SOME THOUGHTS ON CRACK GROWTH

IN HYDRAULIC FRACTURING

by

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INTRODUCTION

A rational approach to predicting the extent and shape of the fracture surface generated by hydraulic fracturing of a well is to make use of linear elastic fracture mechanics. In such an approach the different rock layers may be idealized as homogeneous, isotropic and linear elastic. A crack in a given rock is assumed to advance when the stress intensity factor at the crack tip is equal to the value for which crack growth is observed in laboratory fracture experiments. Even with these idealizations the problem remains extremely difficult because (a) the problem is inherently 3-dimensional, (b) the problem is one of undetermined boundaries in that the location of the crack front is the subject of primary interest, and (c) the problem is complicated by the presence of layers with different elastic moduli. In order to make progress it is clear that further idealizations are necessary. To this end we make the following assumptions:

1. <u>The presence of the bore hole can be neglected</u>. (The circumferential "hoop" stress around the cylindrical cavity is $O(r^{-2})$ so that the perturbation in the stress field due to the hole is highly localized. At a distance of 10 radii from the hole the stress perturbation due to the hole is of the order of 1% of the pressure used for hydraulic fracturing.)

2. <u>The shale layers above and below the oil or gas bearing sandstone layers can be regarded as half-spaces</u>. (This assumption is more tenable if the shale layers are thick relative to the sandstone layer. In any event it appears to be a reasonable first approximation. Once it is made the depth and inclination of the sandstone layer becomes immaterial; inclined bedding planes are no different from horizontal bedding planes.)

3. One of the principal stress directions is perpendicular to the plane of the sandstone layer; the smallest compressive principal stress acts on a plane perpendicular to the sandstone layer. Then, the crack surface initiates on the plane subject to minimum compressive stress and the effective internal pressure driving the crack is the difference between the pressure applied for hydraulic fracturing and the minimum compressive principal stress. (Here we also assume that the in-situ stress field is homogeneous; that the variation of the minimum compressive stress over the distances involved is negligible.)

Based on these assumptions the problem becomes one of finding the locus $\Gamma(x,y) = 0$ of the crack front such that for an effective internal pressure p acting on the crack surface in the plane z = 0, the stress intensity factor K_I on $\Gamma(x,y) = 0$ is everywhere equal to the critical stress intensity factor K_{IC}.

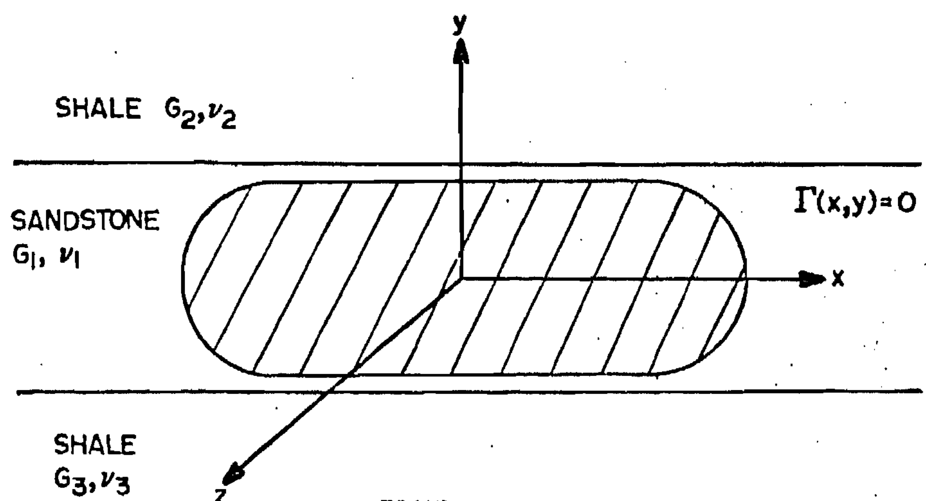

FIGURE 1.

As a first step in understanding the shape $\Gamma(x,y) = 0$ consider the case when the elastic properties of the materials are the same. For a homogeneous, isotropic elastic solid the crack would become penny-shaped and the relationship between the pressure p and the crack radius b would be:

$$K_{IC} = \frac{2}{\pi} p \sqrt{\pi b} \qquad \text{(b=crack radius)} \qquad (1)$$

If the elastic properties are homogeneous, but the critical stress intensity factor is nonuniform (varies with the coordinate y to reflect the layering) then a qualitative understanding of the shape $\Gamma(x,y) = 0$ can be obtained by considering an elliptical crack as shown in Figure 2. The result is that the major axis:minor axis ratio a:b is related to the critical stress intensity factors on the major and minor axes by:

$$\frac{a}{b} = \left(\frac{(K_I)_{\phi=\pi/2}}{(K_I)_{\phi=0}}\right)^2 \tag{2}$$

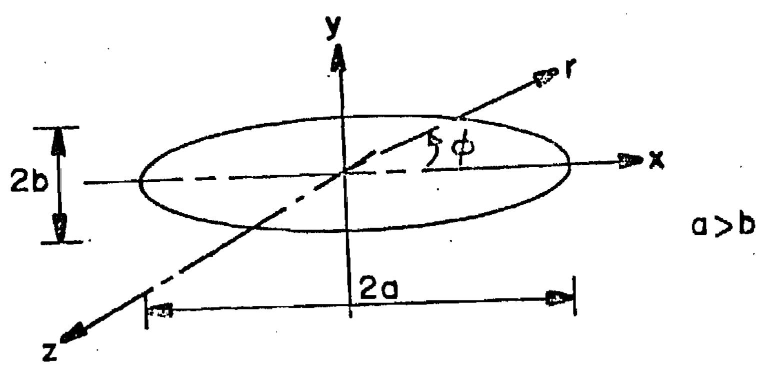

FIGURE 2

STABILITY OF CRACK SHAPES

Stress normal to the crack face (see Figure 2) is given by:

$$\sigma_{zz} = \frac{p}{E\left[(1-(b/a)^2)^{\frac{1}{2}}\right]} \left(\frac{b}{a}\right)^{\frac{1}{2}} (a^2 \sin^2\phi + b^2 \cos^2\phi)^{\frac{1}{4}} \quad \frac{1}{(2r)^{\frac{1}{2}}}$$

where E(s) is the complete elliptic integral of the second kind ($s \equiv \sqrt{1-(b/a)^2}$)

$$\phi = \frac{\pi}{2} \qquad \sigma_{zz} \simeq \frac{p}{E(s)} \left(\frac{b}{a}\right)^{\frac{1}{2}} a^{\frac{1}{2}} \frac{1}{(2r)^{\frac{1}{2}}} = \frac{p\sqrt{b}}{E(s)(2r)^{\frac{1}{2}}}$$

$$\phi = 0 \qquad \sigma_{zz} \simeq \frac{p\sqrt{b}}{E(s)(2r)^{\frac{1}{2}}} \left(\frac{b}{a}\right)^{\frac{1}{2}} \qquad \left\{ \text{Note: } \frac{b}{a^{\frac{1}{2}}} = \sqrt{r_{curv} \text{ at } \phi=0} \right\}$$

$$\text{Thus:} \qquad \frac{(K_I)_{\phi=0}}{(K_I)_{\phi=\pi/2}} = \left(\frac{b}{a}\right)^{\frac{1}{2}} < 1$$

(In a homogeneous isotropic solid the stress intensity factor is greatest at the ends of the minor axes; the crack would grow along the minor axis until the shape of the crack-front became circular.)

Thus, for example, if the shale and sandstone layers had similar elastic moduli, but K_{IC} for the shale is <u>twice</u> the value for the sandstone, then the extent of the crack along the layer (x-direction) could be expected to be <u>four</u> times the extent perpendicular to the layer (y-direction).

In order to gain insight into the effects of the shale and sandstone having different elastic properties consider the case where the crack extends a long distance along the x-axis (5-10 times the depth of the sandstone layer); then, the stress intensity factor at the crack front on the y-axis should be given quite accurately by the stress intensity factor for a plane strain crack approaching a plane interface at normal incidence. The latter problem has been considered by Erdogan & Biricikoglu (*Int. J. Eng. Sci.*, *Vol. 11*, 1973, p. 745-766); numerical results have been given for a crack going from a soft material (epoxy) to a hard material (aluminum) and vice versa. The former case is of interest here because the elastic moduli for the sandstone are less than for the shale. The qualitative behavior is shown in Figure 3.

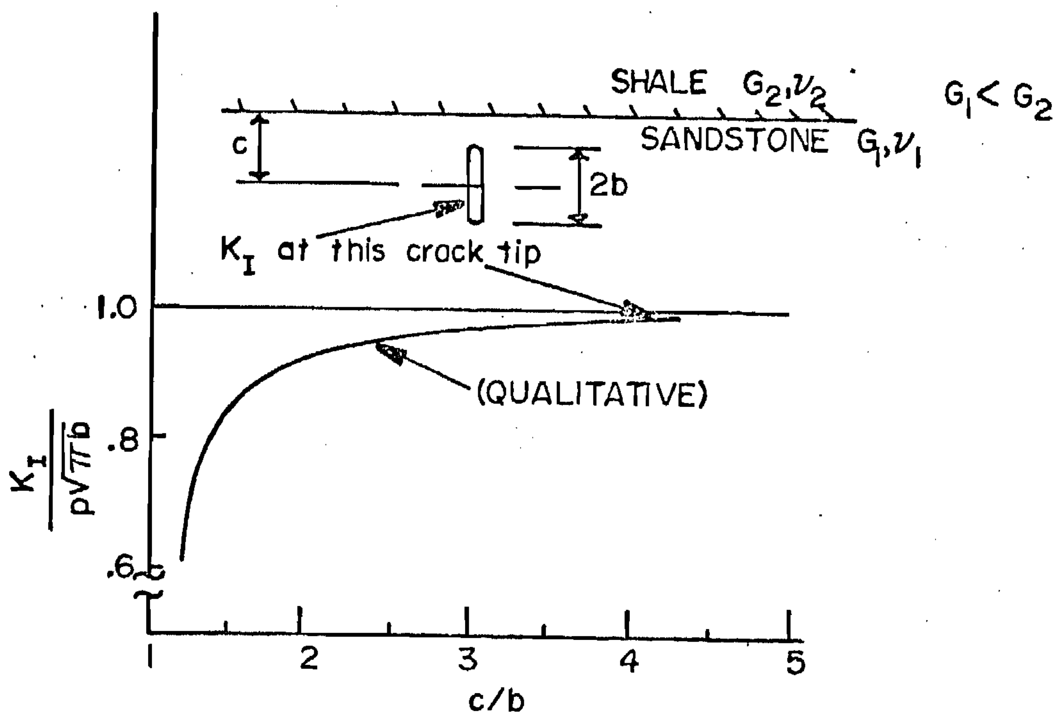

FIGURE 3.

Note that as the crack tip approaches the interface (i.e., as $(c/b) \to 1+$) the stress intensity factor K_I goes to <u>zero</u>. *Therefore, the presence of a layer of higher elastic modulus (i.e., $G_2 > G_1$) resists strongly the extension of cracks from a low modulus layer into a high modulus layer.* Indeed, since the stress intensity factor for the plane strain case is greater than for the case of a curved boundary (as shown in Figure 1) and the former $\to 0$ as the interface is approached, *it appears that it may be impossible for a crack to propagate from a low modulus material into a high modulus material (at least due to internal pressure).*

One additional mechanism for crack extension should be mentioned: As the crack-front approaches the sandstone-shale interface the normal stress, σ_{yy}, acting on the interface becomes infinite. The strength of the singularity depends on the ratio of the elastic moduli; for a crack originating in the sandstone the singularity has the form $\sigma_{yy} \approx r^{-\alpha}$ where $0 < \alpha < \frac{1}{2}$. The stress σ_{yy} at the interface is <u>tensile</u> so that interfacial cracks can be expected to develop as the crack $\Gamma(x,y) = 0$ approaches a sandstone-shale interface. (This failure mechanism is analogous to the delamination that occurs in the failure of fiber reinforced composites; i.e. cracks in the soft matrix do not cut through the hard fibers, but turn along the fibers to separate the fibers from the matrix.) Development of these interfacial cracks would lead to the following in crack geometry:

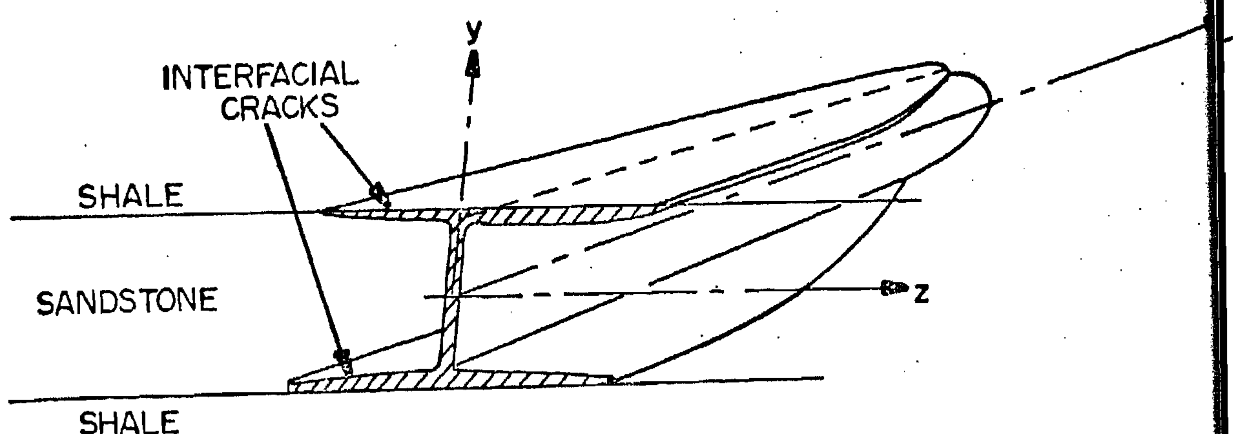

FIGURE 4.

(Actually, the width (in the z-direction) of the interfacial cracks may be shown too wide as the hydraulic fracturing pressure is presumably considerably less than the vertical compressive in-situ stress so that the interfacial cracks probably open only near the intersection of the interfaces with the original crack plane -- i.e. the x-y plane.)

PRELIMINARY CONCLUSIONS

If either (a) the critical stress intensity factor for the shale is considerable greater than for the sandstone, or (b) the elastic moduli for the shale are significantly larger than for the sandstone, it appears that the cracks will be confined to the sandstone. In the case (a) the extent of the cracking possible within the sandstone is given by (2). In the case (b) interfacial cracks are to be expected along the intersection of the original crack with the shale-sandstone interfaces.