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**MASTER**

**FURTHER RESULTS DETERMINING PERMEABILITY AND THICKNESS FOR  
A MULTI-LAYER FIVE SPOT TRACER TEST**

Work Performed for the Department of Energy  
Under Contract DE-AC03-76ET12056

Date Published—February 1981

Stanford University Petroleum Research Institute  
Stanford, California



**U. S. DEPARTMENT OF ENERGY**

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PERMEABILITY AND THICKNESS FOR A MULTI-LAYER  
FIVE SPOT TRACER TEST**

**SUPRI TR-17**

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FURTHER RESULTS DETERMINING  
PERMEABILITY AND THICKNESS FOR A MULTI-LAYER  
FIVE SPOT TRACER TEST

by

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Stanford University

ABSTRACT

This report presents further results obtained using a computer algorithm developed by Dexter Yuen, which gives an indication of the heterogeneity among the layers of a reservoir. Yuen, Brigham and Cinco-Ley presented a match obtained by this program with field data reported by Brigham and Smith.<sup>2</sup>

To find a more accurate fit for these data, the program was modified to allow the selection of up to ten peaks. The results of this more detailed analysis are presented herein.

## I. INTRODUCTION

Yuen's computer algorithm is based upon the dimensionless solution for tracer flow in a single layer for a five-spot pattern developed by Brigham and Smith.<sup>2</sup> Yuen extended this solution through superposition to apply to multi-layer reservoirs. The following assumptions were made:

- (1) The permeability and porosity are constant within each layer
- (2) The dispersion coefficient,  $k$ , is proportional to the fluid velocity
- (3) The mobility ratio is one
- (4) The water saturation is constant in each layer
- (5) There is no crossflow between layers
- (6) The dispersion constant,  $\alpha$ , is equal for all layers
- (7) The distribution of injected fluid entering each layer is proportional to the  $kh$  of each layer
- (8) The distribution of the tracer material entering each layer is proportional to the  $kh$  of each layer.

The following expression was derived for a five-spot pattern:

$$\frac{C}{C_{mp}}(PV) = \int_{.72}^{2.29} 1.302728 \left[ (Q_i - .72)^{-.419} \right] \left[ 10^{-.909913 \{ (Q_i - .72)^{.581} \}} \right] \exp - \left[ \frac{3(Q_i - PV)^2}{\pi^2} \frac{L}{\alpha} \right] dQ_i \quad (1)$$

Yuen did not leave a computer-readable copy of his program, so we had only copies of his listing from which to reenter his original work into the computer. These listings were very difficult to read and this caused several errors while transcribing the program, including an improper indexing within the matrix transposition part of the program. We established that we had successfully transcribed Yuen's original program by rerunning the four peaks he had chosen to match the field data, Figure 1. The chosen peaks were 2264 barrels at 35.1, 2545 barrels at 33.4 ppm. 2768 barrels at 36.9 ppm, and 3092 barrels at 32.2 ppm. These are adjusted values that take into account the amount produced from outside the pattern. (Concentrations were multiplied by 240/205, average producing rate divided by average injection rate; volumes injected were multiplied by 205/240). We then expanded the program to handle ten chosen peaks instead of four as was originally the case with Yuen's program. A current listing of this program is included at the end of this report.

## II. PROCEDURE & RESULTS

Once we had the match for the field data choosing four peaks, we set about to improve the match. As can be seen in Figure 1 the area under the curve for the matched data is less than that for the field data. The early break-through portion, between 2000-2400 barrels produced, has not been incorporated into the match. The first data from the program occurs at 2360 and increases rapidly until the first peak (at 2650 bbls) is reached. Between peaks 1 and 2 (at 2980 bbls) the field data minimum occurs at 2800 barrels at 25 ppm. However, the program's minimum occurs at a much lower concentration, 19.5 ppm, for the same volume produced. Another discrepancy is seen between 3000 and 3200 bbls where the field data increases then decreases at a pivot point of 3100 bbls while the generated data decreases until 3040 bbls then increases. The generated data also drops below the field data significantly between 3340-3480 barrels and 3800-4980 barrels. These differences in the data found when comparing the program and the field data cause the area under the curve to be less for the generated data than for the field data. However, Yuen found k and h values which were consistent with Brigham and Smith's findings. We hoped by choosing more peaks we could improve the program's match, thereby approaching the correct value for the area under the curve and more accurate values for k and h.

We began by choosing nine peaks and realized we could not distinguish the effect that any single was having on the peaks around it. Thus,

we added only one peak at a time to help us determine where to choose the peaks to arrive at an optimum match. Figure 2 shows again the original four peaks with a fifth peak added (2410 bbls at 11.275 ppm), and the curve of the raw data we expected the program to generate. We hoped by adding this early time peak we would get a curve beginning at about 2320 barrels but actually overshooting the field data curve slightly between 2400 and 2651 barrels (the second peak). We expected the rest of the curve to be only slightly different from the four peak trial.

The actual results from the program are shown in Figure 3. We observed two things from this run. First, the first and second peaks were far enough apart that instead of experiencing a steady increase through the first peak as we had hoped, the program decreases slightly after reaching the peak and begins increasing again well below the field data curve. Another observation which became apparent to us since we were generating more data points than had Yuen, was that the chosen peak values were not the actual peak values generated. For each peak, excluding the first peak, the generated peak was shifted approximately 20 barrels to the left and .5 ppm upward. This interference effort is similar to that discussed by Yuen in reference 1.

To these five peaks, a sixth peak was added between the second and third peak. The new peak, 2780 barrels at 28.53 ppm, was chosen to improve the low values of the matched data between 2680 and 2980 barrels. Figure 4 shows the expected outcome from the program. We hoped with this peak to move the generated data slightly below the

field data between 2640 and 2980 barrels, and to cause only a slight decrease after the peak at 2980 barrels with an upward swing from 3040 barrels to the next peak ( at 3240 bbls.).

Figure 5 shows the curve generated by the program. We were able to match the field data more closely between 2680 and 2980 barrels. The match is very close from 2660 barrels to the added peak but then the peaks generated by the program were shifted in relation to the inputted peaks such that the curve fell to the left of where we had expected it. The curve between the peaks at 2980 and 3240 had a much lower minimum than we had anticipated (3080 bbls at 26.5).

From these examples it became apparent to us that it would be necessary to use a trial and error method to obtain the best approximation for the field data. We went through several runs and the best fit for the data was obtained choosing ten peaks. This match is shown in Figure 6. The match is very good from 2000 barrels at 0 ppm until the fourth peak, 2620 barrels at 30 ppm. This was accomplished by taking peaks at 1) 2160 barrels at 3.5 ppm, 2) 2340 barrels at 7.75 ppm, 3) 2470 barrels at 17.5 ppm and 4) 2620 barrels at 30 ppm. The program cannot respond accurately when the peaks are close together, but an attempt was made to get the two areas under the curves as equal as possible. Between the fourth and fifth peaks (2760 bbls, at 26.5 ppm) a dip occurs in the field data at 2660 barrels at 27.6 ppm which does not show up in the generated data. After the sixth peak the generated data falls to the left of the field

data but this was necessary since the sixth and seventh peaks (2930 bbls at 28.0 ppm and 3030 bbls at 28.75 ppm) had to be chosen close together and in such a way as to minimize the superpositioning effects of the peaks. Even so, the match between them and the eighth peak (3260 bbls at 31.5 ppm) is not very good. The field data goes up at 3000 to 3040 barrels then down till 3160 barrels then up again till 3200 barrels. The generated data however goes up slightly after the sixth peak until 2980 barrels, decreases slightly till 3080 barrels and then increases above the field data curve until it reaches the eighth peak. The fit from the eighth to the ninth peak (3620 barrels at 27.5 ppm) is fairly close except that the generated curve deviates slightly from the field data curve between 3360 and 3550 barrels (falling below the curve between 3380 and 3480 bbls, then going above ). The last peak was chosen at 3900 barrels at 20.5 bbls. None of the sharp decreases were matched by the generated data. The main two occur at 3800 barrels at 20 ppm where the generated value is 22.5 ppm, and at 4000 barrels at 12 ppm where the generated value is 18.2 ppm.

Table 1 gives the outputed values for the case with four peaks and the case with ten peaks. The peaks were not taken exactly at the same locations, but for comparison the values from the four peak case have been placed in the table next to the closest value of barrels produced for the peak value of the ten peak case. The values on the table are calculated assuming an average permeability of 1500 md, total thickness of 12 feet, and average porosity of

0.26. The total kh of the system ( $\Sigma kh$ ) is 18000 md-ft. The kh of each layer is calculated by:

$$(kh)_i = \frac{(kh)_i}{\Sigma kh} \quad \Sigma kh = \frac{(kh)_i}{\Sigma kh} (18000) \quad (2)$$

Where  $\frac{(kh)_i}{\Sigma kh}$  is calculated by the program. The height of each layer can

be found by:

$$h_i = \frac{(h\phi)_i}{\phi} = \frac{(h\phi)_i}{.26} \quad (3)$$

where  $(h\phi)_i$  is calculated by the program. Permeability is simply found by:

$$k_i = \frac{(kh)_i}{h_i} \quad (4)$$

Several differences are apparent between the two cases. The total height of the layers for the four peak case is 1.511 ft. while that for the ten peak case is 1.736 ft. A greater height was found in the ten peak case because less area under the curve was lost to the minimums generated by the program than in the four peak case. Also since the area under the curve is greater for the ten peak case than for the four peak case, the total  $kh/\Sigma(kh)$  value calculated by the program is larger for the ten peak case (0.3449 compared to 0.2950). Also, since the early breakthrough has been accounted for in the ten peak case but not the four peak case, a higher permeability is observed in the first zone of the ten peak case (5018 md compared to 4050 md.)

It should be mentioned that it is possible for the computer program to produce negative  $kh/\Sigma(kh)$  values. This occurs when two peaks are chosen such that a large minimum is generated between them.

The matrix manipulation routine does not take into account that the negative values are physically unrealistic. This can be helpful however, in determining where not to choose peaks.

### III. CONCLUSIONS

This algorithm accurately describes the shape of tracer data for multi-layer systems by choosing the peak values. Permeability and height can be found for each layer. The algorithm works best when the spread between peaks is great. As the peaks approach one another, interference causes a shift in the generated peak values. This is described by Yuen in more detail.

Further work now in progress at SUPRI under Abbasgadeh and Brigham includes finding solutions for different pattern sizes and different mobilities, as well as, finding a solution which will allow peaks chosen to be close together, but still accurately match the tracer data.

The final match in Figure 6 using ten layers to depict the tracer breakthrough curve is about as accurate as it is possible to match these tracer data. Although slightly different peak locations could have been chosen depending on the judgement of the program user, the results could not differ significantly from Figure 6. Thus, this algorithm works well even though it requires considerable trial and error by the user of the program.

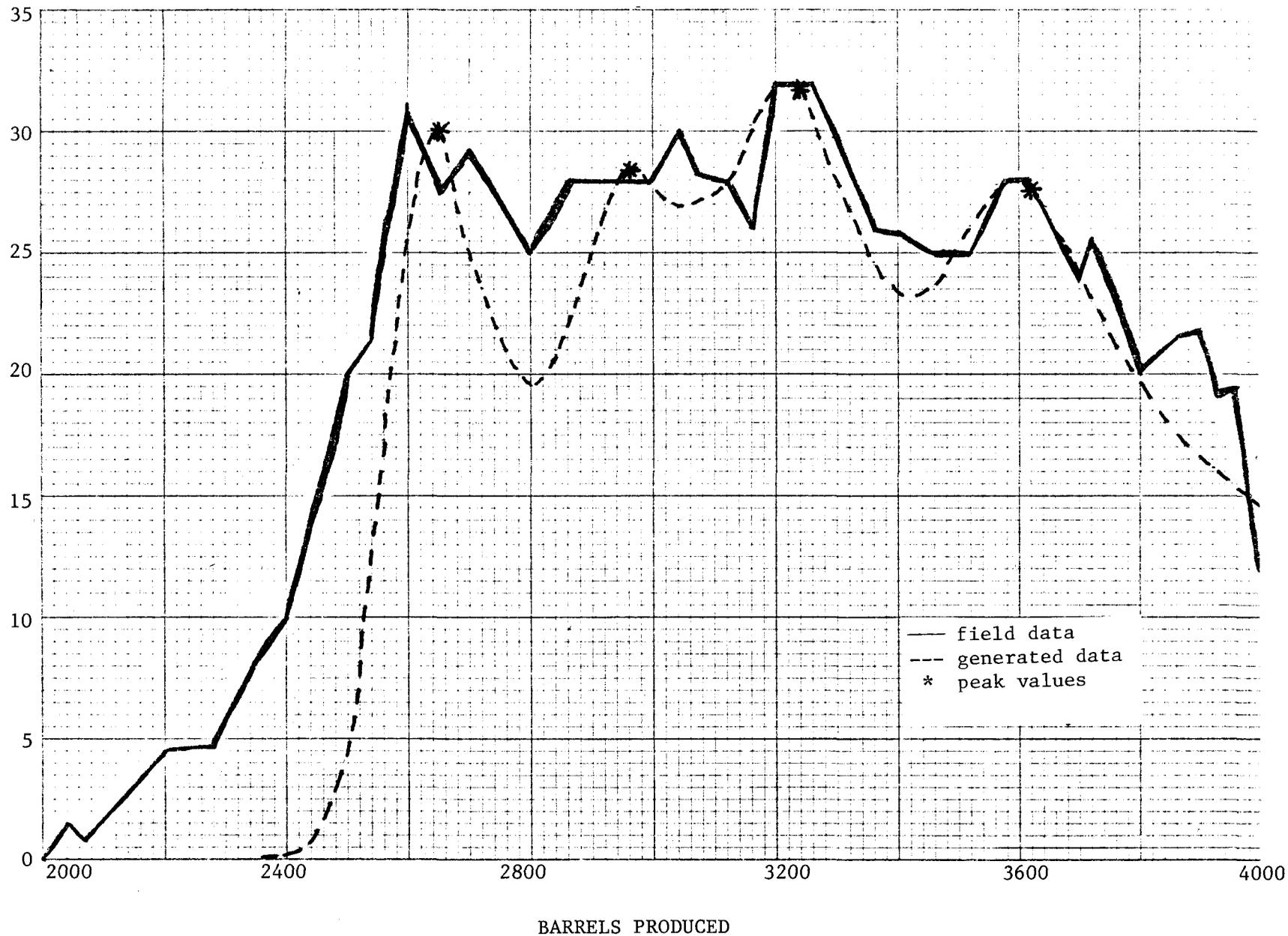


FIGURE 1 ANALYSIS USING FOUR PEAKS

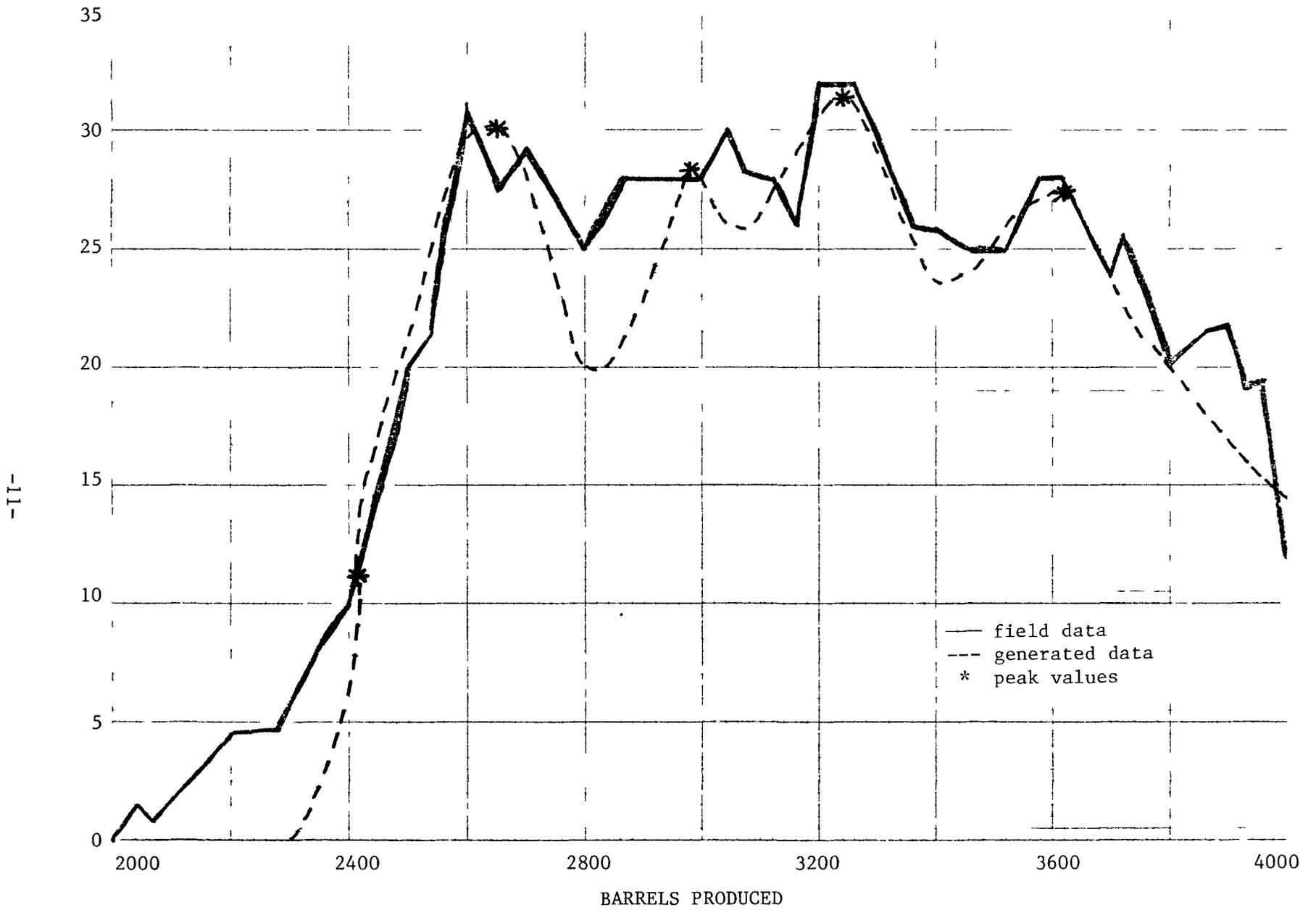


FIGURE 2 ANALYSIS EXPECTED USING FIVE PEAKS

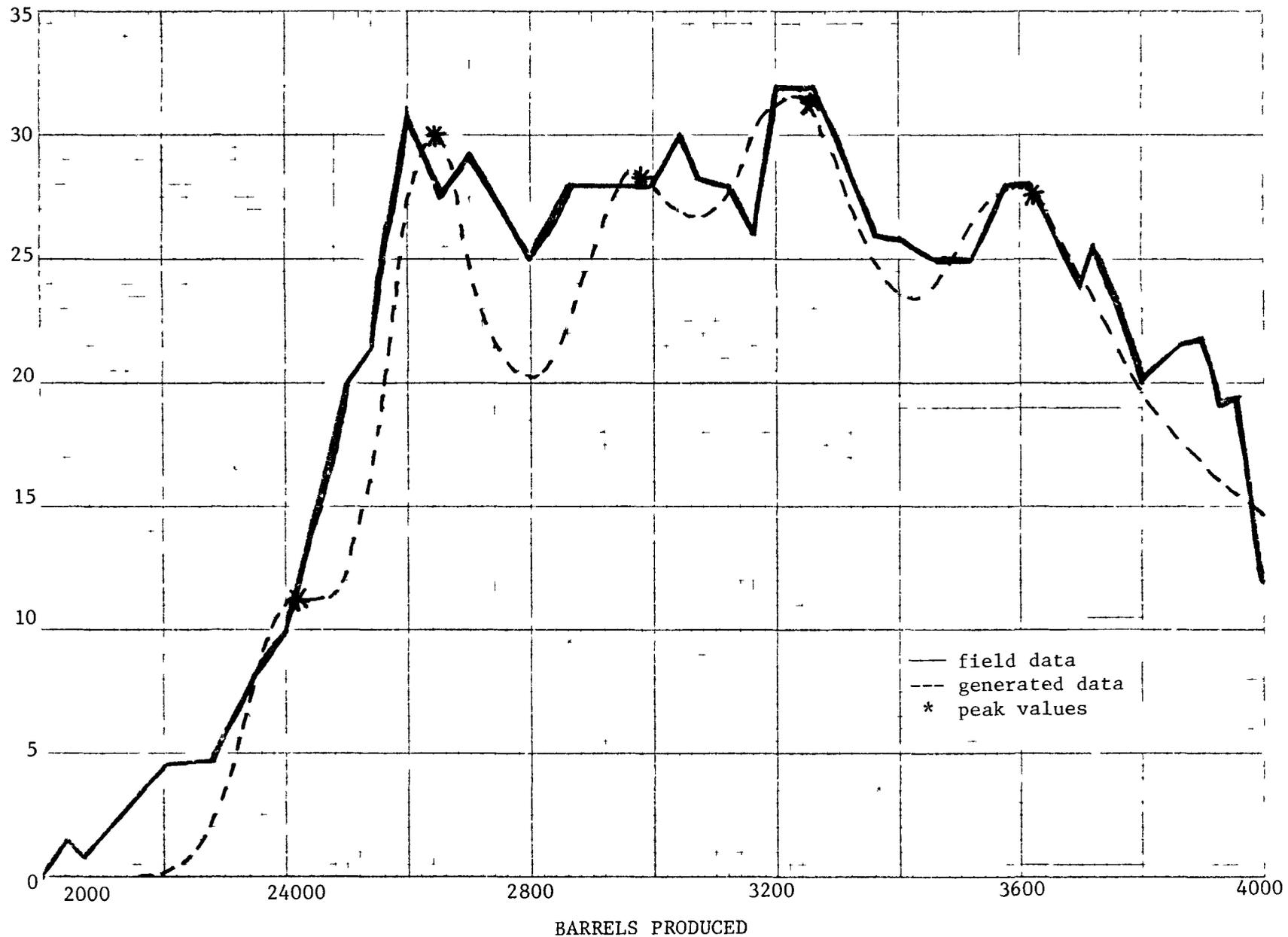


FIGURE 3 ANALYSIS ACHIEVED USING FIVE PEAKS

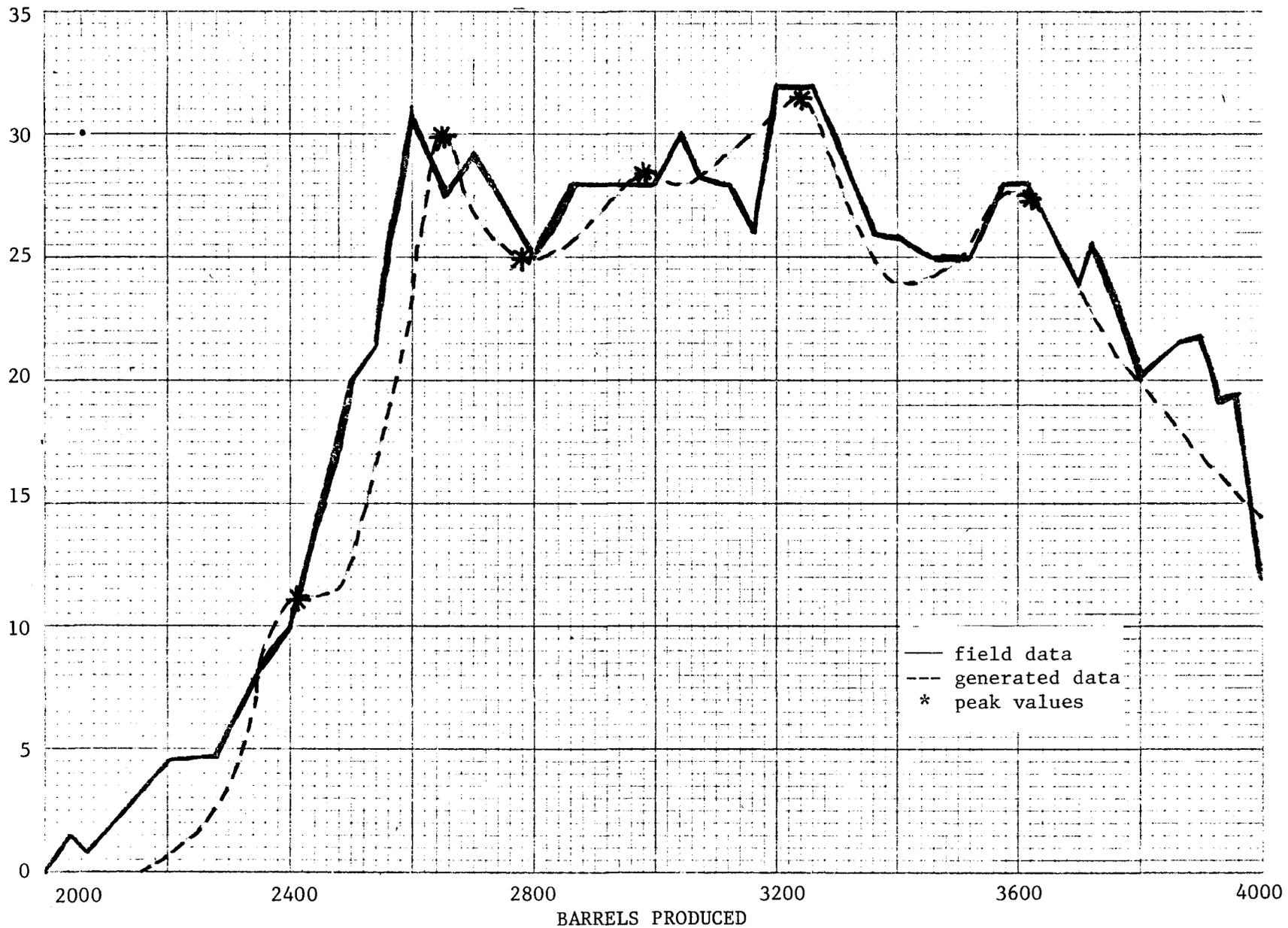


FIGURE 4 ANALYSIS EXPECTED USING SIX PEAKS

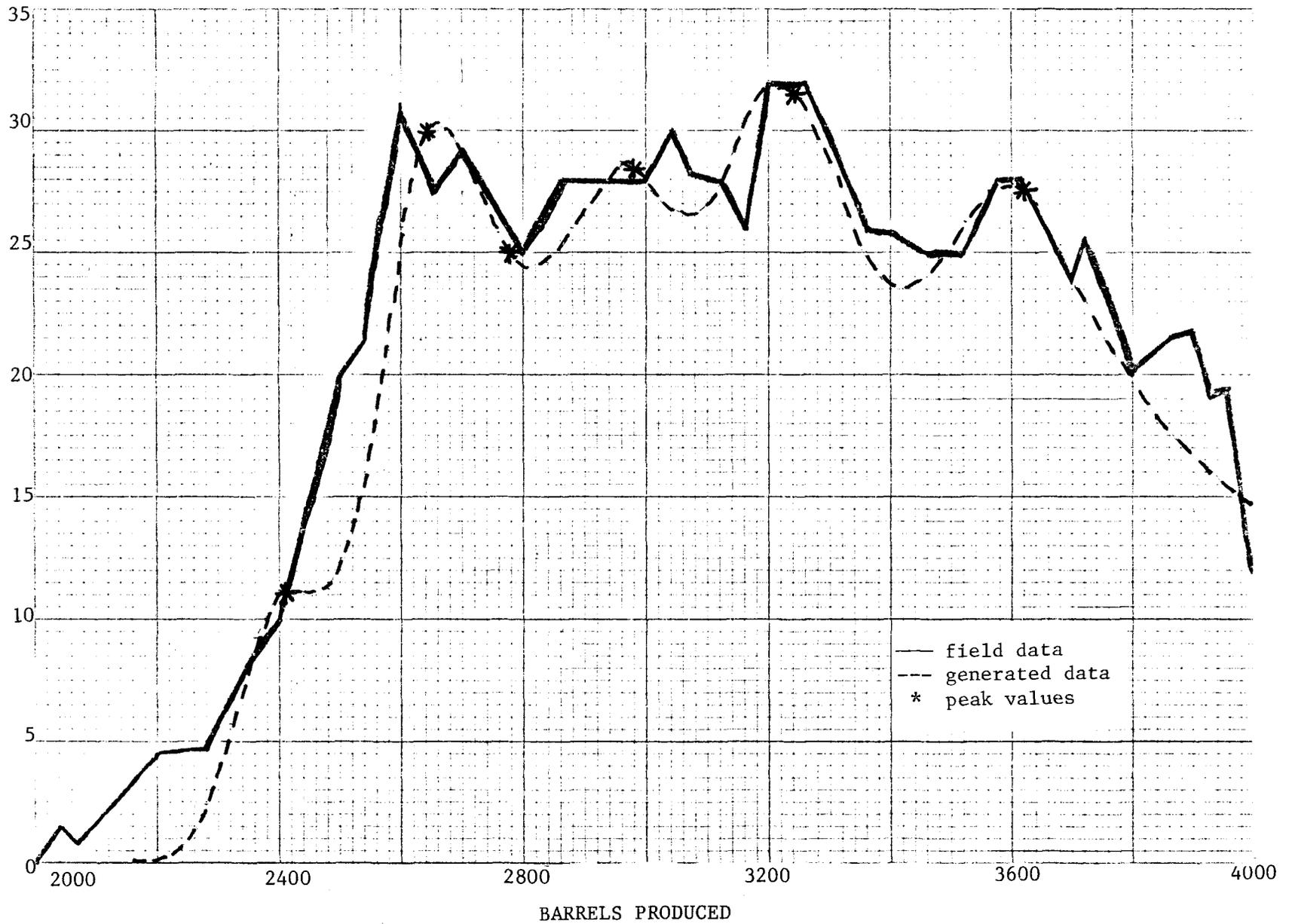


FIGURE 5 ANALYSIS ACHIEVED USING SIX PEAKS

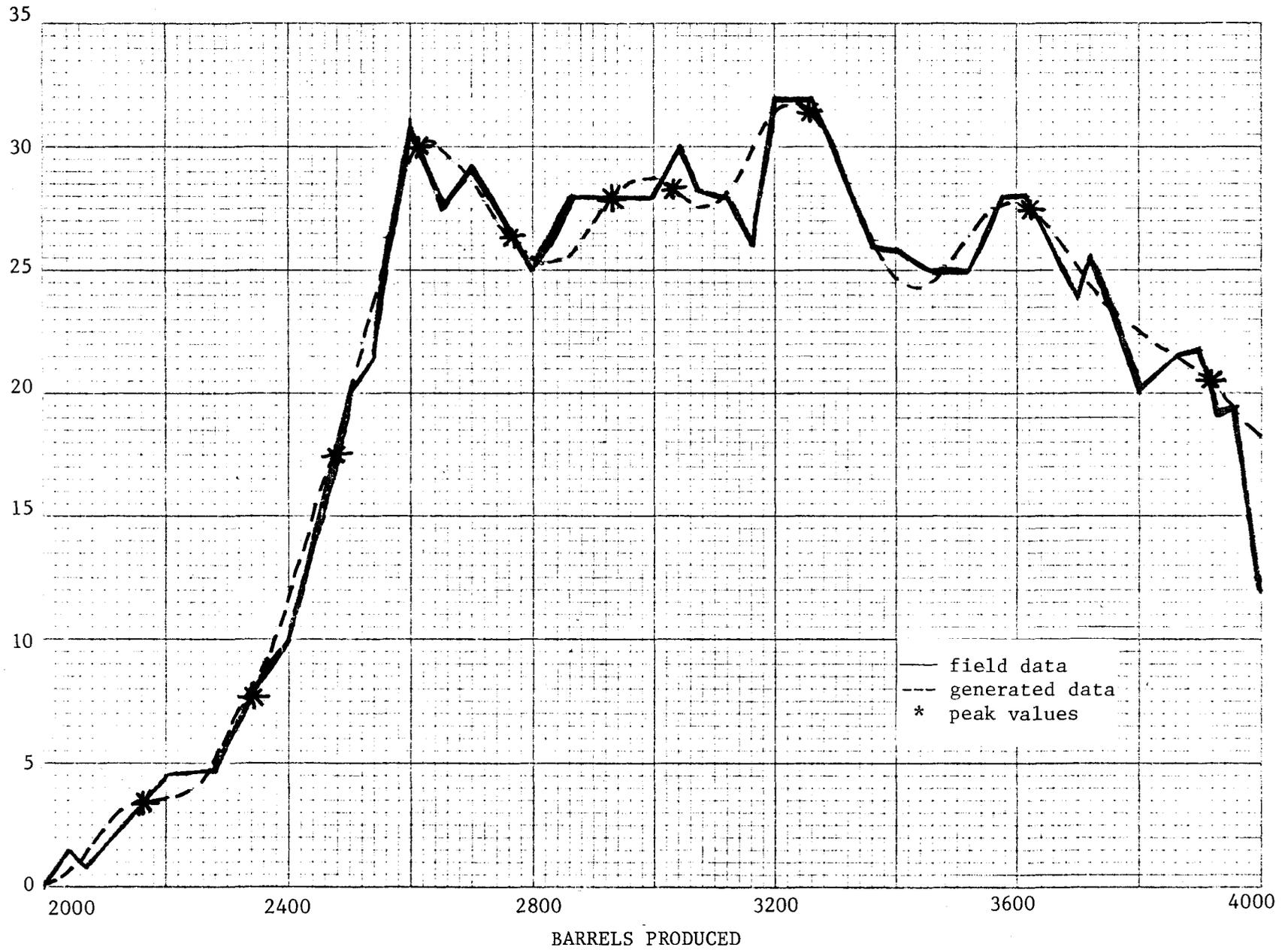


FIGURE 6 ANALYSIS USING TEN PEAKS

TABLE 1

Assuming  $k_{ave} = 1500 \text{ md}$   
 $h_{total} = 12 \text{ ft.}$   
 $\phi_{ave} = 0.26$

<u>Layer</u>	$\frac{kh}{\Sigma kh}$		$h\phi$		$kh$ (md - ft.)		$h$ (ft.)		$k$ (md)	
	<u>10 P</u>	<u>4 P</u>	<u>10 P</u>	<u>4 P</u>	<u>10 P</u>	<u>4 P</u>	<u>10 P</u>	<u>4 P</u>	<u>10 P</u>	<u>4 P</u>
1	0.0092		0.0086		165.6		0.033		5018	
2	0.0106		0.0108		190.8		0.042		4543	
3	0.0336		0.0361		604.8		0.139		4351	
4	0.0657	0.1002	0.0749	0.1157	1182.6	1804	0.288	0.445	4106	4050
5	0.0285		0.0342		513.0		0.132		3886	
6	0.0241	0.0664	0.0307	0.0862	433.8	1195	0.118	0.332	3676	3600
7	0.0373		0.0492		671.4		0.189		3552	
8	0.0636	0.697	0.0902	0.0983	1144.8	1255	0.347	0.378	3299	3320
9	0.0531	0.0587	0.0837	0.0925	955.8	1057	0.322	0.356	2968	2970
10	<u>0.0192</u>	_____	0.0327		345.6		<u>0.126</u>	_____	2743	
	0.3449	0.2950					1.736	1.511		

#### IV. NOMENCLATURE

- $C$  = overall tracer concentration measured in producing well, parts per million
- $C_{mp}$  = the layer's peak tracer concentration in the reservoir in the vicinity of the producing well, parts per million
- $PV$  = displaceable pore volumes of injected fluid
- $L$  = distance from injector to producer, ft.
- $Q_i$  = variable of integration, effective pore volume
- $\alpha$  = dispersion constant, ft.

V. REFERENCES

1. Yuen, Dexter, Brigham, William, and Heber, Cinco-L, Analysis of Five-Spot Tracer Tests To Determine Reservoir Layering, DOE Report February 1979, 60 p.
2. Brigham, W.E., and Smith, D.H. "Prediction of Tracer Behavior in Five-Spot Flow", SPE #1130.

VI. APPENDIX

```

1. //DATANL JOB 'JE.SCB,184,2','SUSAN'
4. // EXEC WATFIV
5. //GO.SYSIN DD *
6. $WATFIV
7. C
8. C THIS PROGRAM ANALZES FIELD-TYPE DATA FROM A FIVE-SPOT TRACER TEST
9. C AND DETERMINES THE DEGREE OF RESERVIOR LAYERING.
10. C
11. C PREPARED BY DEXTER L. YUEN AS PARTIAL FULFILLMENT OF THE
12. C REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN PETROLEUM
13. C ENGINEERING, STANFORD UNIVERSITY
14. C
15. C AUGUST 1978
16. C
17. C FUNDING FOR THIS WORK WAS PROVIDED BY THE STANFORD UNIVERSITY
18. C PETROLEUM RESEARCH INSTITUTE.
19. C
20. C
21. C NOMENCLATURE
22. C ALPH = DISPERSION CONSTANT, FT.
23. C AREA = AREA OF INJECTION PATTERN, SQ.FT.
24. C C = OVERALL TRACER CONCENTRATION MEASURED IN PRODUCING WELL, PPM
25. C CAPAC(L) = H*POROSITY FOR LAYER L, FT.
26. C CNCL(L) = CONCENTRATION OF FLUID FROM LAYER L AS IT ENTERS THE
27. C WELLBORE, PPM
28. C CP(L) = HT. OF L TH CONCENTRATION PEAK IN THE OVERALL CURVE, PPM
29. C C$CMP = DIMENSIONLESS CONCENTRATION RATIO OBTAINED FROM
30. C SOLVING BRIGHAM AND SMITH'S INTEGRAL EQUATION
31. C CP$CMP = PEAK CONCENTRATION RATIO
32. C EL = DISTANCE FROM INJECTOR TO PRODUCER, FT.
33. C ELALPH = EL/ALPH RATIO
34. C M = TOTAL WEIGHT OF TRACER INJECTED, LBS.
35. C N = NUMBER OF LAYERS
36. C PERM(L) = KH/TOTAL KH FOR LAYER L
37. C PV = PORE VOLUMES INJECTED INTO A SPECIFIC LAYER
38. C SW = WATER SATURATION, FRACTION
39. C VMIN. VMAX = THIS SETS THE RANGE AT WHICH THE "OVERALL
40. C CONCENTRATION" VS. "TOTAL INJECTION VOLUME" DATA IS
41. C CALCULATED; 51 VALUES WILL BE GENERATED BETWEEN VMIN AND
42. C VMAX, INCLUSIVE.
43. C V = TOTAL VOLUME INJECTED, BARRELS
44. C ZZ = K/POROSITY*TOTAL KH
44.1 C FAC = FRACTION OF MASS PROD:INJ
45. C
46. C
47. C DOUBLE PRECISION ELALPH, Z(10), ZZ(10), PV, PVP, C$CMP, CP$CMP
48. C DOUBLE PRECISION ERREST, FLAG, Y, M, W,FAC
49. C DOUBLE PRECISION A(10,10), LU(10,10), PERM(10), CAPAC(10)
50. C DIMENSION VP(10), CP(10), IPS(10), CNCL(10)
51. C EXTERNAL FUN
52. C
53. C INPUT PARAMETERS
54. C
54.5 C READ, FAC
55. C READ,EL,ALPH
56. C READ,M,SW,AREA
57. C WRITE (6,100) M,SW,AREA
58. C READ,VMIN,VMAX
59. C READ,N
60. C WRITE (6,200)

```

```

61.          DO 1 L = 1,N
62.          READ,VP(L),CP(L)
63.          WRITE (6,201) L,VP(L),CP(L)
64.          1 CONTINUE
65.          C
66.          C          CALCULATE CONSTANTS
67.          C
68.          IDIM = N
69.          ELALPH = EL/ALPH
70.          WRITE (6,300) EL,ALPH,ELALPH
71.          Y = M/ (.0004*SW*(EL**1.5)*(ALPH**.5))
72.          PVP = .7200D0+.580541D0*(ELALPH**(-.430043D0))
73.          C
74.          C          CALCULATE K/POROSITY*TOTAL KH (=ZZ)
75.          C
76.          DO 2 L = 1,N
77.          Z(L) = PVP/VP(L)
78.          ZZ(L) = Z(L)*AREA*SW/5.615
79.          2 CONTINUE
80.          C
81.          C          SET UP MATRIX ELEMENTS
82.          C
83.          CALL QUANC8(FUN,.7200001D0,2.29D0,1.D-7,1.D-7,CP$CMP,ERREST,
84.          * NOFUN,FLAG,PVP,ELALPH)
85.          DO 5 L = 1,N
86.          DO 4 K = 1,N
87.          IF (L.EQ.K) GOTO 3
88.          PV = VP(L)*Z(K)
89.          CALL QUANC8(FUN,.7200001D0,2.29D0,1.D-7,1.D-7,C$CMP,
90.          * ERREST,NOFUN,FLAG,PV,ELALPH)
91.          A(L,K) = C$CMP*Y*ZZ(K)
92.          GOTO 4
93.          3 A(L,K) = CP$CMP*Y*ZZ(K)
94.          4 CONTINUE
95.          5 CONTINUE
95.4          86 CONTINUE
95.5          85 CONTINUE
96.          C
97.          C          SOLVE MATRIX EQUATION TO GET KH/TOTAL KH (=PERM)
98.          C
99.          CALL DECMP1(N,A,IDIM,LU,IPS,&50,&60)
100.         CALL SOLVE1(N,LU,IDIM,CP,PERM,IPS)
101.         C
102.         C          CALCULATE H*POROSITY (=CAPAC)
103.         C
104.         WRITE (6,400)
105.         DO 6 L = 1,N
106.         CAPAC(L) = PERM(L)/ZZ(L)
107.         WRITE (6,401) L,PERM(L),CAPAC(L)
108.         6 CONTINUE
109.         C
110.         C          GENERATE "OVERALL CONCENTRATION" VS. "TOTAL INJECTION VOLUME"
111.         C          VALUES BETWEEN VMIN AND VMAX
112.         C
113.         WRITE (6,500)
114.         DELTV = (VMAX-VMIN)/50.
115.         DO 11 II = 1,51
116.         W = 0.
117.         V = VMIN+(II-1)*DELTV
118.         DO 10 L = 1,N

```

```

119.      PV = V*Z(L)
120.      CALL QUANC8(FUN, .7200001D0, 2.29D0, 1.D-7, 1.D-7, C$CMP,
121.      * ERREST, NOFUN, FLAG, PV, ELALPH)
122.      W = (C$CMP*ZZ(L)*PERM(L))+W
123.      CNCL(L) = C$CMP*ZZ(L)*Y
124.      10 CONTINUE
125.      C = W*Y
125.1      V=FAC*V
125.2      C=1./FAC*C
126.      WRITE (6,501) V,C,(CNCL(JJ),JJ = 1,5)
127.      11 CONTINUE
128.      STOP
129.      50 WRITE (6,51)
130.      STOP
131.      60 WRITE (6,61)
132.      STOP
133.      51 FORMAT (' ZERO ROW FOUND')
133.5     61 FORMAT (' ZERO PIVOT FOUND')
134.     100 FORMAT (1H1,4X,'AMOUNT OF TRACER (LBS) = ',F7.2,/,5X,'SW = ',
134.1     *      F7.3,/,5X,'AREA (SQ. FT.) = ',F12.2)
134.9     200 FORMAT (//,5X,'LAYER NO.',5X,'LOCATION OF PEAK (BBLS)',5X,
135.     *      'HT. OF PEAK (PPM)')
136.     201 FORMAT (9X,I13,14X,F10.2,16X,F10.2)
137.     300 FORMAT (//,5X,'L(FT) = ',F12.2,7X,'ALPHA(FT) = ',F7.2,7X,
138.     *      'L/ALPHA(FT) = ',F12.2)
139.     400 FORMAT (//,5X,'LAYER NO.',5X,'KH/TOTAL KH',5X,'POROSITY*H(FT)')
140.     401 FORMAT (9X,I3,8X,F10.4,9X,F10.4)
141.     500 FORMAT (//,5X,'INJ. VOL. (BBLS)',5X,'OVERALL TRACER CONC(PPM)',
142.     *      5X,'LAYER 1',5X,'LAYER 2',5X,'LAYER 3',5X,'LAYER 4',
143.     *      5X,'LAYER 5')
144.     501 FORMAT (7X,F10.2,17X,F7.2,14X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2,
145.     *      5X,F7.2)
146.     END
147.     C
148.     C
149.     C      THIS PROGRAM EVALUATES THE INTEGRAL THAT DESCRIBES TRACER
150.     C      CONCENTRATION AT THE PRODUCING WELL AS A FUNCTION OF PORE VOLUMES
151.     C      INJECTED INTO A HOMOGENEOUS RESERVOIR LAYER. A FIVE-SPOT
152.     C      INJECTION PATTERN IS ASSUMED
153.     C
154.     C      QUANC8 WAS OBTAINED FROM THE SOURCE LIBRARY AT THE
155.     C      STANFORD CENTER FOR INFORMATION PROCESSING
156.     C
157.     C      SUBROUTINE QUANC8(FUN,A,B,ABSERR,RELERR,RESULT,ERREST,NOFUN,FLAG,
158.     *      VI,ELALPH)
159.     C
160.     C      DOUBLE PRECISION FUN, A, B, ABSERR, RELERR, RESULT, ERREST, FLAG
161.     C      DOUBLE PRECISION VI,ELALPH
162.     C      INTEGER NOFUN
163.     C
164.     C      ESTIMATE THE INTEGRAL OF FUN(X) FROM A TO B
165.     C      TO A USER PROVIDED TOLERANCE.
166.     C      AN AUTOMATIC ADAPTIVE ROUTINE BASED ON
167.     C      THE 8-PANEL NEWTON-COTES RULE.
168.     C
169.     C      INPUT ..
170.     C
171.     C      FUN THE NAME OF THE INTEGRAD FUNCTION SUBPROGRAM FUN(X).
172.     C      A THE LOWER LIMIT OF INTEGRATION.
173.     C      B THE UPPER LIMIT OF INTEGRATION.(B MAY BE LESS THAN A.)

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174. C RELERR A RELATIVE ERROR TOLERANCE. (SHOULD BE NON-NEGATIVE)
175. C ABSERR AN ABSOLUTE ERROR TOLERANCE. (SHOULD BE NON-NEGATIVE)
176. C
177. C OUTPUT ..
178. C
179. C RESULT AN APPROXIMATION TO THE INTEGRAL HOPEFULLY SATISFYING THE
180. C LEAST STRINGENT OF THE TWO ERROR TOLERANCES.
181. C ERREST AN ESTIMATE OF THE MAGNITUDE OF THE ACTUAL ERROR.
182. C NOFUN THE NUMBER OF FUNCTION VALUES USED IN CALCULATION OF RESULT.
183. C FLAG A RELIABILITY INDICATOR. IF FLAG IS ZERO, THEN RESULT
184. C PROBABLY SATISFIES THE ERROR TOLERANCE. IF FLAG IS
185. C XXX.YYY , THEN XXX = THE NUMBER OF INTERVALS WHICH HAVE
186. C NOT CONVERGED AND 0.YYY = THE FRACTION OF THE INTERVAL
187. C LEFT TO DO WHEN THE LIMIT ON NOFUN WAS APPROACHED.
188. C
189. C DOUBLE PRECISION W0,W1,W2,W3,W4,AREA,X0,F0,STONE,STEP,COR11,TEMP
190. C DOUBLE PRECISION QPREV,QNOW,QDIFF,QLEFT,ESTERR,TOLERR
191. C DOUBLE PRECISION QRIGHT(31),F(16),X(16),FSAVE(8,30),XSAVE(8,30)
192. C DOUBLE PRECISION DABS,DMAX1
193. C INTEGER LEVMIN,LEVMAX,LEVOUT,NOMAX,NOFIN,LEV,NIM,I,J
194. C
195. C *** STATE 1 *** GENERAL INITIALIZATION
196. C SET CONSTANTS.
197. C
198. C LEVMIN = 1
199. C LEVMAX = 30
200. C LEVOUT = 6
201. C NOMAX = 5000
202. C NOFIN = NOMAX - 8*(LEVMAX-LEVOUT+2***(LEVOUT+1))
203. C
204. C CALL TRAPS TO PREVENT INTERRUPTION OF EXECUTION BECAUSE OF
205. C EXPONENT UNDERFLOW (1.2., A NUMBER LESS THAN 10**(-75))
206. C
207. C CALL TRAPS(1,1,2000000000,1,1)
208. C
209. C TROUBLE WHEN NOFUN REACHES NOFIN
210. C
211. C W0 = 3956.0D0 / 14175.0D0
212. C W1 = 23552.0D0 / 14175.0D0
213. C W2 = -3712.0D0 / 14175.0D0
214. C W3 = 41984.0D0 / 14175.0D0
215. C W4 = -18160.0D0 / 14175.0D0
216. C
217. C INITIALIZE RUNNING SUMS TO ZERO.
218. C
219. C FLAG = 0.0D0
220. C RESULT = 0.0D0
221. C COR11 = 0.0D0
222. C ERREST = 0.0D0
223. C AREA = 0.0D0
224. C NOFUN = 0
225. C IF (A .EQ. B) RETURN
226. C
227. C *** STAGE 2 *** INITIALIZATION FOR FIRST INTERVAL
228. C
229. C LEV = 0
230. C NIM = 1
231. C X0 = A
232. C X(16) = B
233. C QPREV = 0.0D0

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234.         FO = FUN(VI,X0,ELALPH)
235.         STONE = (B - A) / 16.0D0
236.         X(8) = (X0 + X(16)) / 2.0D0
237.         X(4) = (X0 + X(8)) / 2.0D0
238.         X(12) = (X(8) + X(16)) / 2.0D0
239.         X(2) = (X0 + X(4)) / 2.0D0
240.         X(6) = (X(4) + X(8)) / 2.0D0
241.         X(10) = (X(8) + X(12)) / 2.0D0
242.         X(14) = (X(12) + X(16)) / 2.0D0
243.         DO 25 J = 2, 16, 2
244.         F(J) = FUN(VI,X(J),ELALPH)
245.     25 CONTINUE
246.         NOFUN = 9
247.     C
248.     C     *** STAGE 3 *** CENTRAL CALCULATION .
249.     C     REQUIRES QPREV,X0,X2,X4,...,X16,F0,F2,F4,...,F16.
250.     C     CALCULATES X1,X3,...X15, F.,F3,...F15,QLEFT,QRIGHT,QNOW,QDIFF,AREA.
251.     C
252.     30 X(1) = (X0 + X(2)) / 2.0D0
253.         F(1) = FUN(VI,X(1),ELALPH)
254.         DO 35 J = 3, 15, 2
255.         X(J) = (X(J-1) + X(J+1)) / 2.0D0
256.         F(J) = FUN(VI,X(J),ELALPH)
257.     35 CONTINUE
258.         NOFUN = NOFUN + 8
259.         STEP = (X(16) - X0) / 16.0D0
260.         QLEFT = (W0*(F0 + F(8)) + W1*(F(1)+F(7)) + W2*(F(2)+F(6))
261.     1 + W3*(F(3)+F(5)) + W4*F(4)) * STEP
262.         QRIGHT(LEV+1)=(W0*(F(8)+F(16))+W1*(F(9)+F(15))+W2*(F(10)+F(14))
263.     1 + W3*(F(11)+F(13)) + W4*F(12)) * STEP
264.         QNOW = QLEFT + QRIGHT(LEV+1)
265.         QDIFF = QNOW - QPREV
266.         AREA = AREA + QDIFF
267.     C
268.     C     *** STAGE 4 *** INTERVAL CONVERGENCE TEST
269.     C
270.         ESTERR = DABS(QDIFF) / 1023.0D0
271.         TOLERR = DMAX1(ABSERR,RELERR*DABS(AREA)) * (STEP/STONE)
272.         IF (LEV .LT. LEVMIN) GO TO 50
273.         IF (LEV .GE. LEVMAX) GO TO 62
274.         IF (NOFUN .GT. NOFIN) GO TO 60
275.         IF (ESTERR .LE. TOLERR) GO TO 70
276.     C
277.     C     *** STAGE 5 *** NO CONVERGENCE
278.     C     LOCATE NEXT INTERVAL.
279.     C
280.     50 NIM = 2*NIM
281.         LEV = LEV+1
282.     C
283.     C     STORE RIGHT HAND ELEMENTS FOR FUTURE USE.
284.     C
285.         DO 52 I = 1, 8
286.         FSAVE(I,LEV) = F(I+8)
287.         XSAVE(I,LEV) = X(I+8)
288.     52 CONTINUE
289.     C
290.     C     ASSEMBLE LEFT HAND ELEMENTS FOR IMMEDIATE USE.
291.     C
292.         QPREV = QLEFT
293.         DO 55 I = 1, 8

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294.      J = -I
295.      F(2*J+18) = (F(J+9))
296.      X(2*J+18) = X(J+9)
297.      55 CONTINUE
298.      GO TO 30
299.      C
300.      C      *** STAGE 6 *** TROUBLE SECTION
301.      C      NUMBER OF FUNCTION VALUES IS ABOUT TO EXCEED LIMIT.
302.      C
303.      60 NOFIN = 2*NOFIN
304.      LEVMAX = LEVOUT
305.      FLAG = FLAG + (B - X0) / (B - A)
306.      GO TO 70
307.      C
308.      C      CURRENT LEVEL IS LEVMAX.
309.      C
310.      62 FLAG = FLAG + 1.0D0
311.      C
312.      C      *** STAGE 7 *** INTERVAL CONVERGED
313.      C      ADD CONTRIBUTIONS INTO RUNNING SUMS.
314.      C
315.      70 RESULT = RESULT + QNOW
316.      ERREST = ERREST + ESTERR
317.      COR11 = COR11 + QDIFF / 1023.0D0
318.      C
319.      C      LOCATE NEXT INTERVAL.
320.      C
321.      72 IF (NIM .EQ. 2*(NIM/2)) GO TO 75
322.      NIM = NIM/2
323.      LEV = LEV-1
324.      GO TO 72
325.      75 NIM = NIM + 1
326.      IF (LEV .LE. 0) GO TO 80
327.      C
328.      C      ASSEMBLE ELEMENTS REQUIRED FOR THE NEXT INTERVAL.
329.      C
330.      QPREV = QRIGHT(LEV)
331.      X0 = X(16)
332.      F0 = F(16)
333.      DO 78 I = 1, 8
334.      F(2*I) = FSAVE(I,LEV)
335.      X(2*I) = XSAVE(I,LEV)
336.      78 CONTINUE
337.      GO TO 30
338.      C
339.      C      *** STAGE 8 *** FINALIZE AND RETURN
340.      C
341.      80 RESULT = RESULT + COR11
342.      C
343.      C      MAKE SURE ERREST NOT LESS THAN ROUND OFF LEVEL.
344.      C
345.      IF (ERREST .EQ. 0.0D0) RETURN
346.      82 TEMP = DABS(RESULT) + ERREST
347.      IF (TEMP .NE. DABS(RESULT)) RETURN
348.      ERREST = 2.0D0*ERREST
349.      GO TO 82
350.      END
351.      C
352.      C      FUN IS THE INTEGRAND
353.      C

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354.      FUNCTION FUN(VI,QI,ELALPH)
355.      DOUBLE PRECISION VI,QI,ELALPH,AA,BB,CC,FUN,EXP
356.      AA = (QI-.72)**(-.419)
357.      BB = 10.**((- .909913)*(QI-.72)**.581)
358.      EXP = (((QI-VI)/3.141592654)**2 )*(-3.)*ELALPH
359.      IF (EXP.LE.(-170.)) GOTO 501
360.      CC = DEXP(EXP)
361.      FUN = 1.302728*AA*BB*CC
362.      GOTO 502
363.      501 FUN = 0.
364.      502 RETURN
365.      END
366.      C
367.      C
368.      C      DECMP1 WAS OBTAINED FROM THE SOURCE LIBRARY AT THE
369.      C      STANFORD CENTER FOR INFORMATION PROCESSING
370.      C
371.      C      LINEAR SYSTEMS (SOBRoutine DECMP1)
372.      C      LIBRARY PROGRAM NUMBER C022
373.      C      JOHN H. WELSCH (SLAC)
374.      C      MARCH 18,1967
375.      C      SUBROUTINE DECMP1(N, A, IDIM, LU, IPS, *, *)
376.      C      INTEGER N, IDIM, IPS(N)
377.      C      DOUBLE PRECISION A(10,10), LU(10,10)
378.      C      DECOMPOSE THE N*N MATRIX A INTO TRIANGULAR L & U SO THAT L*U = A.
379.      C      IPS IS THE ROW PIVOT VECTOR.
380.      C      MATRIX A CAN BE OVERWRITEN BY LU.
381.      C      RETURN 1 FOR ALL ZERO ELEMENTS IN ROW.
382.      C      RETURN 2 FOR ZERO PIVOT.
383.      C      INTEGER I, J, K, IP, KP, KP1, NM1, IDXPIV
384.      C      REAL SCALES(100), EM, BIG, SIZE, PIVOT
385.      C      DOUBLE PRECISION ROWNRM
386.      C      INITIALIZE IPS, LU AND SCALES.
387.      C      DO 5 I = 1,N
388.      C      IPS(I) = I
389.      C      ROWNRM = 0.0D0
390.      C      DO 2 J = 1,N
391.      C      LU(I,J) = A(I,J)
392.      C      ROWNRM = DMAX1(ROWNRM, DABS(LU(I,J)))
393.      C      2 CONTINUE
394.      C      TEST FOR MATRIX WITH ZERO ROW.
395.      C      IF(ROWNRM.EQ.0) RETURN 1
396.      C      SCALES(I) = 1.0/ROWNRM
397.      C      5 CONTINUE
398.      C      GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
399.      C      NM1 = N-1
400.      C      DO 17 K = 1,NM1
401.      C      BIG = 0.0
402.      C      DO 11 I = K,N
403.      C      IP = IPS(I)
404.      C      SIZE = DABS(LU(IP,K))*SCALES(IP)
405.      C      IF(SIZE.LE.BIG) GO TO 11
406.      C      10 BIG = SIZE
407.      C      IDXPIV = I
408.      C      11 CONTINUE
409.      C      TEST FOR ZERO PIVOT.
410.      C      IF(BIG.EQ.0) RETURN 2
411.      C      INTERCHANGE ROW IF NECESSARY.
412.      C      IF(IDXPIV.EQ.K) GO TO 15
413.      C      J = IPS(K)

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414.      IPS(K) = IPS(IDXPIV)
415.      IPS(IDXPIV) = J
416.      15 KP = IPS(K)
417.      PIVOT = LU(KP,K)
418.      KP1 = K+1
419.      DO 16 I = KP1,N
420.      IP = IPS(I)
421.      EM = LU(IP,K)/PIVOT
422.      LU(IP,K) = EM
423.      DO 16 J = KP1,N
424.      LU(IP,J) = LU(IP,J) - EM*LU(KP,J)
425.      C      INNER LOOP. USE MACHINE LANGUAGE CODING IF COMPILER
426.      C      DOES NOT PRODUCE EFFICIENT CODE.
427.      16 CONTINUE
428.      17 CONTINUE
429.      C      TEST FOR ZERO LAST PIVOT.
430.      IF(LU(IPS(N),N).EQ.0) RETURN 2
431.      RETURN
432.      C      LAST CARD OF SUBROUTINE DECMP1
433.      END
434.      C
435.      C
436.      C      SOLVE1 WAS OBTAINED FROM PROFESSOR JOEL H. FERZIGER OF THE
437.      C      DEPARTMENT OF MECHANICAL ENGINEERING DURING THE ME 200A COURSE
438.      C      HE TAUGHT IN FALL QUARTER, 1977
439.      C
440.      SUBROUTINE SOLVE1(N,LU,IDIM,B,X,IPS)
441.      INTEGER N,IDIM,IPS(N)
442.      DOUBLE PRECISION LU(10,10), X(N)
443.      REAL B(N)
444.      C      SOLVE A*X = B USING L*U FROM SUBROUTINE DECMP1
445.      C      IPS IS THE ROW INTERCHANGE VECTOR FROM DECMP1
446.      INTEGER I,J,IP,IP1,IM1,NP1,IBACK
447.      REAL SUM
448.      NP1=N+1
449.      C      FIND U*X = L(-1)*B
450.      X(1)=B(IPS(1))
451.      DO 2 I=2,N
452.      IP=IPS(I)
453.      IM1=I-1
454.      SUM=0.0
455.      DO 1 J=1,IM1
456.      1      SUM=SUM+LU(IP,J)*X(J)
457.      2      X(I)=B(IP)-SUM
458.      X(N)=X(N)/LU(IPS(N),N)
459.      C      FIND X = U(-1)*L(-1)*B
460.      DO 4 IBACK=2,N
461.      I=NP1-IBACK
462.      C      I GOES (N-1).....1
463.      IP=IPS(I)
464.      IP1=I+1
465.      SUM=0.0
466.      DO 3 J=IP1,N
467.      3 SUM=SUM+LU(IP,J)*X(J)
468.      4 X(I)=(X(I)-SUM)/LU(IP,I)
469.      RETURN
470.      C      LAST CARD OF SUBROUTINE SOLVE1
471.      END
472.      $DATA
472.05 1.17073

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472.1	233.3	0.05	
472.2	273	0.55	27225.0
472.3	1708.3	3416.67	
472.4	10		
472.41	1845.0	4.098	
472.45	1998.75	9.07	
472.5	2109.79	20.49	
472.56	2237.92	35.12	
472.6	2357.50	31.02	
472.65	2502.2	32.78	
472.7	2588.125	33.073	
472.8	2784.58	36.88	
472.85	3092.08	32.195	
472.86	3339.79	24.00	
473.	\$STOP		
474.	//		
475.	/*		