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GYPSY FIELD PROJECT IN RESERVOIR CHARACTERIZATION

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# **GYPSY FIELD PROJECT IN RESERVOIR CHARACTERIZATION**

## **Objectives**

The overall objective of this project is to use the extensive Gypsy Field laboratory and data set as a focus for developing and testing reservoir characterization methods that are targeted at improved recovery of conventional oil.

The Gypsy Field laboratory consists of coupled outcrop and subsurface sites which have been characterized to a degree of detail not possible in a production operation. Data from these sites entail geological descriptions, core measurements, well logs, vertical seismic surveys, a 3D seismic survey, crosswell seismic surveys, and pressure transient well tests.

The overall project consists of four interdisciplinary sub-projects which are closely interlinked:

1. Modeling depositional environments.
2. Upscaling.
3. Sweep efficiency.
4. Tracer testing.

The first of these aims at improving our ability to model complex depositional environments which trap movable oil. The second entails testing the usefulness of current methods for upscaling from complex geological models to models which are more tractable for standard reservoir simulators. The third investigates the usefulness of numerical techniques for identifying unswept oil through rapid calculation of sweep efficiency in large reservoir models. The fourth explores what can be learned from tracer tests in complex depositional environments, particularly those which are fluvial dominated.

## Summary of Technical Progress

During this quarter, the main activities involved the "Modeling depositional environments" sub-project. In this report we formulate systems based on finite element approximations to parabolic initial value problems modeling the transient pressure behavior. We then study problems to estimate reservoir parameters (permeability) from measurements of the transient pressure. Of particular interest is the differentiability of the estimated reservoir parameters with respect to data. These considerations allow one to analyze the sensitivity of estimates with respect to perturbations of the data. Tools are developed that will be used to assess the inherent resolution supported by the data.

**The finite dimensional problem.** In this section we present the formulation for the parabolic models. A detailed discussion of finite dimensional approximations to elliptic problems is included in [6]. To fix ideas, let  $\Omega$  be an open bounded domain in  $R^n$  with a Lipschitz boundary  $\partial\Omega$ . Let  $H=L^2(\Omega)$  and  $V=H^1(\Omega)$ . Let

$$f \in L^2(0,T;H) \text{ and } a \in Q \subset L^\infty(\Omega).$$

We assume that there is a positive constant  $\nu$  such that

$$a(x) \geq \nu \text{ almost everywhere in } \Omega$$

Consider the initial boundary value problem given by

$$(1) \quad \frac{\partial u}{\partial t} - \nabla \cdot (a \nabla u) = f \text{ in } \Omega \times (0,T)$$

$$(2) \quad \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega,$$

and

$$(3) \quad u(\cdot, 0) = u_0 \in H$$

with  $f \in L^2(0, T; H)$  and  $a \in Q \subset L^\infty(\Omega)$ . For ease we will take  $u_0 = 0$ . It is well known [2] that there exists a unique solution  $u \in L^2(0, T; V)$ . Furthermore, if  $a_n \rightarrow a$  in  $Q$  for  $a_n \geq v$ , then the sequence of associated solutions  $u(a_n)$  converges weakly to  $u(a)$  in  $L^2(0, T; V)$ , [2]. In formulating a regularized output least squares estimation problem, we suppose that  $Q$  is a Hilbert space that imbeds compactly into  $L^\infty(\Omega)$ .

We study systems of initial value problems obtained from finite element approximations [4]. Suppose that  $\{B_i\}_{i=1}^N$  and  $\{b_j\}_{j=1}^M$  are linearly independent functions in  $V$  and  $Q$ , respectively. Express  $u$  and  $a$  as sums and

$$u(t) = \sum_{i=1}^N c_i(t) B_i$$

and

$$a = \sum_{j=1}^M a_j b_j$$

respectively. Given the coefficient  $a$ , we seek  $u = u(a)$  such that

$$(4) \quad \frac{\partial}{\partial t} \int_{\Omega} u(t) B_i dx + \int_{\Omega} a \nabla u(t) \cdot \nabla B_i dx = \int_{\Omega} f(t) B_i dx, \quad t \in (0, T)$$

for  $i = 1, \dots, N$ . Introducing the representation of  $a$  as the above sum and collecting terms, we define component stiffness matrices as the  $N \times N$  matrices  $G^{(k)}$  with entries

$$G_{ij}^{(k)} = \int_{\Omega} b_k \nabla B_i \cdot \nabla B_j dx$$

for  $k = 1, \dots, M$  and

$$G_{0ij} = \int_{\Omega} B_i B_j dx.$$

Define the column  $N$ -vector valued function  $t \rightarrow F(t)$  with entries

$$F(t)_i = \int_{\Omega} f(t) B_i dx$$

for  $i = 1, \dots, N$  and set

$$\mathbf{a} = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_M \end{bmatrix}$$

and

$$\mathbf{c} = c(t) = \begin{bmatrix} c_1(t) \\ \cdot \\ \cdot \\ \cdot \\ c_M(t) \end{bmatrix}$$

We also write  $c = c(\mathbf{a})$  when it is desirable to emphasize the dependence of  $c$  on  $\mathbf{a}$ . The stiffness

matrix is given as the linear combination of the component matrices

$$G = G(\mathbf{a}) = \sum_{k=1}^M a_k G^{(k)}$$

and the semidiscrete version of the initial boundary value problem (4) is thus given by the equation

$$(5)(i) \quad G_0 \frac{d}{dt} c + G(\mathbf{a})c = F$$

with initial condition

$$(5)(ii) \quad c(0) = 0.$$

Setting

$$(6) \quad S(\mathbf{a})(t) = \exp[tG_0^{-1}G(\mathbf{a})]$$

the solution to (5) may be represented by

$$(7) \quad c(t) = \int_0^T S(\tau-t)G_0^{-1}F(\tau)d\tau.$$

Suppose there are given continuous real-valued linear functionals  $\{\Delta_n\}_{n=1}^{N_0}$  on  $V$  and  $\{\Theta\}_{n=1}^{N_1}$  on  $Q$  to serve as observation functionals, [3]. From these functionals we construct the operators

$$C_0 : L^2(0,T;V) \rightarrow Z_0 = L^2(0,T;\mathbf{R}^{N_0})$$

and  $C_1 : Q \rightarrow Z_1 = \mathbf{R}^{N_1}$  as

$$C_0 v(t) = \begin{bmatrix} \langle \Delta_1, v(t) \rangle \\ \cdot \\ \cdot \\ \cdot \\ \langle \Delta_{N_0}, v(t) \rangle \end{bmatrix}$$

and

$$C_1 \psi = \begin{bmatrix} \langle \Theta_1, \psi \rangle \\ \cdot \\ \cdot \\ \cdot \\ \langle \Theta_{N_1}, \psi \rangle \end{bmatrix}$$

respectively. The minimization problem is formulated by introducing a fit-to-data functional

$$J(a) = \int_0^T \sum_{k=1}^{N_0} (\langle \Delta_k, u(t) \rangle - z_k(t))^2 + \sum_{k=1}^{N_1} (\langle \Theta_k, a \rangle - K_k)^2 + \int_{\Omega} [\gamma_2 |D^2 a|^2 + \gamma_1 |\nabla a|^2] dx$$

where  $\gamma_1$ , and  $\gamma_2 \geq 0$ . The functional  $J(a)$  is to be minimized over an admissible set  $Q_{ad} \subset Q$ . For example,  $Q_{ad}$  may be taken to be

$$(8) \quad Q_{ad} = \{a \in H^2(\Omega): a \geq v > 0\}$$

cf. [5].

The finite dimensional formulation of the fit-to-data functional is obtained by introducing the  $N_0 \times N$  matrix  $\Phi$

$$\Phi_{ij} = \langle \Delta_i, B_j \rangle$$

for  $i = 1, \dots, N_0$  and  $j = 1, \dots, N$ , the  $M \times M$  matrix

$$H_{ij} = \int_{\Omega} [\gamma_1 \nabla b_i \cdot \nabla b_j + \gamma_2 D^2 b_i D^2 b_j] dx$$

for  $i, j = 1, \dots, M$ , the  $N_1 \times M$  matrix

$$\Psi_{ij} = \langle \Theta_i, b_j \rangle$$

for  $i = 1, \dots, N_1$  and  $j = 1, \dots, M$ , the  $N_0$  column vector

$$\mathbf{z} = \begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_{N_0} \end{bmatrix},$$

the  $N_1$  column vector

$$\mathbf{K} = \begin{bmatrix} K_1 \\ \cdot \\ \cdot \\ \cdot \\ K_{N_1} \end{bmatrix},$$

Let

$$\Phi_2 = \Phi^* \Phi \text{ and } \Psi_2 = \Psi^* \Psi$$

where \* denotes matrix transposition. The functional  $J(\cdot)$  may thus be viewed as being defined on  $\mathbf{R}^M$  and is expressed as

$$(9) \quad J(\mathbf{a}) = \int_0^T [c^* \Phi_2 c - 2z^* \Phi c + z^* z] dt + \mathbf{a}^* (H + \Psi_2) \mathbf{a} - 2\mathbf{K}^* \Psi \mathbf{a} + \mathbf{K}^* \mathbf{K}$$

where  $\mathbf{a} \in Q_{ad}^M$  and  $Q_{ad}^M$  serves as an appropriate admissible set in  $\mathbf{R}^M$ .

To study the effect of perturbations of the data on interior optimal estimators, our starting point is the system of equations characterizing optimal estimators. Note the Frechet derivative of  $c$  at  $\mathbf{a}$  with increment  $\mathbf{a}$ ,  $Dc(\mathbf{a})\mathbf{a}$ , satisfies the equation

$$(10) \quad G_0 \frac{d}{dt} [Dc(\mathbf{a})\mathbf{a}] + G [Dc(\mathbf{a})\mathbf{a}] = -G(\mathbf{a})c(\mathbf{a}).$$

with initial condition

$$[Dc(\mathbf{a})\mathbf{a}](0) = 0$$

so that

$$[Dc(\mathbf{a})\mathbf{a}](t) = -\int_0^t S(\tau-t) G_0^{-1} G(\mathbf{a})c(\mathbf{a})(\tau) d\tau.$$

Defining the column N-vectors

$$d_0^{(k)}(\mathbf{a})(t) = \int_0^t S(\tau-t) G_0^{-1} G^{(k)}(\mathbf{a})c(\mathbf{a})(\tau) d\tau$$

and the N x M matrix

$$D_0(\mathbf{a})(t) = [d_0^{(1)}(\mathbf{a})(t), \dots, d_0^{(k)}(\mathbf{a})(t)],$$

we may write

$$[Dc(\mathbf{a})\mathbf{a}'](t) = -D_0(\mathbf{a})(t)\mathbf{a}'.$$

The derivative of  $J$  satisfies

$$(11) \quad \frac{1}{2}D\mathcal{J}(\mathbf{a})\mathbf{a}' = \int_0^T (\Phi_2 c(\mathbf{a}) - \Phi' z)' [Dc(\mathbf{a})\mathbf{a}'] dt + ((H - \Psi_2)\mathbf{a} - \Psi' \mathbf{K})' \mathbf{a}'$$

Introducing the vector  $\pi = \pi(\mathbf{a}, z)$  as the solution of the system,

$$(12) \quad -\frac{d}{dt}G_0\pi + G(\mathbf{a})\pi = \Phi_2 c(\mathbf{a}) - \Phi' z,$$

$$\pi(T) = 0,$$

we see that

$$\int_0^T (\Phi_2 c(\mathbf{a}) - \Phi' z)' [Dc(\mathbf{a})\mathbf{a}'] dt = -\int_0^T \pi(\mathbf{a}, z)' G(\mathbf{a}) c(\mathbf{a}) dt$$

holds. The solution of (12) may be represented by the formula

$$\pi(\mathbf{a}, z)(t) = -\int_t^T S(t-\tau)G_0^{-1}(\Phi_2 c(\mathbf{a})(\tau) - \Phi' z(\tau))d\tau.$$

Define the column  $M$ -vector  $X = X(\mathbf{a}, z)$  with entries,

$$(13) \quad X_k(\mathbf{a}, z) = \int_0^T \pi(\mathbf{a}, z)' G^{(k)} c(\mathbf{a}) dt.$$

The derivative of J may now be expressed by the formula

$$(14) \quad \frac{1}{2}DJ(a)a' = [(H + \Psi_2)a - \Psi'K - X]'a'$$

Thus, the optimality conditions satisfied by an interior solution are given by the following, cf [5].

*If a is an interior local minimum for the estimation problem, then a satisfies the optimality system*

$$(15)(i) \quad G_0 \frac{d}{dt}c(a) + G(a)c(a) = F \text{ in } (0, T)$$

$$c(a)(0) = 0$$

$$(15)(ii) \quad -G_0 \frac{d}{dt}\pi(a, z) + G(a)\pi(a, z) = \Phi_2 c(a) - \Phi'z \text{ in } (0, T)$$

$$\pi(a, z)(T) = 0$$

$$(15)(iii) \quad (H + \Psi_2)a - \Psi'K - X(a, z) = 0.$$

where the components of  $X(a, z)$  are given by (13).

The optimality system in (15) establishes a relationship between the data vectors  $z$  and  $K$  and an optimal estimator  $a$ . We next obtain conditions such that the relation given by the optimality conditions of (15) determines a function  $z \mapsto a(z)$  from  $R^{N_0}$  into  $R^M$ . To this end, define the function

$$F : R^M \times Z_0 \times R^{N_1} \rightarrow R^M$$

by

$$(16) \quad F(a, z, K) = (H + \Psi_2)a - \Psi'K - X(a, K)$$

For the time being we are interested only in the dependence of  $\mathbf{a}$  on  $\mathbf{z}$ . Hence, we view  $\mathbf{K}$  as a constant vector and set

$$F(\mathbf{a}, \mathbf{z}) = F(\mathbf{a}, \mathbf{z}, \mathbf{K}).$$

Of course, existence of an interior solution for data  $\mathbf{z}$  implies that the relation

$$(17) \quad F(\mathbf{a}, \mathbf{z}) = 0$$

holds. At a pair  $(\mathbf{a}_0, \mathbf{z}_0)$  for which  $F(\mathbf{a}_0, \mathbf{z}_0) = 0$ , the implicit function theorem asserts that if the Frechet partial derivatives,  $D_{\mathbf{a}}F(\mathbf{a}_0, \mathbf{z}_0)$  and  $D_{\mathbf{z}}F(\mathbf{a}_0, \mathbf{z}_0)$  exist and  $D_{\mathbf{a}}F(\mathbf{a}_0, \mathbf{z}_0)^{-1}$  exists, then  $\mathbf{z} \mapsto \mathbf{a}(\mathbf{z})$  is

determined as a Frechet differentiable function in a neighborhood of  $\mathbf{z}_0$ , [1].

For any  $\mathbf{a}'$ , with  $D_{\mathbf{a}}\pi = D_{\mathbf{a}}\pi(\mathbf{a}, \mathbf{z})$ , and  $Dc = Dc(\mathbf{a})$ ,

$$(18) \quad -G_0 \frac{d}{dt} [(D_{\mathbf{a}}\pi)\mathbf{a}'] + G(\mathbf{a})[(D_{\mathbf{a}}\pi)\mathbf{a}'] = -G(\mathbf{a}')\pi + \Psi_2[(Dc)\mathbf{a}']$$

and initial condition

$$[(D_{\mathbf{a}}\pi)\mathbf{a}'](T) = 0.$$

Defining the  $N \times M$  matrix  $P(\mathbf{a}, \mathbf{z})(t)$  with columns

$$P_k(t) = P_k(\mathbf{a}, \mathbf{z})(t) = \int_t^T S(t-\tau)G_0^{-1}G^{(k)}\pi(\mathbf{a}, \mathbf{z})(\tau)d\tau$$

for  $k = 1, \dots, M$ , and the  $N \times M$  matrix

$$D(\mathbf{a})(t) = \int_t^T S(t-\tau)G_0^{-1}\Psi_2 D_0(\mathbf{a})(\tau)d\tau,$$

In addition, it is easy to see that for  $D_z \pi = D_z \pi(a, z)$

$$[D_a \pi(a, z) a'] (t) = -(P(a, z)(t) + D(a)(t)) a'$$

$$(19) \quad -G_0 \frac{d}{dt} [(D_z \pi) z'] + G(a) [(D_z \pi) z'] = -\Phi \cdot z'$$

$$[(D_z \pi) z'] (T) = 0,$$

and

$$[(D_z \pi) z'] = -\int_t^T S(t-\tau) G_0^{-1} \Phi \cdot z'(\tau) d\tau.$$

It follows from equation (13) that

$$D_a X_k(a, z) a' = \int_0^T \{ [D_a \pi(a, z) a'] G^{(k)} c(a) + \pi(a, z) \cdot G^{(k)} [Dc(a) a'] \} dt$$

and

$$D_z X_k(a, z) z' = \int_0^T [D_z \pi(a, z) z'] \cdot G^{(k)} c(a) dt.$$

Hence, we obtain the expressions

$$D_a X_k(a, z) a' = -\left[ \int_0^T \{ c(a)(t) \cdot G^{(k)} (P(a, z)(t) + D(a)(t)) \right. \\ \left. + \pi(a, z)(t) \cdot G^{(k)} D_0(a)(t) \} dt \right] a'$$

$$D_z X_k(a, z) z' = \int_0^T \left[ \int_0^\tau c(a)(t) \cdot G^{(k)} S(t-\tau) dt \right] G_0^{-1} \Phi' z'(\tau) d\tau$$

Setting

$$X(a, z)(t) = - \begin{bmatrix} \int_0^t c(a)(\tau) \cdot G^{(1)} S(\tau-t) d\tau \\ \vdots \\ \int_0^t c(a)(\tau) \cdot G^{(M)} S(\tau-t) d\tau \end{bmatrix} G_0^{-1} \Phi'$$

we may write

$$(20) \quad D_z F(a, z) z' = \int_0^T X(a, z)(t) z'(t) dt.$$

Furthermore, define the  $M \times N$  matrices  $K_1$  and  $K_2$  in which the  $k$ -th rows are given by

$$K_1(a, z)_k = \int_0^T c(a)(t) \cdot G^{(k)} (P(a, z)(t) + D(a)(t)) dt$$

and

$$K_2(a, z)_k = \int_0^T \pi(a, z)(t) G^{(k)} D_0(a)(t) dt,$$

respectively, and set

$$K = K(a, z) = K_1(a, z) + K_2(a, z).$$

From (16), we see that

$$D_a F(a, z) = H + \Psi_2 + K(a, z),$$

and from the implicit function theorem we have the following.

Suppose that  $F(a_0, z_0) = 0$ . If matrix  $H + \Psi_2 + K$  is invertible, then there is a neighborhood  $N(z_0)$  such that  $z \rightarrow a(z)$  is defined as a function on  $N(z_0)$ , and

$$Da(z)z' = (H + \Psi_2 + K(a, z))^{-1} \int_0^T X(a, z)(t)z'(t)dt.$$

It is also of interest to calculate the second derivatives of  $z \rightarrow a(z)$ . The following is a consequence of a straight forward calculation.

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