,FLIN,PHOUT,NB,IP);
        PHA[II]:=PHOUT;
      END;
      WRITELN(' # of production streamlines into well is ',NSTR);
    END; {proc AFINDM}

BEGIN {procedure PARTA}
  NTB:=0;
  AFINDM;
  FOR I:=1 TO NSTR DO
    BEGIN
      DLT:=0.02;
      PHIA:=PHA[II];
      XOL:=RAI*SIN(PHIA);
      YOL:=RAI*COS(PHIA);
      J:=1;
      WHILE (XOL<WX/2) AND (YOL<HY/2) DO
        BEGIN
          R2AND4(XOL,YOL);
          VX:=- (RAT*XOL)/RSQ-(2*XOL*YOL*RAS)/R4;
          VY:=- (RAT*YOL+RSQ-RAS)/RSQ-(2*YOL*YOL*RAS)/R4;
          XNU:=XOL-DLT*VX;
          YNU:=YOL-DLT*VY;
          {moves points in direction opposite to streamlines}
          IF (XNU*XNU+YNU*YNU)<RAS THEN WRITELN('in A J=',J,'on line ',I);
          IF YNU>=HY/2 THEN
            BEGIN
              NTB:=I;
              FTB[II]:=FLUX(XNU,0,YNU);
            END;

          MOVE;
          J:=J+1;
        END;
      WRITE(' ');
    END;
    CNNECT(XTZ,HY/2,XTZ,HY/2-0.065);
    WRITELN(' PARTA COMPLETE, NTB= ',NTB);
  END; {procedure PARTA}

PROCEDURE PARTB; {computes injection streamlines, starting at well
  perimeter at south pole, moving forward on streamlines.}

VAR TH,THOUT,FLIN,THZ,THIB : REAL;
    THB,THBB,FLB : ARY;
    I,IP,II : INTEGER;

FUNCTION FNL(TH:REAL):REAL; {total inj flux from s pole to TH}

```

H3RC:WELLSTRM.TEXT

```

BEGIN
  FNL:= 2*SIN(TH)-AT*TH;
END;

PROCEDURE BFINDM; {fills needed values of THB[I], starting at
                  south pole, for injection streamlines.}
BEGIN
  THZ:=PI-PHIZ; {angular boundary pt. between prod. & inj. streamlines}
  FOR I:=1 TO NB DO
    BEGIN
      TH:=(I-1)*THZ/NA;
      THB[I]:=TH;
      FLB[I]:=FNL(TH);
    END;
  TNFC:=FLB[NB]; {total injection flux between south pole and THZ}
  NETFLIN:=TFC-TNFC; {net flux into well}
  IF AT=0 THEN NETFLIN:=0;
  II:=1;
  THOUT:=0;
  FLIN:=0;
  WHILE FLIN<(TNFC-DFLX) DO
    BEGIN
      FLIN:=(II-0.5)*DFLX;
      NABERP(FLB, THB, FLIN, THOUT, NB, IP);
      THB[II]:=THOUT;
      NNST:=II;
      II:=II+1;
    END;
  WRITELN('# of injection streamlines from well is ', NNST);
END; {procedure BFINDM}

BEGIN {procedure PARTB }
  BFINDM;
  NBB:=0;
  FOR I:=1 TO NNST DO
    BEGIN
      DLT:=0.02;
      THIB:=THB[I];
      XOL:=RAI*SIN(THIB);
      YOL:=-RAI*COS(THIB);
      J:=1;
      WHILE (XOL<WX/2) AND (-YOL<HY/2) DO
        BEGIN
          XCS:=XOL;
          R2AND4(XOL, YOL);
          VX:=- (RAT*XOL)/RSQ- (2*XOL*YOL*RAS)/R4;
          VY:=- (RAT*YOL+RSQ-RAS)/RSQ- (2*YOL*YOL*RAS)/R4;
          XNU:=XOL+DLT*VX;
          YNU:=YOL+DLT*VY;
          {moves point in direction of streamlines}
          IF (XNU*XNU+YNU*YNU)<RAS THEN WRITELN('in B J=', J, 'on line ', I);
          IF -YNU>=HY/2 THEN
            BEGIN
              NBB:=I;
              FBB[I]:=FLUX(XNU, 0, YNU)+NETFLIN;
            END;

          MOVE;
          J:=J+1;
        END;
      WRITE(' ');
    END;
  END;

```

```
H3RC:WELLSTRM.TEXT
```

```
END;
NBBB:=NBB;
WRITELN(' PARTB COMPLETE,NBB= ',NBB);
END; {procedure PARTB}
```

```
PROCEDURE PARTC; {computes streamlines that don't intersect wellbore,
starting from bottom frame boundary (beginning at XCS, that first
misses circle) and moving upward, backwards along streamlines.}
```

```
VAR XCC,FLC : ARY;
XC : ARRAY[1..50] OF REAL;
XX,XBS,TFLX,FLMX : REAL;
I,IP,II : INTEGER;
```

```
FUNCTION FLX(XN,XC:REAL):REAL;{gives the integral of flux out of
lower frame boundary (at Y=-HY/2) between XN and XC}
```

```
VAR XS1,XS2 : REAL;
BEGIN
XS1:=XN*XN+HY*HY/4;
XS2:=XC*XC+HY*HY/4;
FLX:=(XN-XC)/RA + AT*(ARTAN(-2*XN/HY)-ARTAN(-2*XC/HY)) +
RA*(XN/XS1-XC/XS2);
END;
```

```
PROCEDURE CFINDM; {fills needed values of XC[I], as places to
start streamlines, from lower frame boundary}
```

```
BEGIN
FOR I:=1 TO NB DO
BEGIN
XX:=(I-1)*WX/(2*NA);
XC[I]:=XX;
FLC[I]:=FLX(XX,0)+NETFLIN;
END; {filling array for NABERP to work from}
FLMX:=FLC[NB]; {is flux at right boundary}
NABERP(FLC,XCC,TFC,XBZ,NB,IP);
IF IP=NB THEN XBZ:=WX/2;
{Now to fill XC[I]}
XBS:=XBZ;
TFLX:=TFC-DFLX/2;
II:=0;
WHILE (XBS<WX/2) AND (TFLX<FLMX-DFLX) DO
BEGIN
II:=II+1;
NBBB:=NBBB+1;
TFLX:=TFLX+DFLX;
NABERP(FLC,XCC,TFLX,XBS,NB,IP);
IF NBBB<=NB THEN FBB[NBBB]:=TFLX
ELSE BEGIN
WRITELN(' IN CFINDM, NBBB= ',NBBB);
WRITELN('Streamlines are too densely spaced!');
EXIT(PROGRAM);
END;
XC[II]:=XBS;
END; {loop on II,getting start pts. for partc streamlines}
NOX:=II;
WRITELN('# of outer streamlines is ',NOX);
END; {procedure CFINDM}
```

H3RC:WELLSTRM.TEXT

```

BEGIN {procedure PARTC}
CFINDM;
FOR I:=1 TO NOX DO
  BEGIN
    DLT:=0.02;
    XOL:=XC[I];
    YOL:=-HY/2;
    J:=1;
    WHILE (YOL<HY/2) AND (XOL<WX/2) DO
      BEGIN
        R2AND4(XOL,YOL);
        VX:=- (RAT*XOL)/RSQ-(2*XOL*YOL*RAS)/R4;
        VY:=- (RAT*YOL+RSQ-RAS)/RSQ-(2*YOL*YOL*RAS)/R4;
        XNU:=XOL-DLT*VX;
        YNU:=YOL-DLT*VY;
        {moves points in direction opposite to streamlines}
        IF (XNU<XNU+YNU*YNU)<RAS THEN WRITELN('in C J=',J,'on line ',I);
        IF YNU>=HY/2 THEN
          BEGIN
            NTB:=NTB+1;
            FTB[NTB]:=FLUX(XNU,0,YNU);
          END;

        MOVE;
        J:=J+1;
      END; {computing Ith streamline}
    WRITE('!');
  END; {loop on I over NOX streamlines}
  CNNECT(XBZ,-HY/2,XBZ,0.065-HY/2);
  WRITELN(' PARTC complete, NTB IS NOW ',NTB,', and NBBB= ',NBBB);
END; {procedure PARTC}

PROCEDURE PRINTSTUF;
VAR I : INTEGER;
BEGIN
  REWRITE(PR,'#6:');
  WRITELN(PR);
  WRITELN(PR,' ':23,'PROGRAM WELLSTREAM, WITH AT=',AT:4:2);
  WRITELN(PR,' ':24,'Total flux entering well = ',TFC:6:4);
  WRITELN(PR,' ':24,'Total flux leaving well = ',TNFC:6:4);
  WRITELN(PR,' ':24,'NET flux entering well = ',NETFLIN:6:4);
  WRITELN(PR,' ':16,'Flux change between adjacent streamlines = ',DFLX:5:4);
  IF (CHP='Y') OR (CHP='y') THEN
    BEGIN
      WRITELN(PR);
      WRITELN(PR,' ':20,'FLUX AT TOP AND BOTTOM FRAME BOUNDARIES');
      WRITELN(PR,' ':27,'I      FTOP      FBOT');
      FOR I:=1 TO NTB DO
        WRITELN(PR,' ':20,I:8,FTB[I]:13:3,FBT[I]:13:3);
      END;
    CLOSE(PR,NORMAL);
  END;

BEGIN {MAIN}
NA:=NB-1;
INITASK;
XCS:=0;
WRITELN(PR);
CIRC(0,0,RA);

```

H3RC:WELLSTRM.TEXT

```
TIC;
PARTA;
IF AT<2 THEN PARTB
      ELSE WRITELN('NO INJECTION STREAMLINES FOR THIS CASE');
PARTC;
WRITELN('total flux entering well is TFC= ',TFC:6:4);
WRITELN('flux leaving well is -TNFC=-',TNFC:6:4);
WRITELN('NET flux entering well is NETFLIN= ',NETFLIN:6:4);
MAPFIT;{drives epson to make MAP,& closes printer file}
WRITELN;
WRITE('Is printout of flux at top and bottom desired? (Y/N) ');
READLN(CHP);
PRINTSTUF;
END.
```

APPENDIX C - TRACER DISTRIBUTION IN RADIAL FLOW

Introduction

Displacement of one fluid by another in porous rock can be a very complex subject if all of the details of the actual physical flow system are taken into account. As was pointed out in Appendix A, useful solutions can be obtained of the mathematically simple cases in which permeability and porosity are assumed to be constant. Another simplifying assumption that was made in the calculations given in that Appendix was that the injected fluid displaced completely and with no dispersion the fluid originally in place. In justification, one could say that the "sharp front" given by those calculations represents the central isoconcentration line in a front which is spread out or dispersed. But no attempt was made in those calculations to consider the effects of a dispersed or dispersing front.

The assumption of a sharp front is relaxed in this Appendix, which discusses the combined influence of dispersion and radial flow geometry on the spreading of a band of tracer injected into the formation. Because tracer material is generally used at low concentration, it can safely be assumed that there are no significant density or viscosity differences between the fluid that carries tracer in solution and the fluid which is pumped into the well prior to or subsequent to the tracer injection. Thus, there is no coupling between the Darcy equation for the velocity of the fluid and the convection-dispersion (C-D) equation to calculate the evolution of any concentration distribution.

The Flow Equations

In vector form the C-D equation is

$$\frac{\partial C}{\partial t} = - \frac{\vec{U}}{\phi} \cdot \nabla C + \nabla \cdot \tilde{\mathcal{D}} \cdot \nabla C \quad (C1)$$

Here the Darcy velocity is \vec{U} , ϕ is the formation porosity, C is the tracer concentration, and t represents time. The symmetric tensor $\tilde{\mathcal{D}}$ is the dispersivity, which has in this case only two independent components, \mathcal{D}_L and \mathcal{D}_t . The longitudinal dispersivity is greater than the transverse by a factor of about 30 and applies to dispersion in the direction of flow. [Bear, 1961; de Josselin de Jong, 1958; Blackwell, 1962; Perkins & Johnston, 1963].

In this problem, all flow is from the well at the center of coordinates. The Darcy velocity is in the radial direction and inversely proportional to the distance from the well--which for convenience is then taken as the center of coordinates.

$$\vec{U} = \hat{a}_r \frac{Q}{2\pi hr} + \hat{a}_\theta \cdot 0 + \hat{a}_z \cdot 0 \quad (C2)$$

Where $(\hat{a}_r, \hat{a}_\theta, \hat{a}_z)$ are the unit vectors in a cylindrical polar coordinate system. There is no variation in the azimuthal or vertical directions of either the velocity or of the concentration. The latter condition follows if we consider only those concentration patterns that can evolve from variations of tracer content injected at the well. Thus, Eq. (C1) becomes

$$\frac{\partial C}{\partial t} = \frac{Q}{2\pi hr\theta} \frac{\partial C}{\partial r} + \frac{\partial}{\partial r} \left(\mathcal{D}_L \frac{\partial C}{\partial r} \right) \quad (C3)$$

Dispersion

It has been shown in a number of laboratories [Blackwell, Rayne & Terry, 1959; Blackwell, 1962; San Filippo & Guckert, 1977] that at very low flow rates through a porous medium the value of \mathcal{D}_L (and of \mathcal{D}_t as well) are equal to the molecular diffusion coefficient divided by a number characteristic of the rock. At higher flow rates, a second term appears in the dispersivity that is approximately proportional to the displacement velocity itself. In this second term, the proportionality constant is a distance that equals some measure of the pore structure of the rock. The relationship is summed up in the equation [Blackwell, 1962; Heller, 1963]:

$$\mathcal{D}_L = \frac{D_m}{\phi F} + \frac{\sigma_L d_p}{\phi} U \quad (C4)$$

Here, D_m is the molecular diffusion coefficient of the fluid, ϕ is the porosity of the rock, and F its electrical "formation resistivity factor." The value d_p is a characteristic particle diameter or other pore-scale distance, and σ_L is a dimensionless constant that indicates the microscopic heterogeneity.

Near the wellbore, the flow velocity is large, and the second term becomes quite important. If the value of U from Eq. (C-2) is inserted, and Eq. (C4) is differentiated, one obtains

$$\frac{\partial \mathcal{D}_L}{\partial r} = - \frac{\sigma_L d_p Q}{2\pi h \phi r^2} \quad (C5)$$

Then the full equation for the evolution of a radial concentration distribution becomes

$$\frac{\partial C}{\partial t} = - \frac{Q}{2\pi h \phi} \left(\frac{1}{r} + \frac{\sigma_L d_p}{r^2} \right) \frac{\partial C}{\partial r} + \left(\frac{D_m}{\phi F} + \frac{\sigma_L d_p Q}{2\pi h \phi r} \right) \frac{\partial^2 C}{\partial r^2} \quad (C6)$$

No analytic solution of this equation is known to the writer. The numerical solution of the equation furthermore poses unusual difficulties for standard finite-difference calculations. Because of the changing velocity, the proper choices of grid block and time-step sizes at the beginning of the

problem are not appropriate later on in the computation. This is especially troublesome because during the very early stages of the flow described by this equation, the geometric constraints cause any concentration profile to become steeper with increasing time. It is only later in the displacement that the influence of the dispersion term (containing the second radial derivative of C) becomes dominant, and the steepness of the radial concentration profile declines as the front continues to move outward. This behavior makes even more difficult any appropriate discretization.

The technique that is used for calculation of tracer band width in this Appendix does not use a stationary grid system, but rather a method by which particular isoconcentration lines or "fronts" are followed during the flow process. The technique is based on the calculation of the velocity of the isoconcentration lines by use of an expression derived from Eq. (C6).

The Isoconcentration Line Velocities

Because the tracer concentration is represented by a function $C(r, t)$ that depends only on the radial coordinate and time, the differential of C is

$$dC = \frac{\partial C}{\partial r} dr + \frac{\partial C}{\partial t} dt \quad (C7)$$

On an isoconcentration line this differential is zero, a fact that enables Eq. (C7) to be turned into a relation between the differentials of r and t on such a line. The ratio of these, dr/dt, is the normal velocity of the isoconcentration line.

$$v_{iso} = \frac{dr}{dt} = - \frac{\partial C / \partial t}{\partial C / \partial r} \quad (C8)$$

This form is particularly useful because we have available, in the partial differential equation of (C6), an expression for the partial derivative ($\partial C / \partial t$). Combining the Eqs. (C8) and (C6) we obtain

$$v_{iso} = \frac{Q}{2\pi h \phi} \left(\frac{1}{r} + \frac{\sigma_L d_p}{r^2} \right) - \left(\frac{D_m}{\phi F} + \frac{\sigma_L d_p Q}{2\pi h \phi r} \right) \frac{\partial^2 C}{\partial r^2} / \frac{\partial C}{\partial r} \quad (C9)$$

The analytic expression is used directly in the simulation technique described here. An initial set of isoconcentration lines (circles of different radii) are first identified by the concentration levels they mark. For each, the outward velocity is calculated. In the absence of dispersion, each of the circles would expand at a rate inversely proportional to its radius, leading to a continual crowding together or steepening of the radial concentration gradient. With dispersion, this initial steepening is halted and reversed at some critical radius.

Computation of Tracer Widths

The calculation would be quite straightforward except for the fact that at each of the isoconcentration lines, values for both the first and the second radial derivatives must be computed. This computation is only approximate, of course--a circumstance that would leave the entire result in doubt except for an interesting fact. This is that the calculation, like the process of dispersion itself, is stable so long as the chosen time increments are not too large. A local inaccuracy (a "bump" on the concentration profile), caused by a previous error in the calculation of velocity, will become smaller in the next iteration. For this reason, a simple difference scheme can be used for calculation of these derivatives.

The computation has been coded in PASCAL, and a listing is included as Fig. C1. It describes the motion of a tracer band, in which the concentration is a maximum at the center. It is presumed that the actual tracer band is initially introduced as a flat-topped "slug" of tracer, with approximate profiles of the concentration and its first two radial derivatives as sketched in Fig. C2a, b and c.

The shape of the initial slug avoids any physically impossible features. Vertical sides of the concentration slug would imply an infinite concentration gradient, and sharp corners at the top of the slug would indicate infinite values of the second derivative. Even if the engineer managing the tracer injection were able to approach such conditions closely, they could not endure in the face of any dispersion whatsoever. Instead, it is not only more realistic, but computational difficulties are eased, if an assumed initial slug shape is designed as shown, with discontinuities only in the second derivative curve. It turns out that because of the stability of the calculation, the final results are not appreciably influenced by the assumed values of the initial ramp widths at the front and rear edges of the initial concentration slug.

As the flow proceeds, two distinct changes take place in this profile. Firstly, because of the geometric effect, the isoconcentration line at r_1 moves more rapidly than that at r_3 , so that the distance between them decreases. Secondly, the effect of dispersion can also be described in a qualitative way from the same Figure, and by referring to the second term on the RHS of Eq. (C9). The velocity of an isoconcentration line at a radius $(r_1 - \delta)$ will move more slowly than does the one at r_1 . At $(r_1 + \delta)$, on the other hand, the second derivative has become negative while the first derivative is still positive--so that this isoconcentration line at $(r_1 - \delta)$ moves more rapidly than does the line at r_1 . If this widening tendency is more powerful than the geometric sharpening, the slope of the inward side of the flattop concentration at r_1 , will decrease. The relative strengths of the opposing tendencies depend on Q and on the radial distance, and on the magnitude of the dispersion coefficient.

Similarly, the outward edge of the slug is subject to the same influences. It can be seen that as flow proceeds and the center of the slug at r_2 moves outwards, a time will be reached when the dispersive effects will become dominant. Then the widening of the positive and negative spikes of $\partial C / \partial r$ will proceed unchecked until they begin to influence each other. At that time, the flattop will disappear, and the maximum concentration of the slug will start to decrease.

H3RV: UMURK . TEXT

listed on 10/12/84. This is the Aug 31 1984 version.

```

PROGRAM NUMURKY; { To compute growth of miscible tracer band as it is
  displaced outward from the injection well. Includes effects of both
  the radial geometry and of dispersion, and follows the decrease of
  peak concentration after original flattop distribution changes into
  an approximate Gaussian. 8/22/84 version}
USES SCREENCONTROL; {to be able to get date of run}
CONST PI=3.1415927; CFPB=5.6146; DEL=0.001; DIN=0.4; CR=1.414;
  QU=9; {this is basis of number of iso lines.}
  NHH=18; {this is calc'd as 2*QU, and equal to NH}
  NIS=37; {this is calc'd as 4*QU+1, equal to NISO}
  KJ=100;

TYPE FRONT = RECORD {main front profile record. One of these records
  is stored onto disk after every two feet of tracer band travel}
  SERNO : INTEGER; {month &day ; plus serial # of day's run}
  TM : REAL; {time in days}
  MXCON : REAL; {conc. at peak}
  TBNDWID: REAL; {width of conc. dist'n at half conc points}
  VM : REAL; {recalc volume of tracer band}
  VKRCT : REAL; {value of vol corr'n factor last time}
  RAD : ARRAY[1..NIS] OF REAL; {radiof isoconcentration lines}
  CC : ARRAY[1..NIS] OF REAL; {values of conc. at those lines}
  NIT : INTEGER; {# of iters to get from previous record entry}
END;
DISTNS = FILE OF FRONT;

SHEET = RECORD {"statrecord" stored on disk once, after calc'ns}
  STGONE : ARRAY[1..KJ] OF INTEGER;
  STTIM : ARRAY[1..KJ] OF REAL;
  STPK : ARRAY[1..KJ] OF REAL;
  STRPK : ARRAY[1..KJ] OF REAL;
  STBW : ARRAY[1..KJ] OF REAL;
  STVOL : ARRAY[1..KJ] OF REAL;
END;
BOOK = FILE OF SHEET;

VAR C,R,PD1,PD2 : ARRAY[1..NIS] OF REAL;
  T,Q,H,PHI,FF,LAM,DFC,DLT,RFAR,TD,RF,CHALF,VCP,
  VCOF,RCEN,SW,DRMX,RBIAS,CFK,BW,RTAR,BLR,ALP,B75,
  SOR,WT,AA,BB,CC,VOLR,VOLIN,G,RMAX,VC,VRATIO : REAL;
  I,J,NH,SN,RNUM,ITL,ITH,K,KK,GONE,NLEFT,NFIT,LFB,
  IST,NREDONE,NISO,RPB,JL,JR,OLGONE : INTEGER;
  MM: 0..12; DD: 0..31; YY: 0..99; DRN:0..9;
  IX : ARRAY[1..KJ] OF INTEGER;
  A1,A2,A3,A4,A5 : ARRAY[1..KJ] OF REAL;
  TFRONT : FRONT;
  CONDIST : DISTNS;
  SKRIP : STRING;
  CH : CHAR;
  FLAT,PRINPA,RFR,FRDI : BOOLEAN;
  PULF : BOOK;
  PIECE : SHEET;

PROCEDURE CENTROID; FORWARD;

```

```
SEGMENT PROCEDURE INIT; {Accepts initializing info, calculates a starting
concentration distribution, opens the file CONDIST, puts in the initial
front and closes the file again}
```

```
PROCEDURE STARDIS; {initial concentration distribution}
VAR RH,DR,E,DC : REAL;
BEGIN
  WRITE('ENTER DURATION OF INITIAL TRACER INJECTION (days) ');
  READLN(TD);
  RF:=SQRT(TD/WT+DIN*(0.5+0.25*DIN)+1) - 1.5*DIN - 0.5;
  RH:=0.5+DIN+RF/2;
  {DIN is radial thickness of front and back ramps on concentration
distribution. RF is radial thickness of flat top portion}
  DR:=DIN/(NH+1);
  DC:=1/(NH+1);
  FOR J:=1 TO NH DO
    BEGIN
      R[J]:=0.50+J*DR;
      C[J]:=J*DC;
    END;
  R[NH+1]:=RH;
  C[NH+1]:=1.0;
  FOR J:=NH+2 TO NISO DO
    BEGIN
      R[J]:=0.5+DIN+RF+(J-NH-1)*DR;
      C[J]:=1.0-(J-NH-1)*DC;
    END;
  RCEN:=(R[NH]+R[NH+2])/2;
  R[NH+1]:=RCEN;
  T:=WT*RCEN*RCEN; {this T is starting value not 0, but to center
of initial tracer slug.}
END; {proc STARDIS in seg.proc INIT}
```

```
PROCEDURE ASK;
BEGIN
  WRITE('ENTER FLOW RATE IN STRATUM (BFD) ');
  READLN(Q);
  WRITE('ENTER THICKNESS OF STRATUM (FEET) ');
  READLN(H);
  WRITE('ENTER POROSITY OF STRATUM ');
  READLN(PHI);
  WRITE('ENTER RESIDUAL OIL SATURATION ');
  READLN(SOR);
  WRITE('ENTER FORMATION RESISTIVITY FACTOR ');
  READLN(FF);
  WRITE('ENTER MOLECULAR DIFFUSION COEF. (SQ FT/DAY) ');
  READLN(DFC);
  WRITE('ENTER CHAR. LENGTH FOR DISPERSIVITY (FEET) ');
  READLN(LAM);
  WRITE('ENTER LARGEST RADIUS FOR COMPUTATIONS (FEET) ');
  READLN(RFAR);
  WT:=PI*H*PHI*(1-SOR)/(Q*CFPB);
  WRITELN('WT:= ',WT:8:6);
  BB:=DFC/(PHI*FF);
  AA:=1/(2*WT) - BB;
  CC:=LAM*PHI*(1-SOR)/(2*WT);
  VCOF:=2*PI*H*PHI*(1-SOR)/CFPB;
  WRITE('ENTER SERIAL NUMBER FOR FILE (0..9) ');
  READLN(DRN);
```

```

WRITE('Want conc. distribution listed in WRITPROF?(Y/N) ');
READLN(CH);
PRDI:=(CH='Y') OR (CH='y');
END; {procedure ASK}

BEGIN {INIT}
G:=0.35; {initial value of correction factor for end shapes}
NH:=2*QU;
NISO:=4*QU+1;
GONE:=0;
NLEFT:=NISO; {while GONE is zero}
NREDONE:=0;
VC:=1E-8;
ALP:=1;
ASK;
STARDIS;
CENTROID;
WRITE('ENTER NAME FOR FILE OF FRONTS ');
READLN(SKIP);
REWRITE(CONDIST,SKIP);
CLOSE(CONDIST,LOCK);
DATE(MM,DD,YY);
SN:=MM*1000+DD*10+DRN;
RNUM:=1;
WITH TFRONT DO
BEGIN
SERNO:=SN;
MXCON:=1;
TBNWDID:=RF+DIN;
TM:=T;
VKRCT:=0;
VM:=VOLR;
FOR J:=1 TO NISO DO
BEGIN
RAD[J]:=R[J];
CC[J]:=C[J];
END;
NIT:=1;
END;
RESET(CONDIST,SKIP);
SEEK(CONDIST,RNUM);
CONDIST^:=TFRONT;
PUT(CONDIST);
CLOSE(CONDIST);
FOR IST:=1 TO KJ DO BEGIN
IX[IST]:=999;
A1[IST]:=0;
A2[IST]:=0;
A3[IST]:=0;
A4[IST]:=0;
A5[IST]:=0;
END;

IST:=1;
JL:=NH+1;
JR:=NH+1;
END; {segment procedure INIT}

PROCEDURE WRITPROF; {lists values of conc.dist'n and derivatives}
VAR PR:TEXT;

```

```

      CH: CHAR;
      ITC: INTEGER;

BEGIN
  ITC:=ITL;
  REWRITE(PR,'#6:');
  IF ITC=1 THEN
    BEGIN
      WRITELN(PR);
      WRITELN
        (PR,'S/N ',SN,' ',SKRIP,' inj.duration=',TD:5:3,'days. GONE= ',GONE);
      WRITELN(PR,
        'Q= ',Q:5:1,' bpd H= ',H:5:2,' ft PHI=',PHI:4:3,' SOR=',SOR:4:3);
      WRITELN(PR,
        'F= ',FF:5:1,' Dmol= ',DFC:6:5,' sq.ft/day Ldisp= ',LAM:6:5);
      WRITELN(PR,'Specified maximum radius= ',RFAR:4:1,' ft.');
```

END;

```

  WRITELN(PR,'ITC= ',ITC,' ',T=',T:7:4,
    ', VOLR=',VOLR:7:4,' ',GONE=',GONE,' ',NREDONE= ',NREDONE);
  IF PRDI THEN
    BEGIN
      WRITELN(PR,
        'J      R[CJ]      C[CJ]      FD1[CJ]      FD2[CJ] ');
      FOR J:=1 TO NISO DO IF C[CJ]>0 THEN WRITELN(PR,
        J:2,R[CJ]:8:3,C[CJ]:15:6,FD1[CJ]:14:6,FD2[CJ]:14:4,' ');
      WRITELN(PR);
    END;
  CLOSE(PR,NORMAL);
END; {procedure writprof}

PROCEDURE CENTROID; {Integrates C(R) and R*C(R),gets tracer band
                    volume VOLR}
  VAR  S,SR,SG,SRG,RC,RLT,RRT,AF,BF,CF,RLP,
        LP2,LP3,LP4,RRP,RP2,RP3,RP4      : REAL;
        SPART,SRPART                      : ARRAY[1..7] OF REAL;
        QH,IP                              : INTEGER;

  {PROCEDURE INTFRNT;}
  {diagnostic: to print partial integrals from proc. centroid.}
  (VAR  PR      : TEXT;
    II      : INTEGER;
  BEGIN
    REWRITE(PR,'#6:');
    WRITELN(PR);
    WRITELN(PR,' ':10,'GONE= ',GONE);
    WRITELN(PR,' IP      S      SR ');
    FOR II:=1 TO 7 DO
      WRITELN(PR,II:3,' ',SPART[II]:9:5,' ',SRPART[II]:9:5,' ');
    WRITELN(PR,'TOT ',S:9:5,' ',SR:9:5,' ');
    WRITELN(PR,'T=',T:7:4,' ',VOLR=',VOLR:8:4,' ');
    WRITELN(PR);
    CLOSE(PR,NORMAL);
  END;){proc INTFRNT}

PROCEDURE ADJUST; {Called from proc centroid, while GONE is 0
                  (ie while tracer band is still flat-topped)
                  and evaluates RLT and RRT for area and volume calcs.}

```

```

VAR RL2,RL1,RPK,RR1,RR2      : REAL;
BEGIN
  RR2:=R[NH+3];
  RR1:=R[NH+2];
  RPK:=R[NH+1];
  RL1:=R[NH];
  RL2:=R[NH-1];
  RLT:=RL1+CR*(RL1-RL2);
  RRT:=RR1-CR*(RR2-RR1);
END; {proc adjust}

```

```

PROCEDURE FITPARB; {get coeffs to fit central zone to parabola}
VAR S1,S2,S3,S4,C1,C2,C3,RSQ,RR,CC,DEN,ADN,BDN,CDN,RB,CX : REAL;
    I,II      : INTEGER;
    CH       : CHAR;

```

```

{TABLE OF VALUES, examples if QU = 5 :

```

	GONE	NLEFT	LPB	RFB	NFIT	NH+1-GONE	NH+1-GONE
						=JL	=JR
	0	21	6	16	12	11	11
	1	20	6	16	10	10	12
	2	18	5	17	10	9	13
	3	16	5	17	8	8	14
	4	14	4	18	8	7	15
	5	12	4	18	6	6	16
	6	10	3	19	6	5	17 }

```

PROCEDURE SUMS; {called by proc fitparb}
BEGIN
  S1:=0; S2:=0; S3:=0; S4:=0;
  C1:=0; C2:=0; C3:=0;
  RBIAS:=R[JL];
  FOR I:=LPB TO RFB DO
    IF (I<=JL) OR (I>=JR)
      THEN {the sum will not contain the isoconcentration lines that
            have disappeared, so that the parabolic fit will ignore them}
      BEGIN
        RR:=R[I]-RBIAS;
        CC:=C[I];
        RSQ:=RR*RR;
        S1:=S1+RR;
        S2:=S2+RSQ;
        S3:=S3+RR*RSQ;
        S4:=S4+RSQ*RSQ;
        C1:=C1+CC;
        C2:=C2+CC*RR;
        C3:=C3+CC*RSQ;
      END;
  END; {procedure SUMS}

```

```

BEGIN {proc fitparb}
SUMS;
DEN:=NFIT*(S2*S4-S3*S3)-S1*(S1*S4-S2*S3)+S2*(S1*S3-S2*S2);
ADN:=C1*(S2*S4-S3*S3)-C2*(S1*S4-S2*S3)+C3*(S1*S3-S2*S2);
BDN:=NFIT*(C2*S4-C3*S3)-S1*(C1*S4-S2*C3)+S2*(C1*S3-C2*S2);
CDN:=NFIT*(S2*C3-S3*C2)-S1*(S1*C3-S3*C1)+S2*(S1*C2-S2*C1);
AF:=ADN/DEN;
BF:=BDN/DEN;
CF:=CDN/DEN;

```

{These 3 parameters describe the best fit parabola for the concentration distribution around the center of the tracer band. The fit is used for calculating volume, and only after the original flat top has disappeared, omitting the isoconcentration lines that have vanished as the peak of the band decreases in concentration.}

```

IF PRINPA THEN
BEGIN
  WRITELN('AF= ',AF,', BF= ',BF,', CF= ',CF);
  WRITELN(' J      R      C');
  FOR J:=LPB TO RPB DO
    IF (J<=JL) OR (J>=JR) THEN
      BEGIN
        RR:=R[J]-RBIAS;
        CC:=AF+RR*(BF+RR*CF);
        RR:=RR+RBIAS;
        WRITELN(J:2,RR:11:6,CC:11:6);
      END;
  PRINPA:=FALSE;
  READLN(CH);
  IF CH='Q' THEN EXIT(PROGRAM);
END; }

END; {procedure FITFARB}

```

```

PROCEDURE NTGRAND; {Called by both INTFLAT and INTAFTER, in proc
centroid. SG and SRG are the integrands CdR and CRdR, from J to J+1}
BEGIN
  RC:=(C[J]+C[J+1])*(R[J+1]-R[J])/2;
  SG:=SG+RC;
  SRG:=SRG+RC*(R[J]+R[J+1])/2;
END;

```

```

PROCEDURE INTFLAT;
{Called by centroid to integrate tracer band while GONE=0}
VAR JJ : INTEGER;
BEGIN
  S:=0;
  SR:=0;
  SPART[1]:=CR*C[1]*(R[2]-R[1])/2;
  SRPART[1]:=SPART[1]*(R[2]+R[1])/2;
  SG:=0; SRG:=0;
  FOR J:=1 TO NH-1 DO NTGRAND;
  SPART[2]:=SG;
  SRPART[2]:=SRG;
  RC:=(C[NH]+1)*(RLT-R[NH])/2;
  SPART[3]:=RC;
  SRPART[3]:=RC*(R[NH]+RLT)/2;
  SPART[4]:=(RRT-RLT);
  SRPART[4]:=(RRT*RRT-RLT*RLT)/2;
  RC:=(1+C[NH+2])*(R[NH+2]-RRT)/2;
  SPART[5]:=RC;
  SRPART[5]:=RC*(RRT+R[NH+2])/2;
  SG:=0; SRG:=0;
  FOR J:=NH+2 TO NISO-1 DO NTGRAND;
  SPART[6]:=SG;
  SRPART[6]:=SRG;
  SPART[7]:=CR*C[NISO]*(R[NISO]-R[NISO-1])/2;
  SRPART[7]:=SPART[7]*(R[NISO]+R[NISO-1])/2;

```

```

FOR JJ:=1 TO 7 DO
  BEGIN
    S:=S+SPART[JJ];
    SR:=SR+SRPART[JJ];
  END;
END; {proc intflat}

PROCEDURE INTAFTER;
  {Called by centroid to integrate tracer band after GONE>0}
  VAR JJ : INTEGER;
  BEGIN
    NLEFT:=NISO+1-2*GONE;
    QH:=NLEFT DIV 4;
    LPB:=QH+1;
    RPB:=NISO-QH;
    NFIT:=RPB-LPB+2-2*GONE;
    FITPARB;
    S:=0;
    SR:=0;
    SPART[1]:=CR*C[1]*(R[2]-R[1])/2;
    SRPART[1]:=SPART[1]*(R[2]+R[1])/2;
    SG:=0; SRG:=0;
    FOR J:=1 TO LPB-1 DO NTGRAND;
      SPART[2]:=SG;
      SRPART[2]:=SRG;
      RLP:=R[LPB]-RBIAS;
      RRP:=R[RPB]-RBIAS;
      LP2:=RLP*RLP;
      LP3:=RLP*LP2;
      LP4:=RLP*LP3;
      RP2:=RRP*RRP;
      RP3:=RRP*RP2;
      RP4:=RRP*RP3;
      SPART[4]:=AF*(RRP-RLP)+BF*(RP2-LP2)/2+CF*(RP3-LP3)/3;
      SRPART[4]:=AF*RBIAS*(RRP-RLP)+(AF+BF*RBIAS)*(RP2-LP2)/2
        +(BF+CF*RBIAS)*(RP3-LP3)/3+CF*(RP4-LP4)/4;
      SG:=0; SRG:=0;
      FOR J:=RPB TO NISO-1 DO NTGRAND;
        SPART[6]:=SG;
        SRPART[6]:=SRG;
        SPART[7]:=CR*C[NISO]*(R[NISO]-R[NISO-1])/2;
        SRPART[7]:=SPART[7]*(R[NISO]+R[NISO-1])/2;
      FOR JJ:=1 TO 7 DO
        BEGIN
          S:=S+SPART[JJ];
          SR:=SR+SRPART[JJ];
        END;
      END;
    END; {proc intafter}

BEGIN {proc centroid}
  S:=0;
  SR:=0;
  FOR IP:=1 TO 7 DO BEGIN
    SPART[IP]:=0;
    SRPART[IP]:=0;
  END;

  IF GONE=0
    {The initial flat top of concentration distribution still exists}
  THEN BEGIN

```

```

        ADJUST;
        INTFLAT;
        END
    ELSE INTAFTER; {The decay of peak concentration has started}
    VOLR:=VCOF*SR; {barrels of fluid in tracer slug}
    END; {procedure centroid}

PROCEDURE DERIVS; {Computes 1st and 2nd derivatives of C with R}
VAR DL,DR,RDL,RDR,PD1MX,SX,BX,RDY,LDY,CZZ : REAL;
    L : INTEGER;

PROCEDURE AFTER; {Called from proc derivs after GONE>0}
BEGIN
    PD1MX:=0;
    FOR J:=2 TO NISO-1 DO
        IF (J<JL) OR (J>JR)
            THEN
                BEGIN
                    RDL:=R[J]-R[J-1];
                    RDR:=R[J+1]-R[J];
                    DL:=(C[J]-C[J-1])/RDL;
                    DR:=(C[J+1]-C[J])/RDR;
                    PD1[J]:=(DL+DR)/2;
                    IF ABS(PD1[J])>PD1MX THEN PD1MX:=PD1[J];
                    PD2[J]:=2*(DR-DL)/(RDL+RDR);
                    IF J=2 THEN BEGIN
                        PD1[1]:=DL/2;
                        PD2[1]:=DL*G/RDL;
                    END;
                    IF J=NISO-1 THEN BEGIN
                        PD1[NISO]:=DR/2;
                        PD2[NISO]:=-DR*G/RDR;
                    END;
                END;
            SX:=R[J]-R[JL];
            BX:=R[J+1]-R[JL-1];
            RDY:=C[J]-C[J+1];
            LDY:=C[JL]-C[JL-1];
            CZZ:=4/(BX*BX-SX*SX);
            PD1[JL]:=LDY*CZZ*SX;
            PD1[J+1]:=-RDY*CZZ*SX;
            PD2[JL]:=-2*LDY*CZZ;
            PD2[J+1]:=-2*RDY*CZZ;
        END; {proc AFTER}

BEGIN {procedure DERIVS}
    PD1MX:=0;
    IF GONE=0
        THEN BEGIN
            FOR J:=2 TO (NISO-1) DO
                BEGIN
                    RDL:=R[J]-R[J-1];
                    RDR:=R[J+1]-R[J];
                    IF J=NH THEN RDR:=CR*RDL;
                    IF J=NH+2 THEN RDL:=CR*RDR;
                    DL:=(C[J]-C[J-1])/RDL;
                    DR:=(C[J+1]-C[J])/RDR;
                    PD1[J]:=(DL+DR)/2;
                    IF J=NH+1 THEN BEGIN
                        PD1[J]:=0;

```

```

                PD2[J]:=0;
                END;
            IF ABS(PD1[J])>PD1MX THEN PD1MX:=PD1[J];
            PD2[J]:=2*(DR-DL)/(RDL+RDR);
            IF J=2 THEN BEGIN
                PD1[1]:=DL/2;
                PD2[1]:=DL*G/RDL;
            END;
            IF J=NISO-1 THEN BEGIN
                PD1[NISO]:=DR/2;
                PD2[NISO]:=-DR*G/RDR;
            END;
        END;
    END
ELSE AFTER;
    {G is correction factor for 2nd derivative approximations at the
    ends - ie., for J=1 and NISO. value of G is 0.35}
    IF PD1MX<1 THEN KK:=10;
END; {procedure derivs}

FUNCTION VEL(LN:INTEGER):REAL;
    {Computes displacement velocity of isoconc. lines}
    VAR A,B,RI : REAL;
    BEGIN
        RI:=1/R[LN];
        A:=(1+LAM*PHI*(1-SOR)*RI)*RI/(2*WT);
        B:=BB + CC*RI;
        IF (ABS (PD1[LN])>DEL) THEN VEL:=A-B*PD2[LN]/PD1[LN]
            ELSE VEL:=A;
    END;

PROCEDURE DELTIME; {Calculates maximum safe time increment for
                    iteration. Also prints out a,b or c as indicator.}
    VAR DTA,DTB,DTC,DRMM,FRMX,DF,DISP,RAV : REAL;
        JX,QH : INTEGER;

    BEGIN
        QH:=NISO DIV 4;
        DTA:=0.02*WT*SQR(R[QH]);
        {Criterion A to keep quadratic Taylor term small}
        DRMM:=100;
        RAV:=(R[J]+R[JL])/2;
        FOR J:=3 TO JL DO
            BEGIN
                DF:=(R[J]-R[J-1]);
                IF DF < DRMM THEN DRMM:=DF;
                JX:=J;
            END;
        {DRMM is now least distance between adjacent isoconc lines}
        IF R[JX]<RAV THEN RAV:=R[JX];
        DISP:=BB+CC/RAV;
        DTB:=0.1*DRMM*DRMM/DISP;
        {Criterion B is to maintain arithmetic stability}
        DTC:=0.010;
        {Criterion C sets max time increment to be permitted}
        IF (DTA < DTB) AND (DTA < DTC)
            THEN BEGIN WRITE('a'); DLT:=DTA; END
            ELSE BEGIN IF (DTB < DTC) THEN BEGIN WRITE('b'); DLT:=DTB; END

```

```

ELSE BEGIN WRITE('c'); DLT:=DTC; END;
END;
IF DLT<1E-7 THEN BEGIN WRITPROF; EXIT (PROGRAM); END;
END; {procedure deltime}

PROCEDURE PRGONE; FORWARD;

PROCEDURE MOVE; {Computes new positions of isoconc lines, and also a new
value of RCEN as center of conc. distribution, and also new T}
VAR DLR,SDR,RL,RDL,RDM,RDR,RDH,RCG,DC : REAL;

PROCEDURE FLADJUST; {Compares spacing of points around peak, to det-
ermine whether GONE should be set from 0 to 1. Also,
while GONE is 0 (ie while tracer band is still flat)
evaluates RLT and RRT for area and volume calcs.}

VAR RL2,RL1,RPK,RR1,RR2,RLT,RRT : REAL;
BEGIN
RR2:=RINH+3J;
RR1:=RINH+2J;
RPK:=RINH+1J;
RL1:=RINHJ;
RL2:=RINH-1J;
RLT:=RL1-CR*(RL1-RL2);
RRT:=RR1-CR*(RR2-RR1);
RL:=(RL1+RR1)/2;
RINH+1J:=RL;
IF (RRT<=RLT)
THEN BEGIN
WRITELN('RLT= ',RLT:7:4,' RPK= ',RPK:7:4,' RRT= ',RRT:7:4);
WRITELN(' ** ONE GONE ** CR=',CR:5:4);
GONE:=1;
JL:=NH;
JR:=NH+2;
PRINPA:=TRUE;
RINH+1J:=(RL1+RR1)/2;
CINH+1J:=-1;
END;
END; {proc fladjust}

PROCEDURE REDO;
{Called form proc move. Redoubles the number of R and C points,
after GONE has reached QU+1}
VAR RT,CT : ARRAY[1..NHH] OF REAL;
K,KK,LK : INTEGER;

BEGIN
WRITELN(' Entering PROCEDURE REDO...');
FOR K:=1 TO QU DO
BEGIN
RT[K]:=R[K];
CT[K]:=C[K];
END;
FOR K:=QU+1 TO NH DO
BEGIN
LK:=NISO-2*QU+K;
RT[K]:=R[LK];
CT[K]:=C[LK];
END;
END;

```

```

FOR LK:=QU DOWNT0 1 DO
  BEGIN
    KK:=2*LK;
    K:=KK-1;;
    IF LK>1
      THEN BEGIN
        R[KK]:=RT[LK];
        R[K]:=(RT[LK]+RT[LK-1])/2;
        C[KK]:=CT[LK];
        C[K]:=(CT[LK]+CT[LK-1])/2;
      END
    ELSE BEGIN
      R[2]:=RT[1];
      R[1]:=RT[1]-CR*(R[3]-R[2]);
      C[2]:=CT[1];
      C[1]:=CT[1]/2;
    END;
  END;
FOR LK :=QU+1 TO NH DO
  BEGIN
    KK:=2*LK;
    K:=KK+1;
    IF LK<NH
      THEN BEGIN
        R[KK]:=RT[LK];
        R[K]:=(RT[LK]+RT[LK+1])/2;
        C[KK]:=CT[LK];
        C[K]:=(CT[LK]+CT[LK+1])/2;
      END
    ELSE BEGIN
      R[NISO-1]:=RT[NH];
      R[NISO]:=RT[NH]+CR*(R[NISO-1]-R[NISO-2]);
      C[NISO-1]:=CT[NH];
      C[NISO]:=CT[NH]/2;
    END;
  END;
GONE:=1;
JL:=NH;
JR:=NH+2;
WRITELN('REDONE');
NREDONE:=NREDONE+1;
END; {proc REDO}

BEGIN {proc move}
  OLGONE:=GONE;
  SDR:=0;
  DRMX:=0;
  FOR J:=1 TO NISO DO
    IF (J<=JL) OR (J>=JR)
      THEN BEGIN
        DLR:=VEL(J)*DLT;
        IF ABS(DLR)>DRMX THEN DRMX:=DLR;
        SDR:=SDR+DLR;
        R[J]:=R[J]+DLR ;
      END;
  IF GONE=0
    THEN FLADJUST
    ELSE BEGIN
      RL:=(R[JL]+R[JR])/2;
      RDM:=(R[JR]-R[JL]);
    END;

```

```

RDL:=R[JL]-R[JL-1];
RDR:=R[JR+1]-R[JR];
RDH:=(RDR+RDL)/4;
DC:=C[JL]-C[JL-1];
RCG:=SQRT(0.015*DC/BLR); {BLR defined in PARAM}
IF (R[JR]-R[JL])<RCG {GONESTEP criterion}
  THEN BEGIN
    R[JL]:=RL; C[JL]:=-C[JL];
    R[JR]:=RL; C[JR]:=-C[JR];
    GONE:=GONE+1;
    JL:=NH+1-GONE;
    JR:=NH+1+GONE;
    PRINPA:=TRUE;
    WRITELN('GONE=',GONE:2,', R[JL]=',
            R[JL]:7:4,', RL=',RL:7:4,', R[JR]=',R[JR]:7:4);
  END;
END;
T:=T+DLT;
IF OLGONE<>GONE THEN PRGONE;
RCEN:=RL;
IF (GONE>QU) AND (RDM<RDH) THEN REDO;
END; {procedure move}

PROCEDURE CRECTV; { To keep roundoff and other errors from changing
  initial value of tracer band volume too far. Acts by adjusting
  values of R.}
VAR RM : REAL;
BEGIN
  VCF:=VC;
  VC:=1-VOLR/VOLIN;
  RM:=(R[JR]+R[JL])/2;
  FOR J:=1 TO NISO DO
    IF (J<=JL) OR (J>=JR) THEN
      R[J]:=R[J]+ALF*VC*(R[J]-RM);
  END; {proc CRECTV}

PROCEDURE PARAM; {to calculate maxconc (CPK) and tracer bandwidth (BW)}
VAR CK,CR1,CL1,CR2,CL2,RR1,RL1,RR2,RL2,BL,BR,RC : REAL;

PROCEDURE FIND;
VAR KF : INTEGER;
    RLH,RRH : REAL;
BEGIN
  KF:=0;
  REPEAT
    KF:=KF+1;
    CK:=C[KF];
  UNTIL CK>CHALF;
  RLH:=R[KF-1]+(CHALF-C[KF-1])*(R[KF]-R[KF-1])/(C[KF]-C[KF-1]);
  KF:=NISO+1;
  REPEAT
    KF:=KF-1;
    CK:=C[KF];
  UNTIL CK>CHALF;
  RRH:=R[KF+1]+(CHALF-C[KF+1])*(R[KF]-R[KF+1])/(C[KF]-C[KF+1]);
  BW:=RRH-RLH;
END; {procedure FIND}

```

```

BEGIN {procedure param}
  IF GONE=0
  THEN BEGIN
    CPK:=1;
    CHALF:=CPK/2;
    FIND;
  END
  ELSE BEGIN
    RC:=(R[JJR]+R[JLJ])/2;
    RR1:=R[JJR];
    CR1:=C[JJR];
    RR2:=R[JJR+1];
    CR2:=C[JJR+1];
    RL1:=R[JLJ];
    CL1:=C[JLJ];
    RL2:=R[JLJ-1];
    CL2:=C[JLJ-1];
    BR:=(CR1-CR2)/(SQRT(RC-RR2)-SQRT(RC-RR1));
    BL:=(CL1-CL2)/(SQRT(RC-RL2)-SQRT(RC-RL1));
    BLR:=(BL+BR)/2;
    CPK:=(CL1+BL*SQRT(RC-RL1) + CR1+BR*SQRT(RC-RR1))/2;
    CHALF:=CPK/2;
    FIND;
  END;
END; {procedure param}

PROCEDURE PRGONE; {To list values of T,GONE,CPK,BW, and VOLR}
  VAR PR : TEXT;
  BEGIN
    REWRITE(PR,'#6:');
    PARAM;
    IF RPR THEN WRITE(PR,RNUM:2,' ')
      ELSE WRITE(PR,' ');
    WRITELN
    (PR,' at T=',T:9:5,' , GONE= ',GONE:2,' ,CPK=',
    CPK:7:5,' ,BW=',BW:9:6,' ,V=',VOLR:8:5);
    CLOSE(PR,NORMAL);
    IF IST<=KJ THEN
      BEGIN
        IX[IST]:=GONE;
        A1[IST]:=T;
        A2[IST]:=CPK;
        A3[IST]:=(R[JLJ]+R[JJR])/2;
        A4[IST]:=BW;
        A5[IST]:=VOLR;
      END;
    IST:=IST+1;
  END;

PROCEDURE STATF; {To be called after calculations are finished,
                  to save "statrecords" on disk.}
  VAR SKRYP : STRING;
      FT,T75 : REAL;

  BEGIN
    SKRYP:=SKRIP;
    T75:=5625*WT;
    B75:=0;
    WITH PIECE DO

```

```

FOR I:=1 TO KJ DO
  BEGIN
    STGONE[I]:=IX[I];
    STTIM[I]:=A1[I];
    STPK[I]:=A2[I];
    STRPK[I]:=A3[I];
    STBW[I]:=A4[I];
    STVOL[I]:=A5[I];
    IF (A1[I]>T75) AND (B75=0)
      THEN BEGIN
        FT:=(T75-A1[I-1])/(A1[I]-A1[I-1]);
        B75:=FT*A4[I]+(1-FT)*A4[I-1];
      END;
    END;
    J:=POS(' .DATA',SKRYP);
    IF J<>0 THEN DELETE(SKRYP,J,5);
    SKRYP:=CONCAT('STAT',SKRYP);
    REWRITE(PULP,SKRYP);
    SEEK(PULP,0);
    PULP^:=PIECE;
    PUT(PULP);
    CLOSE(PULP,LOCK);
  END;
END;

PROCEDURE TXFILE: (to update a summary textfile with info about all runs)
CONST SUDX='*:SUMX.TEXT';
VAR HEADA,HEADB,HEAD : STRING;
    LINE : ARRAY[0..25] OF STRING;
    SUMREC : TEXT;
    IOR : INTEGER;
BEGIN
  { $I- } RESET(SUMREC,SUDX);
  IOR:=IORRESULT;
  CLOSE(SUMREC,NORMAL);
  { $I+ }
  IF IOR=0
  THEN WRITELN('File ',SUDX,' is already on disk ok.')
  ELSE IF IOR=10
  THEN BEGIN
    REWRITE(SUMREC,SUDX);
    HEADA:=' FILENAME SERNO No.Fronts Dmcl ';
    HEADB:=' Lambda Inj.Dur. TBW at 75ft';
    HEAD:=CONCAT(HEADA,HEADB);
    WRITELN(SUMREC,HEAD);
    CLOSE(SUMREC,LOCK);
  END
  ELSE BEGIN
    WRITELN(' IORRESULT= ',IOR);
    EXIT(PROGRAM);
  END;
  I:=0;
  RESET(SUMREC,SUDX);
  WHILE NOT EOF(SUMREC) DO
    BEGIN
      READLN(SUMREC,LINE[I]);
      I:=I+1;
    END;
  J:=I-1;
  CLOSE(SUMREC,PURGE);
  REWRITE(SUMREC,SUDX);

```

```

FOR I:=0 TO J DO WRITELN(SUMREC,LINE[I]);
WRITELN(SUMREC,' ',SKRIP:10,SN:7,RNUM:9,DFC:11:6,LAM:10:6,
TD:9:3,B75:11:3);
CLOSE(SUMREC,LOCK);
END;

PROCEDURE SIGNAL; {to signal completion of run}
BEGIN
FOR J:=1 TO 10 DO
BEGIN
FOR K:=1 TO 1000 DO
RINHJ:=RINHJ+1.02;
WRITE(CHR(07));
END;
END;

BEGIN {MAIN}
INIT;
KK:=1;
DERIVS;
BLR:=0.1;
ITL:=1;
PRINPA:=FALSE;
CENTROID;
VOLIN:=VOLR;
WRITEPRF;
WRITELN('RCEN= ',RCEN:7:4);
RPR:=TRUE;
PRGONE;
RPR:=FALSE;
WHILE RCEN<RFAR DO
BEGIN {major 'while' loop}
RTAR:=RCEN + 2;
ITL:=1;
K:=0;
DELTIME;
WHILE RCEN<RTAR DO
BEGIN {minor 'while' loop}
K:=K+1;
ITL:=ITL+1;
DERIVS;
IF K=KK THEN BEGIN
K:=0;
DELTIME;
END;
MOVE;
IF ITL MOD 4 =0 THEN WRITE('!');
IF ITL MOD 20 =0 THEN
BEGIN
WRITELN;
CENTROID;
PARAM;
CRECTV;
IF (VC*VCP<0) AND (ALP>0.05)
THEN ALP:=ALP*0.5
ELSE IF ABS(VC)>0.0005
THEN ALP:=ALP*1.25;
WRITELN('DLT= ',DLT,' RCEN= ',
RCEN:7:4,' T= ',T:7:4,' V=',VOLR:7:4,

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        ',VC=',VC:7:5,',BW=',BW:6:3);
        WRITELN('ALP=',ALP:7:6);
        END;
    END; {while still iterating to get next filable conc.distribution}
WRITE(' ');
    WRITELN('ready: RCEN=',RCEN:7:4,', RTAR=',RTAR:7:4);
CENTROID;
PARAM;
WRITELN('GONE= ',GONE,', CPK=',CPK:7:4,', BW=',BW:8:4);
WITH TFRONT DO
    BEGIN
        SERNO:=SN;
        MXCON:=CPK;
        TBNDWID:=BW;
        TM:=T;
        VM:=VOLR;
        VKRCT:=VC;
        FOR J:=1 TO NISO DO
            BEGIN
                RAD[J]:=R[J];
                C[C[J]]:=C[J];
            END;
        NIT:=10*ITL+K;
    END;
    RNUM:=RNUM+1;
    RPR:=TRUE;
    PRGONE;
    RPR:=FALSE;
    IF RNUM MOD 10=0 THEN WRITPROF;
    RESET(CONDIST,SKRIP);
    SEEK(CONDIST,RNUM);
    CONDIST^:=TFRONT;
    PUT(CONDIST);
    CLOSE(CONDIST);
    WRITELN('RECNUM ',RNUM,' written');
END; {while last file entry's not yet made}
STATF;
TXFILE;
SIGNAL;
WRITELN('IST=',IST:4); {number of items in "statrecord"}
END.

```

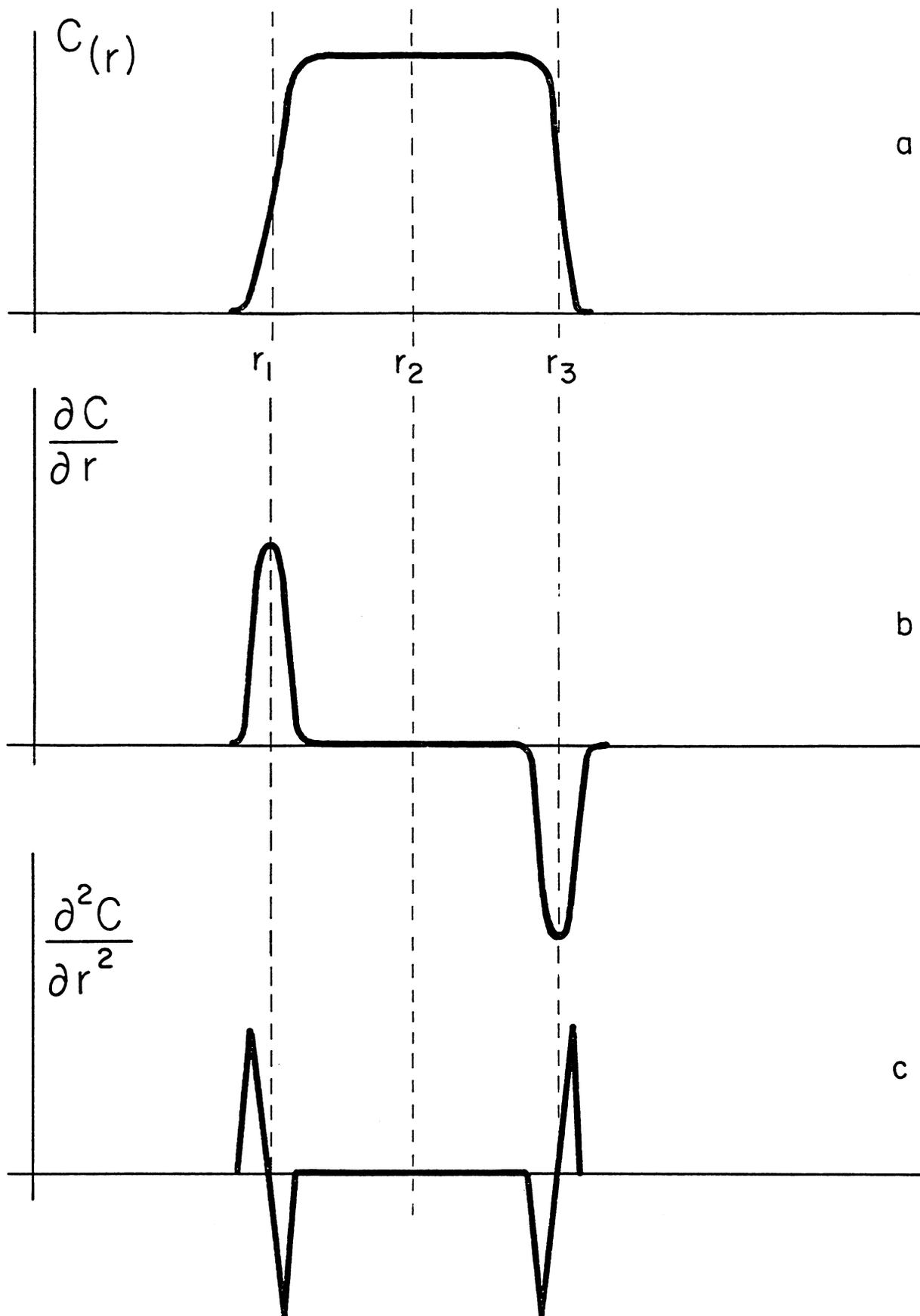


Fig. C2. Tracer slug and its derivatives during early flattop stage.

After the flattop stage has been ended, and the peak concentration starts to decrease, the concentration and its two derivatives will resemble the sketch of Fig. C3. The same general description of the two mechanisms is appropriate, with one addition. For this case, the second derivative at the center of the tracer band is no longer zero, while the first derivative goes through zero at r_2 . Referring again to Eq. (C9), this means that the isoconcentration velocity at r_2 is indeterminate. This corresponds to the fact that this particular isoconcentration line simply disappears as the peak concentration decreases below the value it had denoted.

In the program listed in Fig. C1, the computation is initiated with 37 isoconcentration lines describing a concentration slug like that shown in Fig. C2. A special variable, named GONE, retains its initial value of zero so long as the original flattop is present on the concentration peak.

As the computation proceeds, the middle isoconcentration lines gradually approach each other, and round the "corners" of this concentration distribution, simulating the actual flow of the tracer band. Eventually the flattop disappears, and GONE is set to one. Then as the "flow" proceeds, peak concentration decreases still further and more isoconcentration lines are lost. A procedure REDO keeps track of the variable GONE--when it has declined to a critical value, it doubles the number of isoconcentration lines by putting new ones between each of the remaining lines.

One further topic that bears discussion concerns the movement of the isoconcentration lines. Their velocities may, of course, be calculated by Eq. (C9). But the radial increment by which the line is to be moved depends also on the time increment DT . Obviously, the computation proceeds more rapidly if DT is as large as possible. At the same time, there are maximum values that must not be exceeded, lest various errors become large. The first of these constraints, called DTA , is a limit imposed by the need to keep the time increment small enough so that the change in velocity from one end to the other of the resulting radial increment, will be no larger than, say, 2%.

The second limit, DTB , is imposed by the need for arithmetic stability, so that any bumps in the concentration distribution caused by round-off errors will be reduced in each subsequent iteration. (This superstability was discussed previously and is analogous to the diffusion process itself that is represented by the equation). A third limit DTC is imposed only for convenience, so that the time increment will not grow so large in the later stages of the computer run that detail is lost in the simulation.

Various other procedures have also been inserted into the program, to request input information, to manage the calculation, and to record the results on several disk files and the printer.

Results of the Tracerband Calculations

Several runs of the program described have been run with different values of input parameters. For convenience in modeling the needs of the injection program at the Rock Creek field (where it had appeared likely that radioactive tracers might be injected at well PI-2, for detection at the observation well OB-2), the runs were usually carried out to a radial distance of 80 feet, and with an introduced flow rate of 200 bpd. Values of horizon thickness,

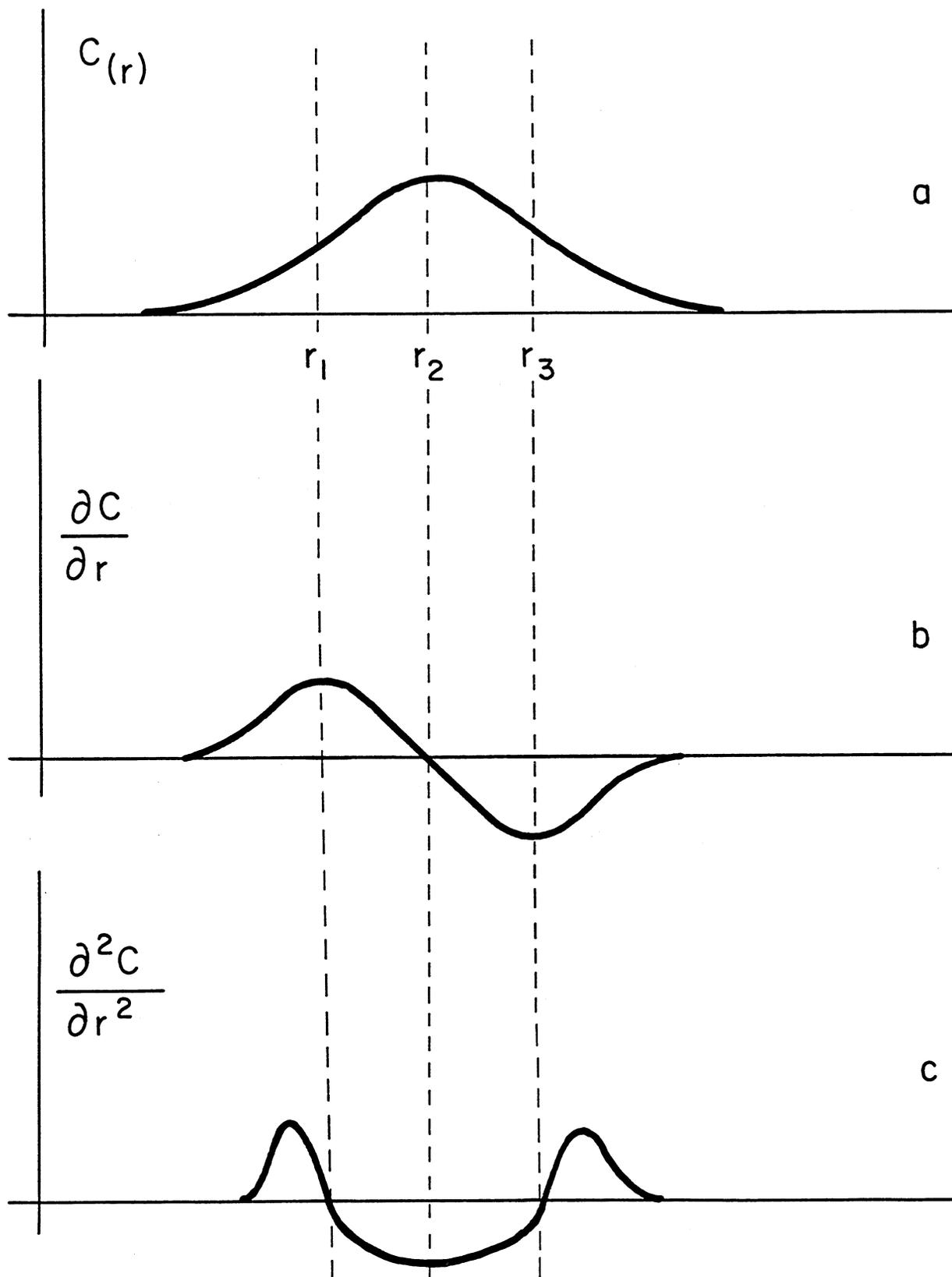


Fig. C3. Tracer slug and its derivatives after flattop stage is past.

porosity, and residual oil saturation appropriate to Rock Creek were also used.

Of the two independent terms of the dispersion equation, the one containing molecular diffusion coefficient, did not permit much variation. Several widely varying values were used of the characteristic dispersion distance σ_{Ld_p} (labelled LAM in the program), however. Some of these results are shown in Fig. C4, which shows several successive concentration profiles, and in Fig. C5, which shows both the variation of tracer bandwidth and peak concentration as a function of radial distance.

Conclusions

Results of these calculations show the widths of the tracer band to be expected with various values of dispersion coefficient, in a relatively near-well situation. Perhaps the most striking feature is that even with large values of the dispersivity distance σ_{Ld_p} , the slug width predicted by this program, to be observed at 75 feet away, is no greater than 5 or 6 feet.

It might be expected that a depth-averaged observation in an actual field situation might be made considerably greater than this by the variation in velocity at different levels in the formation. The detection of such artificial broadening would be a major benefit to be obtained from the frequent use of a gamma ray logging-tool, following the injection of a slug of radioactive isotope into the injector.

The advantage of logging a shut-in, or non-producing observation well would be that the observations could be made of the actual concentration peak as it passed by. Production of an appreciable quantity of fluid from the well, however, would cause major changes in the shapes of the observed peaks, even if, by frequent logging, the response could be recorded as a function of depth.

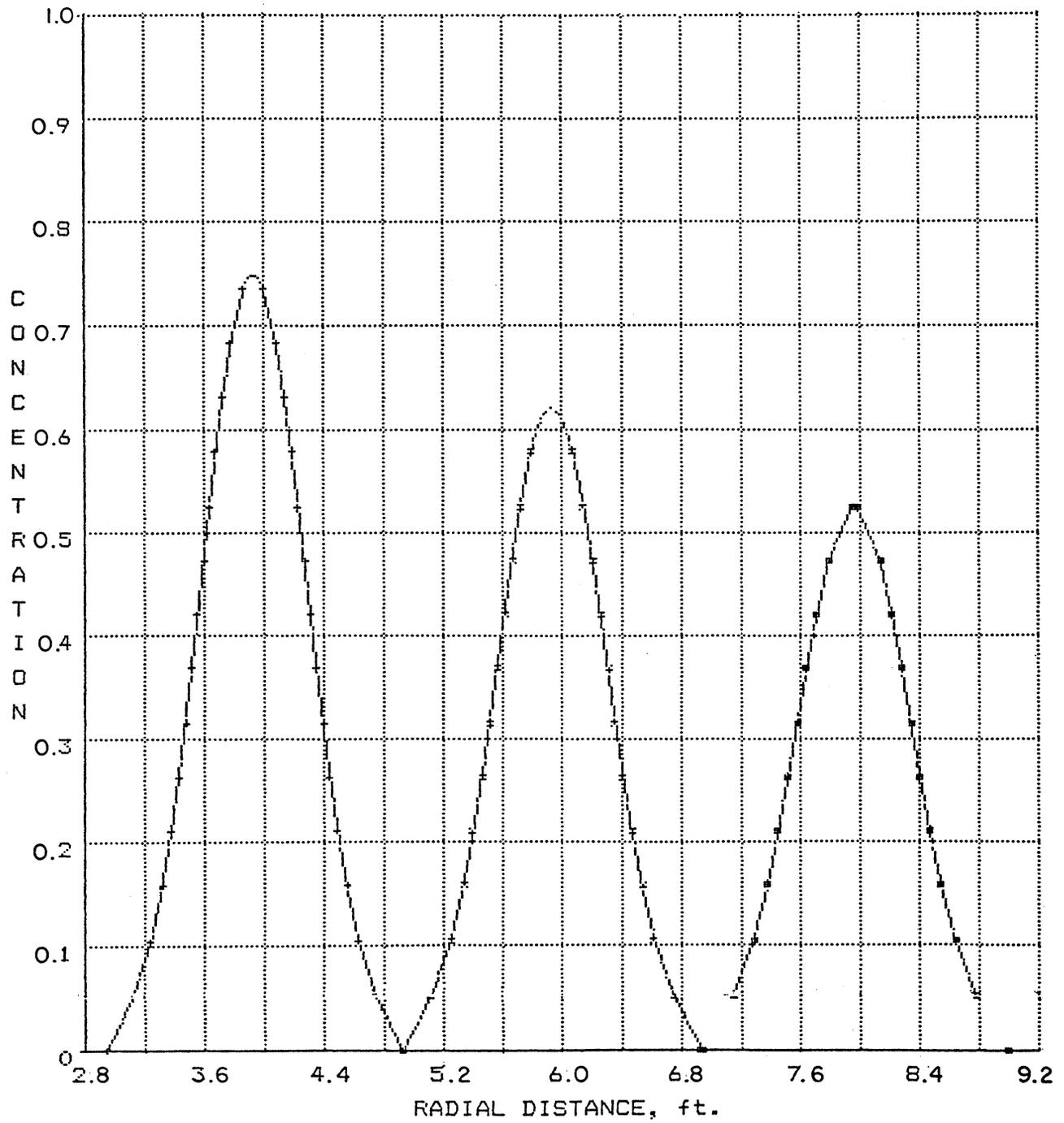


Fig. C4. Successive peaks after disappearance of flattop stage.

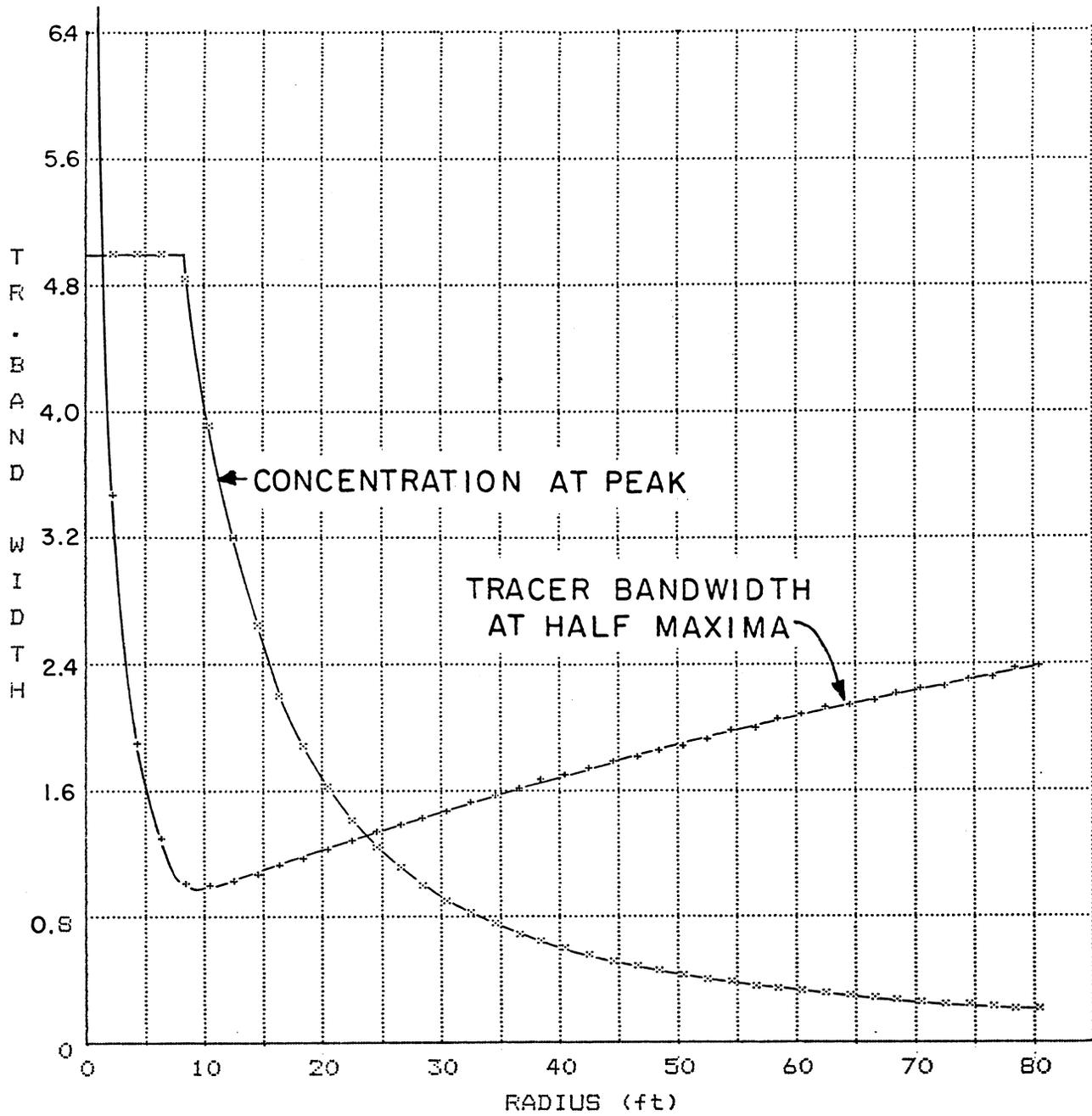


Fig. C5. Peak concentration and tracer bandwidth

$$D_m = .0011 \text{ ft}^2/\text{day}$$

$$\sigma_{Lp} d = 0.018 \text{ ft}$$

Duration of tracer injection: 0.167 days

