High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs

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Principal Investigator: Franklin M. Orr, Jr.
Department of Petroleum Engineering
Stanford University

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Abstract

This report outlines progress in the first quarter of the second year of the DOE project "High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs". The application of the analytical theory for gas injection processes, including the effects of volume change on mixing, has up to now been limited to fully self-sharpening systems, systems where all solution segments that connect the key tie lines present in the displacement are shock fronts. In the following report, we describe the extension of the analytical theory to include systems with rarefactions (continuous composition and saturation variations) between key tie lines. With the completion of this analysis, a completely general procedure has been developed for finding solutions for problems in which a multicomponent gas displaces a multicomponent oil.

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1. Executive Summary

This report presents a new approach for constructing approximate analytical solutions for 1D, multicomponent gas displacement problems including effects of volume change on mixing. The solution to mass conservation equations governing 1D dispersion-free flow in which components partition between two equilibrium phases is controlled by the geometry of key tie lines. It has previously been proven that for systems with an arbitrary number of components, the key tie lines can be approximated quite accurately by a sequence of intersecting tie lines. As a result, analytical solutions can be constructed efficiently for problems with constant initial and injection compositions (known in the mathematical literature as Riemann problems). For fully self-sharpening systems, in which all key tie lines are connected by shocks, the analytical solutions obtained are rigorously accurate, while for systems where some key tie lines are connected by continuous variations in composition and saturation (also known as spreading waves or rarefactions), the analytical solutions are approximations, but accurate ones. Detailed comparison between analytical solutions with both coarse- and fine-grid compositional simulations indicates that even for systems with nontie-line rarefactions, approximate analytical solutions predict composition profiles far more accurately than coarse-grid numerical simulations. Because of the generality of the new approach, approximate analytical solutions can be obtained for any system whose phase behavior can be modeled by an equation of state. The construction of approximate analytical solutions is shown to be orders of magnitude faster than the comparable finite difference compositional simulation. Therefore, the new approach is valuable in areas where fast compositional solutions to Riemann problems are required. One such area of application is the utilization of analytical 1D solutions in streamline/streamtube based compositional simulation.

2. Introduction

A substantial body of mathematical theory now exists for construction of analytical solutions to the dispersion-free 1D flow problem.[1–6] Those investigations considered four-component systems primarily, but special case solutions for systems with more than four components were reported for fully self-sharpening displacements by pure injection gases. The first systematic attempts to describe multicomponent gas/oil systems were restricted to calculation of the minimum miscibility pressure (MMP).[7-9] Those calculations were based on identifications of the key tie lines that control miscibility. Calculation of the full analytical solutions for multicomponent oils and gases was not required to determine the MMP and was not attempted. In the work of Jessen *et al*.[10], the results of previous works was integrated in a new approach allowing automatic generation of 1D solutions for the special case of no volume change on mixing. The restriction of no volume change on mixing was later relaxed for fully self-sharpening systems.[11,12]

In this report, we complete the analytical theory of gas injection processes by combining the work of Ermakov[11] and Jessen[12] with an approach for describing systems with nontie-line rarefactions. The resulting tool allows automatic generation of 1D solutions to multicomponent two-phase flow including effects of volume change on mixing.

3. Mathematical Background

The mass conservation equations for multicomponent, dispersion-free two-phase flow in one dimension can be written as [6]

$$\frac{\partial G_i}{\partial \tau} + \frac{\partial H_i}{\partial \xi} = 0 \quad , \quad i = 1,..,n_c \, , \tag{1}$$

where G_i is the molar concentration of component i

$$G_i = x_i \rho_{xD} (1 - S) + y_i \rho_{yD} S \, , \tag{2}$$

and H_i is the molar flux of component i

$$H_i = u_D \left(x_i \rho_{xD} (1 - f) + y_i \rho_{yD} f \right) . \tag{3}$$

Eqs. (1)-(3) are given in dimensionless form. The dimensionless form is obtained by introducing the variables

$$\tau = \frac{u_{inj} t}{\phi L}, \quad \xi = \frac{z}{L}, \quad \rho_{jD} = \frac{\rho_j}{\rho_{ini}}, \quad u_D = \frac{u}{u_{inj}}, \tag{4}$$

where u_{inj} is the injection velocity, t is the time, ϕ is the porosity and L is the overall length of the porous medium. The distance from the inlet is given by z and the molar density of the initial fluid is denoted ρ_{ini}. The phase equilibrium of the fluids are introduced in the flow equations by the molar density of phase j and the corresponding equilibrium vapor (y_i) and liquid (x_i) compositions of component i. Finally, S is the volumetric vapor phase saturation and f is the fractional flow of vapor related to S through

$$f = \frac{S^n}{S^n + \mu_r (1 - S - S_{or})^n}. \tag{5}$$

In Eq. (5), μ_r is the ratio of vapor to liquid viscosity and the exponent n is a constant depending on the system of interest. In the following examples, the Soave-Redlich-Kwong equation of state is used to calculate phase equilibrium and the Lohrenz-Bray-Clark[13] correlation is used to predict phase viscosities.

4. Solution Construction

Analytical solutions to Eqs. 1 through 5 are constructed by solving the eigenvalue problem associated with the mass conservation equations[1-6]. In composition space, the corresponding problem is to identify the correct (unique) route that connects the initial oil composition and the injection gas composition. The composition route that describes the analytical solution geometrically is subject to the following requirements.

The composition route must have characteristic wave velocities in the two-phase region that increase monotonically from upstream to downstream locations. This condition is known as the velocity rule. If the velocity rule would be violated by a continuous variation (rarefaction), then a shock must be introduced to insure that the solution remains single-valued. The shock must satisfy the integral form of the mass conservation equations.

$$\Lambda = \frac{H_i^u - H_i^d}{G_i^u - G_i^d} \quad , \quad i = 1,..,n_c, \tag{6}$$

where Λ = the shock velocity. Upstream and downstream sides of the shock are denoted u and d respectively. Eq. 6 is known as a Rankine-Hugoniot condition. Any shock present in a solution must satisfy an entropy condition, which requires the shock to be stable in the presence of a small amount of dispersion. In addition, solutions must satisfy a continuity condition with respect to initial and injection data. In other words, small perturbations to the initial or injection compositions must result in small changes in the solution.

Two types of continuous variation are found in the analytical solutions. The first type is a continuous variation along a tie line. This type of rarefaction is easily included into the solution as the key tie lines can be located by the intersection approach[7-9]. In the second type of continuous variation, spreading waves connect neighboring key tie lines. For this type of rarefaction, the eigenvalue problem must be solved for the composition path, where eigenvalues are characteristic wave velocities of a given overall composition subject to composition variation in the eigenvector direction. In the following section we address the eigenvalue problem and demonstrate how to trace nontie-line rarefactions. A method, referred to as the envelope rule[10], is now available for the prediction of nontie-line rarefactions in a given 1D-displacement process. This tool allows us design a general algorithm for automatic generation of analytical solutions including volume change on mixing. The algorithm is:

1. Locate all key tie lines by the tie-line intersection approach.[7-9]
2. Apply the envelope rule for each neighboring pair of key tie lines. If no rarefactions are predicted, switch to the simplified algorithm for fully self-sharpening systems.[11,12]
3. For each predicted rarefaction, locate the equal-eigenvalue point and integrate the eigenvector to obtain the corresponding nontie-line path.
4. Locate the primary key tie line (the shortest tie line) and start the shock construction downstream. Switch points between the nontie-line paths and the tie-line paths are introduced in the solution requirements in parallel with the velocity rule. The downstream solution is traced until the initial oil composition is reached.
5. Continue constructing the upstream solution by the approach of Step 4 until the injection gas composition is reached.

The only remaining question is how to trace the non-tie line paths.

5. Integration of Non-Tie Line Paths

In this section we return to the conservation equations to outline the approach for including non-tie line rarefactions in the analytical solutions. The mass conservation equations can be rewritten as an eigenvalue problem by introducing a self-similarity variable η

$$\eta = \frac{\xi}{\tau} \quad . \tag{7}$$

After variable substitution and rearrangement, the resulting eigenvalue problem takes the form[6]

$$\left(\underline{\underline{A}} - \lambda \underline{\underline{B}}\right)\underline{X} = \underline{0}, \tag{8}$$

where the coefficients of the matrices A and B are evaluated as

$$A_{ji} = \begin{cases} \dfrac{\partial H_j}{\partial Z_i} & , j = 1,..,n_c,\ i = 1,..,n_c - 1, \\ \dfrac{\partial H_j}{\partial u_D} & , j = 1,..,n_c\ , i = n_c, \end{cases} \tag{9}$$

and

$$B_{ji} = \begin{cases} \dfrac{\partial G_j}{\partial Z_i} & , j = 1,..,n_c,\ i = 1,..,n_c - 1, \\ 0 & , j = 1,..,n_c\ , i = n_c. \end{cases} \tag{10}$$

The elements of the eigenvector X are given by

$$\underline{X}^T = \left[\frac{dZ_1}{d\eta}, ..., \frac{dZ_{n_c-1}}{d\eta}, \frac{du_D}{d\eta} \right]. \tag{11}$$

Z_i is the overall composition in mole fractions of component i along the solution path in compositional space. From a numerical point of view, the eigenvalue problem stated in Eqs. (8)-(11) suffers from the fact that one eigenvalue, associated with the total velocity, will be infinite. This is a result of the fact that changes in velocity and density propagating instantaneously throughout the system. Dindoruk[6] developed a method to decouple the total flow velocity u_d and overcome this problem. The decoupling of the total flow velocity facilitates the evaluation of eigenvalues and is the method implemented in the current project.

The decoupling of the total velocity result in the following form of the eigenvalue problem[6]

$$\begin{pmatrix} \underline{\underline{C}} - \lambda^* \underline{\underline{D}} & \underline{0} \\ \underline{E} - \lambda^* \underline{F} & \dfrac{1}{u_d} \end{pmatrix} \underline{X} = \underline{0}, \tag{12}$$

where

$$C_{ji} = \frac{\partial H_j^*}{\partial Z_i} - \frac{H_j^*}{H_{n_c}^*} \frac{\partial H_{n_c}^*}{\partial Z_i}\ ,\ i = 1,..,n_c - 1\ ,\ j = 1,..,n_c - 1, \tag{13}$$

$$D_{ji} = \frac{\partial G_j}{\partial Z_i} - \frac{H_j^*}{H_{n_c}^*} \frac{\partial G_{n_c}}{\partial Z_i}\ ,\ i = 1,..,n_c - 1\ ,\ j = 1,..,n_c - 1\ , \tag{14}$$

with

$$H_i^* = \frac{H_i}{u_d}, \quad \text{and} \quad \lambda^* = \frac{\lambda}{u_d}. \tag{15}$$

The entries of the vectors $\underline{E}$ and $\underline{F}$ are given by

$$E_j = \frac{1}{H^*_{n_c}} \frac{\partial H^*_{n_c}}{\partial Z_j} \ , \ j = 1,..,n_c - 1, \tag{16}$$

and

$$F_j = \frac{1}{H^*_{n_c}} \frac{\partial G_{n_c}}{\partial Z_j} \ , \ j = 1,..,n_c - 1. \tag{17}$$

The decoupled eigenvalue problem stated in Eq. 12 can now be solved in two steps. First step is to evaluate the eigenvalues/eigenvectors from

$$\left(\underline{\underline{C}} - \lambda^* \underline{\underline{D}}\right)\underline{e} = \underline{0} \tag{18}$$

Eq. 18 is solved independently from the total velocity u_D with $\underline{e}$ given by

$$\underline{e}^T = \left[\frac{dZ_1}{d\eta},..,\frac{dZ_{n_c-1}}{d\eta}\right]. \tag{19}$$

The new overall composition along the nontie-line path is obtained by taking a small step δ in the eigenvector direction

$$Z_i^{new} = Z_i^{old} + \delta e_i \ , \ i = 1,..,n_c - 1, \tag{20}$$

and

$$Z_{nc}^{new} = 1 - \sum_{i=1}^{n_c-1} Z_i^{new} \ . \tag{21}$$

Subsequently the change in the total velocity can be evaluated from

$$u_D^{new} = u_D^{old} e^{-\gamma\delta} \ , \tag{22}$$

where the exponent γ is evaluated as

$$\gamma = \left(\underline{E} - \lambda^* \underline{F}\right)\underline{e}. \tag{23}$$

After updating the overall composition and the total flow velocity, the stepwise procedure is repeated for the new overall composition and the nontie-line path is constructed. For details on the decoupling procedure, the reader is referred to Dindoruk (sec.3.4.1,pp. 35)[6.]

In the following, we assume that all key tie lines have been determined by the intersection approach. If a pair of neighboring key tie line is connected by a nontie-line

rarefaction, according to the envelope rule, the integration of the nontie-line path begin on the shortest of the two tie lines. The equal-eigenvalue points of interest (n_c-2) are located by the approach outlined by Dindoruk[6]. The equal-eigenvalue points are points where a tie line path can change to a nontie-line path without violation of the velocity rule. Only one of the equal-eigenvalue points is associated with the appropriate nontie-line path that connects the two key tie lines in question. For regularly S-shaped fractional flow curves, the values of the nontie-line eigenvalues stay ordered on either side of the inflection point (maximum value of df/dS). As the solution for oil-gas systems is restricted to the region of $f>S$., this fact enables us to select the correct equal-eigenvalue point by evaluating the eigenvectors for composition points between two equal-eigenvalue points. Between two equal-eigenvalue points on a key tie line, only one eigenvector will point in the direction of the neighboring key tie line. Hence, the ordering of eigenvalues (eigenvectors) allows selection of the proper equal eigenvalue point from which the nontie-line path integration is started. Once the appropriate equal-eigenvalue point has been determined, stepwise numerical integration of the eigenvectors is used to trace the nontie-line path. In the vicinity of the equal-eigenvalue point, small steps must be used to ensure appropriate accuracy, whereas larger steps can be used away from this point. The typical behavior of a nontie-line path integration is shown in Fig. 1.

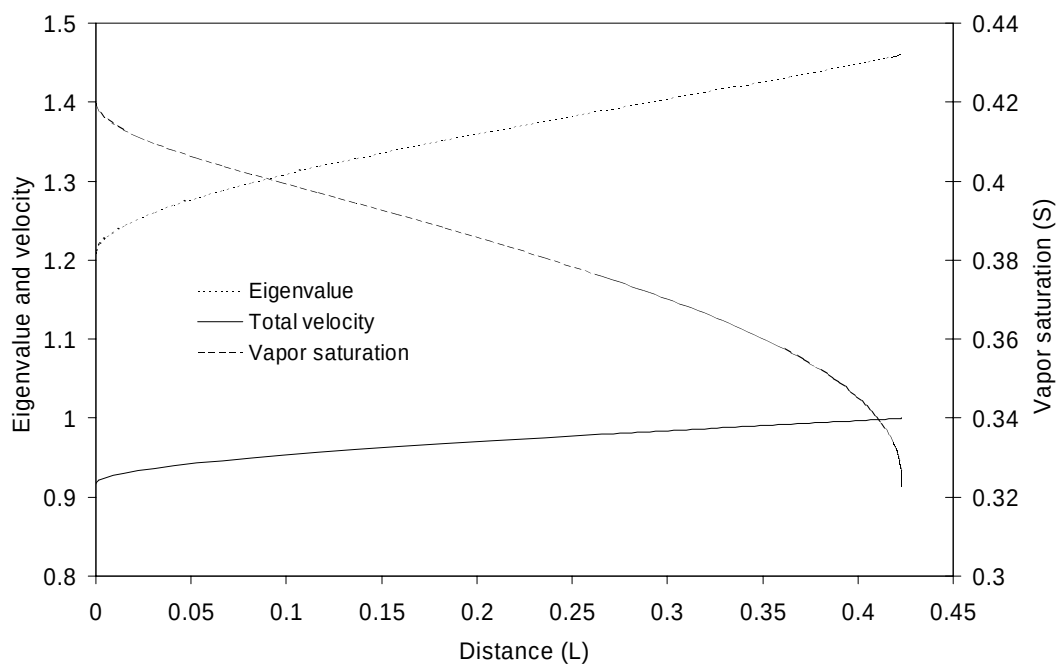

Fig. 1. Variation of wave speed, saturation and total velocity along a non-tie line path.

Fig.1 shows the variation in wave velocity (eigenvalue), vapor saturation and total velocity along a nontie-line path. The location on the nontie-line path is given as the distance L at a given location on the path to the target tie line (neighboring tie line). The distance is defined as

$$L = \sum_{i=1}^{n_c} \left(x_{i,target} - x_{i,current} \right)^2 + \left(y_{i,target} - y_{i,current} \right)^2 \tag{24}$$

In Fig.1, the stepwise integration of the non-tie line path starts from the right hand side of the figure and terminates on the neighboring key tie line at the left hand side. The slope of the eigenvalue vs. distance curve is steeper in the vicinity of the equal eigenvalue point. Hence, using small initial steps minimizes the error introduced by the stepwise integration.

As the total flow velocity varies along nontie-line paths, special care must be taking in including this effect in the analytical solution. For each nontie-line rarefaction, the total flow velocity at the equal-eigenvalue point is assumed to be unity. The total velocity along the nontie-line path can then be rescaled later following the approach outlined in Ermakov[11] and Jessen[12].

6. Example Solutions

In this section we demonstrate some results obtained by the general algorithm for generating analytical solutions including volumetric effects. Two examples are reported.

1. Pure N_2 displacing a mixture of CH_4, C_4 and C_{10} at 344 K and 108.4 atm. In this process the parameter *n* entering the fractional flow function (Eq. 5) is set to 3 and the residual oil saturation S_{or} is set to 0.05.

2. Pure N_2 displacing a multicomponent reservoir fluid (represented by 15 pseudo components) at 387 K and 275 atm. In this process $n = 2$ and $S_{or} = 0.2$.

Input to the thermodynamic model can be found in Refs. 6 and 12 respectively.

6.1 Quaternary System

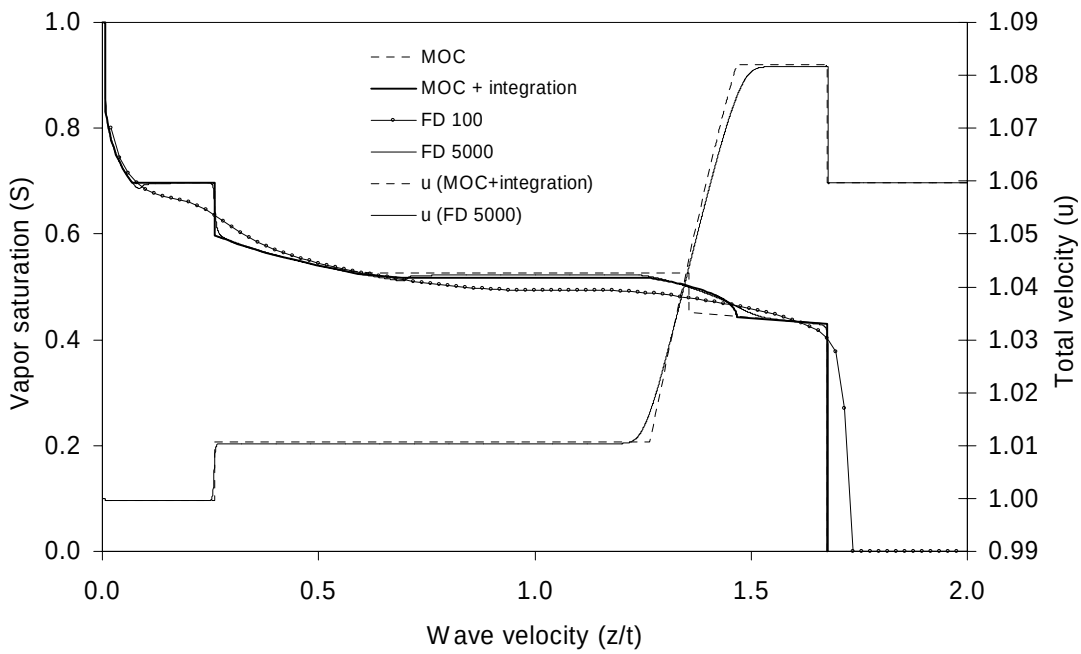

Fig. 2 shows the result of applying the new algorithm to the quaternary system. Four different solutions a reported:

1. the MOC solution, which is obtained by assuming that all key tie lines are connected by shocks (fully self-sharpening system);

2. the MOC solution combined with integration of the nontie-line rarefaction connecting the initial tie line to the first cross over tie line;

3. a coarse grid finite difference (FD) solution obtained by using 100 grid blocks and a $\Delta\zeta/\Delta\tau = 10$ in a one point upstream formulation of the conservation equations; and

4. a fine grid FD solution obtained by using 5000 grid blocks and $\Delta\zeta/\Delta\tau = 10$.

The analytical solution combined with integration of nontie-line paths is in excellent agreement with the fine grid FD simulation. Another important observation is that the fully self-sharpening solution is a better approximation to the exact solution than the coarse grid FD simulation. The coarse grid numerical solution describes the general variation of the dispersion-free solution but fails to capture the details of the shock fronts

and the non-tie line rarefaction. This is due to the strong effects of numerical dispersion in coarse grid numerical simulations.

6.2 Multicomponent System

Next we apply the general analytical approach for the displacement of a real reservoir fluid by pure N_2. The result of this test example is reported in Fig. 3.

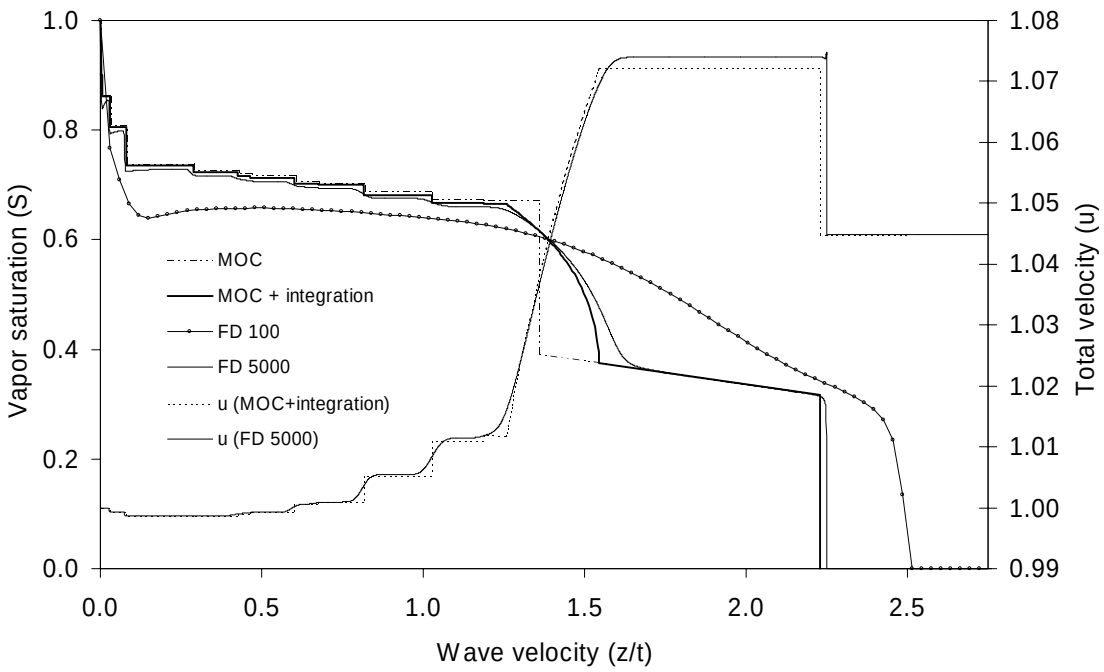

Fig. 3. Comparison of analytical solutions and FD simulations for a 15 component system.

Analytical solutions with and without integration of nontie-line rarefactions are compared with numerical simulations using 100, 1000 and 5000 grid blocks in Fig.3.
Fig. 3 demonstrates again that the fully self-sharpening analytical solution is a far better approximation to the true dispersion-free solution (MOC + integration) than the coarse (100 grid block) grid FD solution. An increasing amount of the detail in the dispersion-free solution is captured as the number of grid blocks is increased. However, the CPU requirement increases rapidly as Table 1 shows.

Example	MOC (shock)	MOC (general)	FD 100	FD 5000
1	<1	<1	4	1506
2	1	7	3	5965

Table 1: Comparison of CPU (sec.) requirement for the MOC and FD approaches. All

7. Discussion and Conclusions

A general approach for automatic generation of dispersion-free solutions for 1D-gas injection processes has been developed and demonstrated. The previous restriction of no volume change as components transfer between phases has been relaxed without adding substantial complexity to the algorithm. The CPU time required for generating the analytical solutions is still small compared to the time required for FD simulations of comparable accuracy.

A comparison of the FD simulator performance for the two test examples demonstrates the fact that a system becomes increasingly sensitive to numerical dispersion as the displacement pressure approaches the minimum miscibility pressure. This behavior strongly suggests that dispersion-free 1D solutions should be used in connection with streamline and streamtube simulators when the transfer of components between phases interacts strongly with two-phase flow to determine displacement efficiency. It is just those situations where the effects of numerical dispersion cause FD simulation results to be misleading.

The examples and analysis presented in this report establish that:

1. The analytical theory of multicomponent gas injection processes has been extended to include systems with nontie-line rarefactions connecting key tie lines.

2. Results of fine grid numerical simulations based on one point upstream weighting are in excellent agreement with the presented algorithm.

3. Coarse grid numerical simulations are strongly affected by numerical dispersion and may lead to a misleading interpretation of the displacement process.

4. The analytical approach is orders of magnitude faster than fine-grid numerical simulations. Coarse grid numerical simulations can be faster than the general analytical approach but at a significant loss of accuracy.

5. The new general approach for generating 1D dispersion-free solutions is highly suitable for combination with streamline simulators, which would allow very fast assessment of displacement performance.

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