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MASSIVE HYDRAULIC FRACTURING: IDENTIFICATION OF CRITICAL TECHNICAL ISSUES FOR APPLICATION IN INCREASING GAS PRODUCTION IN THE WESTERN UNITED STATES

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Abstract

Massive hydraulic fracturing has been shown in a Federal Power Commission study to be at least as effective as nuclear explosives for stimulating the tight gas reservoirs in the Western United States. A survey of the literature and visits to the petroleum stimulation groups of several of the major petroleum companies and petroleum service companies have indicated problem areas in the process where research is required. This research, if pursued, can provide information and understanding which can assist in the application of massive hydraulic fracturing for stimulation of the tight gas reservoirs. A program which addresses the fracturing phenomena, the effects of lithology, the fluid flow in the fracture, the porous flow in the surrounding rock, and interpretation is specified. Theoretical analyses required by the program will be performed by solving the governing continuum equations by utilizing numerical techniques.

Introduction

In 1972, the Federal Power Commission formed a special Natural Gas Technology Task Force which issued a preliminary report on 1 April 1973.[1] That report discusses three major gas deposits in the Rocky Mountain region which cannot be exploited with current techniques, but which could be developed by either nuclear explosive or massive hydraulic fracturing stimulation. The regions considered were: the Green River Basin, Wyoming; the Piceance Basin, Colorado; and the Uinta Basin, Utah. The report compares the nuclear explosive and massive hydraulic fracturing stimulation techniques in an extensive variety of ways, particularly with respect to costs and expected production. It states that while the cost estimates "appear to favor massive hydraulic fracturing, the range of uncertainty is larger than the cost differential between the two methods. Therefore, both methods need to be thoroughly tested and evaluated."

Massive hydraulic fracturing appears to be a viable technique for stimulating tight gas reservoirs to permit economic production of the resource. Predictions indicate that these techniques are at least as effective as nuclear explosives when applied to the tight gas reservoirs. An analysis performed by Holditch and Morse[2] predicts that a pair of vertical fractures 1000 ft long provide production characteristics comparable to a nuclear cavity of 80 ft radius.

Elkins[3] described two massive fracture treatments in very tight pays at depths below 7500 ft, which were successful in production enhancement. Gas production capacity was compared with theoretical analysis and related to the size of each treatment. The theoretical productivity trends were calculated to match the actual production by varying the fracture length and flow capacity with reservoir rock characteristics held constant. By proceeding in this manner, his analysis predicted a range of fracture lengths of up to 2500 ft from the well bore.

The 1973 Natural Gas Task Force report,[1] the analysis of Holditch and Morse,[2] and the actual experience reported by Elkins[3] provide us with the incentive for pursuring research of the hydraulic fracturing process.

In this report, we review the findings of the Natural Gas Technology Task Force, we discuss the current state of the art and problem areas as determined from literature review and discussions with the industry, and we outline research activities that should be undertaken as a part of a proposed LLL Gas Stimulation/ Massive Hydraulic Fracturing Program. The goal of this proposed program is to develop theoretical models which can be used to analyze some of the facets of the process that are not understood. A laboratory program will be closely coupled to the theoretical effort to obtain additional verification of the theory and to improve the models. Both the theoretical and laboratory efforts will be closely coupled to the field programs.

The ultimate goal of this program is to optimize the recovery of natural gas from the tight, low-permeability reservoirs with this technique through the development of well-tested predictive models which can be used to (1) analyze and understand various phenomena associated with hydraulic fracturing; (2) assist in the design and analysis of future field experiments; and (3) help optimize the recovery of gas by this technology.

Section I: Noncommercially Recoverable Gas Reserves

The Natural Gas Technology Task Force determined the location of major gas resources not commercially recoverable with existing technology.[1] Only locations within the "48 states" were considered, and areas remote enough to be considered for stimulation by both nuclear and hydraulic fracturing were sought. For a reservoir to be included in the resource base, it had to meet the following additional criteria: contain at least 100 ft of net pay, at least 15% of the gross productive interval must be pay, and the pay sand must be located at depths between 5,000 and 15,000 ft. It must also be at least 12 square mi in area, and the pay sands must not be interbedded with high-permeability aquifers. The primary reservoir resources established using these criteria are located in the Piceance Basin in western Colorado, the Green River Basin in southwestern Wyoming, and the Uinta Basin in eastern Utah. The location and amount of these reserves is shown in Fig. 1 and Table 1, which are taken from the Task Force's report.

As indicated in Table 1, the Piceance Basin contains about 207 Tcf (trillion cubic feet) of gas-in-place, the Green River Basin about 240 Tcf, and the Uinta Basin about 149 Tcf for a total of about 600 Tcf. If well stimulation processes can effectively be employed, perhaps 40 to 50% of this gas-in-place ultimately can be recovered. Therefore, at a price of \$0.50 per thousand ft^3, the 600 Tcf of gas-in-place would represent a potential value of \$150 billion. The Task Force report indicates that: "there appears to be a potential gas supply of 2 to 5 Tcf/yr that could be developed by the 1990's. This could be sustained, or perhaps increased, for 10 or 20 yr in the 21st century with a long declining production rate thereafter." For comparison, the current consumption rate of natural gas in the U. S. is about 24 Tcf/yr.

The Task Force's report makes the following statements about the information in Table 1. These basins "are judged to be the primary areas in which such reservoirs exist and it is believed that reservoirs in these basins constitute a large fraction of all such reservoirs in the United States. The resource base might be expanded if other intermountain basins of the Rocky Mountains were investigated as thoroughly as were the three primary

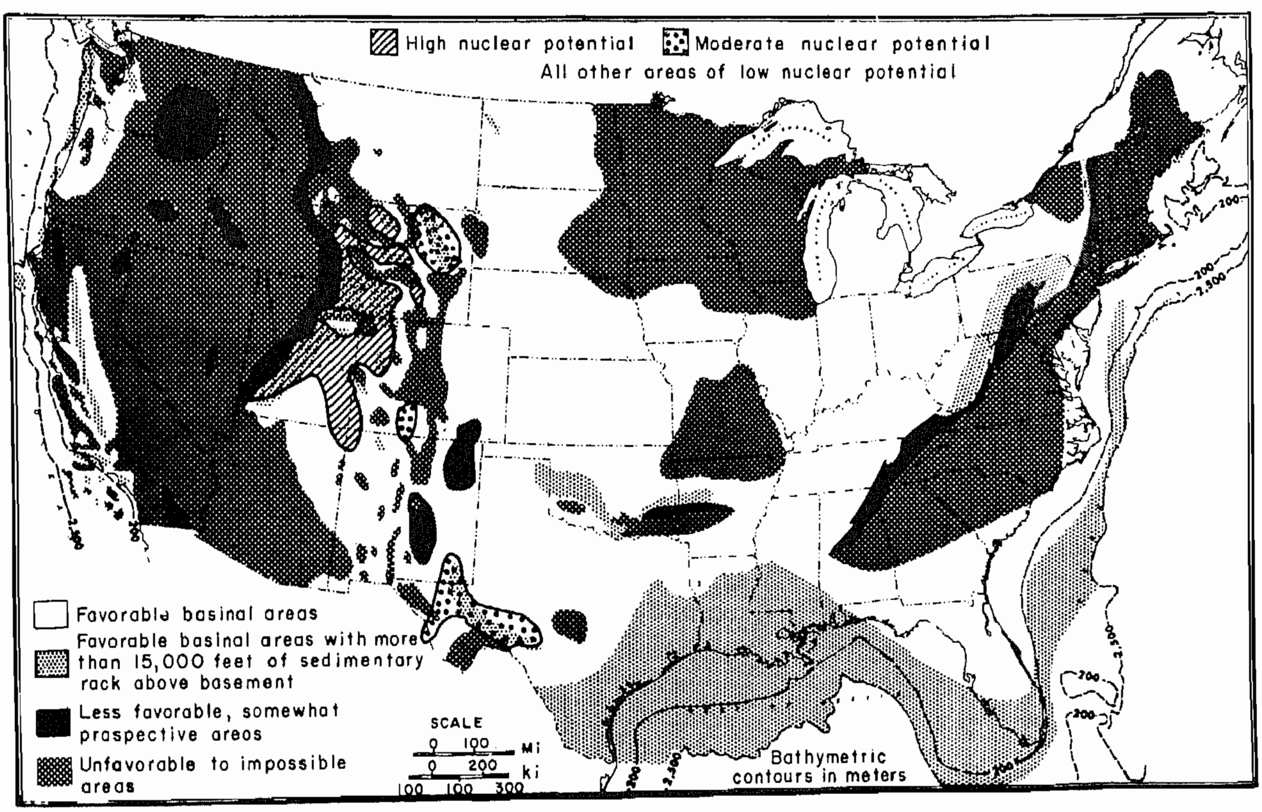

Fig. 1. Map of the United States (excluding Alaska and Hawaii) showing areas favorable for gas and oil exploration. (Taken from FPC Natural Gas Technology Task Force Study.[1])

Table 1. Reservoir characteristics and productive areas. (Taken from Ref. 1.)

Reservoir and location	Productive area, sq. mi.	Depth below surface, ft	Gross interval, ft	Net pay, ft	Porosity	Water saturation	Effective permeability, mD	Initial average BHP, psia	Average BHT, °F	Area, sq. mi.	Gas-in-place: Per sq. mi. Bcf	Gas-in-place: Total Tcf
Piceance Basin, Colorado[a]												
Category 1 (Essentially proved: Based on data from nearby wells)												
Fort Union (North half of basin)	200	5,600-6,200	600	200	0.100	0.50	0.025	2,360	170	—	—	—
Mesaverde I (North half of basin)	300	6,200-7,400	1,200	300	0.095	0.50 }		2,720	187	200	239.6	47.9
Mesaverde II (North half of basin)	300	7,400-8,500	1,100	165	0.095	0.50 }	0.007-0.015	3,180	206	100	192.3	19.2
Mesaverde III (North half of basin)	300	8,500-9,700	1,200	300	0.095	0.50 }		3,640	230			
Mesaverde (south half)	250	6,250-8,750	2,500	625	0.090	0.45	0.020	2,750	204	250	144.4	36.1
Category 2 (Gas in place inferred from geological interpretation)												
Fort Union (North half of basin)	50	same characteristics as category 1	—	—	—	—	—	—	—	100	148.8	14.9
Mesaverde I (North half of basin)	250		—	—	—	—	—	—	—	50	239.6	12.0
Mesaverde II (North half of basin)	150		—	—	—	—	—	—	—	100	192.3	19.2
Mesaverde III (North half of basin)	250		—	—	—	—	—	—	—			
Mesaverde (south half)	400		—	—	—	—	—	—	—	400	144.4	57.8
											Basin total	207.1
Green River Basin, Wyoming												
Category 1												
Fort Union	140	8,000-12,000	4,000	700	0.092	0.54	0.0034	6,820	203	140	264.7	37.1
Category 2												
Fort Union and Mesaverde	300	11,500-13,000	1,500	320	0.090	0.55	0.0034	8,000	225	300	121.5	36.4
		13,000-14,300		0								
		14,300-15,800	1,500	340	0.090	0.55	0.0034	9,800	260		135.8	40.7
Mesaverde	200	9,500-12,000	2,500	600	0.080	0.55	0.015	4,700	200	200	156.4	31.3
Category 3 (Speculative)												
Fort Union and Mesaverde	500	9,000-12,000	3,000	500	0.092	0.54	0.0034	6,820	203	500	189.1	94.5
											Basin total	240.0
Uinta Basin, Utah												
Category 1												
Mesaverde	300	8,000-11,000	3,000	1,000	0.10	0.50	0.007-0.015	4,300	200	300	338.8	101.6
Category 2												
Mesaverde	200	8,000-11,000	3,000	700	0.10	0.50	0.007-0.015	4,300	200	200	237.2	47.5
											Basin total	149.1
											Total	596.2

[a]Mesaverde I, II, and III have identical areal configurations in category 1 and line up vertically so that any well will penetrate all three sands. The Fort Union lies above the Mesaverde sands and any well in it can penetrate the three Mesaverde sands. The same situation exists for category 2 with the exception that Mesaverde II has reduced areal extent, but still is within the boundary of I and III. All productive areas in the Green River and Uinta basins are geographically separated. Refer to Ref. 1 for further information.

basins. This would require subsurface information available only by drilling and testing, which is beyond the scope of this study."

A number of areas which are not remote enough to be considered for nuclear fracturing were not included in the reserves shown in Table 1. These additional reserves are located in the Arkoma Basin, Oklahoma, the Ouachita Mountain province, the Western Gulf Basin, Texas, and the Appalachian province. These areas are known to contain noncommercially exploitable gas reservoirs that might be developed by massive hydraulic fracturing. A firm estimate of these reserves has not been made at this time.

Section II: Economic Analysis — Production and Cost Estimates

This section briefly describes the production and cost estimates compiled by the Natural Gas Technology Task Force. Their report[1] should be consulted for a more detailed description of these estimates.

PRODUCTION ESTIMATES

Predictions of well productivity following stimulation are particularly dependent on the three reservoir characteristics: drainage area of the sand lenses, effective net pay thickness, and permeability. The Task Force's report presents geometrical models for the sand lenses which are based on currently available geologic information. Both radial and lenticular models are developed for the three major basins. Effective net pay thicknesses estimated for the three basins are: Piceance, 765 ft; Uinta, 1000 ft; and Green River, 700 ft. Permeabilities, which were arrived at to derive high and low production rates and recovery estimates for the three basins, are given in Table 2.

Reservoir performance calculations, in current use by companies represented by various Task Force members, were used to determine production and recovery estimates. Table 3 gives estimates for all three basins of the cumulative recovery of gas-in-place (GIP) for 25 yr for wells stimulated by massive hydraulic fracturing. These estimates were made by assuming 80-acre spacing of the wells and fracture treatments which produced 500-ft-long fractures.

EFFECT OF WELL SPACING AND FRACTURE LENGTH ON RECOVERY

Increasing the well spacing (decreasing the number of wells) normally decreases the recovery. The 80-acre spacing for hydraulic fracturing was chosen since it offers efficient development of these low-permeability lenticular sands. For example, if the well spacing is 160 acres,

Table 2. Range of permeabilities.[1]

	Permeability, mD	
Basin	High	Low
Piceance	0.015	0.007
Uinta	0.015	0.007
Green River	0.0034	0.0034

Table 3. Recovery of gas reserves through massive hydraulic fracturing, 25-yr cumulative production.[1]

Basin	Gas-in-place Bscf/sq mi[b]	25-yr recovery[a] Low production Bscf/sq mi[b]	Low production % of GIP	High production Bscf/sq mi[b]	High production % of GIP
Piceance	192	43	22	72	38
Uinta	339	78	23	127	37
Green River	265	73	28	73	28

[a]With 8 wells per section (80-acre spacing).

[b]Bscf — billions of standard cubic feet per square mile.

the 25-yr cumulative production in the Green River Basin is 44.5 Bscf/mi^2 or 16.9% of the GIP. This compares with 28% of the GIP that could be produced under the same conditions if wells were drilled on 80-acre spacing. On the other hand, gas reservoirs with higher permeabilities are usually efficiently developed on 320- or 640-acre spacing.

It is also possible to increase the potential recovery by increasing the fracture length. For example, if wells at 160-acre spacing in the Green River Basin were fractured with a treatment that creates a 1000-ft-long fracture, the 25-yr cumulative production would be approximately 122 Bscf/sec as compared to 93 Bscf/sec for wells stimulated with a 500-ft-long fracture.

POTENTIAL MARKETABLE SUPPLY

Potential marketable supply projections were made by the Task Force for each of the three basins. Two cases, one using high production and the other low production, were developed for each basin. The results of the high production projections are probably optimistic both as to marketable supply and wellhead gas costs. The results of the low production projections probably are conservative, and marketable gas supply could exceed the projections.

These projections are compared in Table 4. The marketable gas supply projections shown in Table 4 are

Table 4. Summary and comparison of projected stimulation results, composite of three basins (assuming 80-acre spacing for both high and low production).[1]

	High production	Low production
Wells/year	480	240
Year	**Production rate/year, Tcf**	
1980	2.178	0.667
1990	4.634	1.541
2000	5.10	1.811
2025	3.33	1.85
2050	0.14	1.48
Cumulative status		
By 2000		
Production, Tcf	93.5	31.2
Wells completed	11,180	5,590
By 2025		
Production, Tcf	201.2	77.3
Wells completed	20,940	11,350
By 2050		
Production, Tcf	231.9	121.8
Wells completed	22,720	16,970

principally directed to the year 2000. Employing the high development rates and optimistic reservoir assumptions, annual marketable supply would peak at about 5-7 Tcf/yr around 2000 to 2010, then steadily decline for another 50 yr.

INVESTMENT AND OPERATING EXPENSES

The investment costs for various cases are presented in Table 5. The various investment amounts required reflect the difficulty of drilling in the different basins, the depth required, the well diameter, and the cost of stimulation and well completion.

The hydraulic fracturing treatment costs are based on the estimated amount of fluid required for the desired length of fracture and height of producing interval. All fractures are assumed to extend out 500 ft from the wellbore each way in opposite directions, and it was assumed that 80% of the net sands could be fractured with a staged treatment covering 50% of the gross interval. A cost of \$0.50 per gallon of total fluid injected was used to approximate the total cost of treatment. This can vary but represents a good average covering horsepower, fluids, propping agents, and other service company charges.

The investment expense factor, 0.8, represents that portion of the investment which is expensed rather than capitalized. The dry hole allowance shown in Table 5 assumes that one well in ten is abandoned due to serious difficulties in drilling and that one well in ten finds insufficient gas to warrant nuclear or hydraulic stimulation.

CALCULATION OF WELLHEAD COST OF GAS

The Task Force developed a mathematical code to calculate the wellhead cost of gas. Costs, taxes, well development schedules, and anticipated production are input to the code for a single well. The costs were escalated at rates suggested by the National Gas Survey Coordinating Committee, namely, 4.0% per year through 1975, 3.5% per year 1976 through 1980, and 3.0% per year after 1980. The code uses a discounted cash flow method and balances the present value of cash outflows against net revenues from the sale of gas by a convergence process.

Included in the calculation are an ad valorem tax rate of 5.5%, a depletion

Table 5. Investment costs per well for hydraulic fracturing (in thousands of 1972 dollars).[1]

	Basin		
	Piceance	Green River	Uinta
Investment	1,200	1,725	1,300
Investment expense factor	0.8	0.8	0.8
Dry hole allowance	70	180	90
Workovers (years 10 and 20)	75	75	75
Operation and maintenance (per year)	3	3	3
Administration and general (per year)	1.5	1.5	1.5

rate of 22%, and a royalty rate of 12.5%. It is assumed that the producer has enough other revenues to offset losses in the first years of development and thus effect a tax savings. Details of this rather sophisticated calculation are given in Ref. 1.

The wellhead cost of gas was calculated for a 24-yr period for the high and low production cases for each basin. Discounted cash flow (DCF) rates of return of 10, 15, and 20% were considered. The results of these calculations are shown in Table 6.

Even though the range of wellhead cost of gas is wide, these costs lie between current area rates for conventionally produced gas and currently published cost estimates for gas from unconventional sources, such as, coal gasification, gas from the Arctic, LNG, and reformer gas. The present uncertainty on reservoir characteristics in these basins makes it apparent that initial development of these areas is a high-risk investment compared to investment in developed technologies (such as reformer gas production from naptha feed stock). Consequently, higher expected DCF rates of return than the range used here will probably be required to attract investment in the early stages of development of these low-permeability gas reservoirs. The costs quoted do not include the cost of surface facilities nor credits for liquids produced.

Table 6. Escalated[a] wellhead cost (cents per Mcf).[1]

	Massive hydraulic fracturing				
Basin	Piceance		Green River	Uinta	
Production case	Low	High	Both	Low	High
10% rate of return					
1980	62	33	54	37	20
1990	84	45	73	50	27
2000	113	61	98	67	37
Leveled[b]	64	35	56	38	21
15% rate of return					
1972[c]	54	29	47	32	17
1980	72	38	63	43	23
1985	84	44	73	50	27
1990	97	51	85	58	31
2000	131	69	114	78	42
Leveled[b]	75	39	65	45	24
20% rate of return					
1980	82	42	72	49	26
1990	110	57	97	66	35
2000	148	77	130	89	47
Leveled[b]	84	44	74	51	27

[a]Escalated at 4% through 1975, 3.5% through 1980, and 3% thereafter.

[b]Level at which technological advances are assumed to balance inflation. The year shown is 1981 for massive hydraulic fracturing and 1982, 1984, and 1987 for nuclear stimulation in the Piceance, Green River, and Uinta Basins respectively.

[c]Entries on this line (only) represent costs in 1972 constant dollars. All others have inflated costs built in as explained in footnote a.

A significant parameter of primary concern for hydraulic fracturing is the length of the fracture. The Task Force report shows the relationship between wellhead cost and fracture length for hydraulic fracturing, which is given in Fig. 2. The investment amount used for a well completed with 80% of the net pay

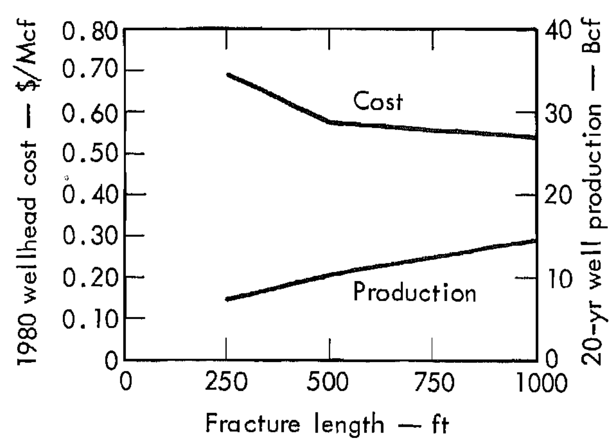

Fig. 2. Sample of effect of fracture length on cost and production, for massive hydraulic fracturing in Green River Basin.[1]

sand fractured a distance of 500 ft was $1,750,000 in Green River. A 15% DCF rate of return was arbitrarily chosen and a 160-acre well spacing was assumed. While this relationship may not hold for all cases and reservoirs, it does suggest a 500-ft fracture as an appropriate length for this study.

It is emphasized that the statistics and analyses given in Secs. I and II were performed by the Natural Gas Technology Task Force and were included here for background to our study.

Section III: Current Status and Problem Areas

Hydraulic fracturing has been the subject of analysis since its inception as a petroleum well cleanup and stimulation technique. Theoretical techniques have been developed so that most designs have a theoretical basis. Although the method has been the subject of extensive analysis, there still remains much that is not known. A primary reason is that the fracturing usually takes place deep within the earth. Thus, measurement and observation of the process is difficult if not impossible. There is little visual evidence of what happens during the fracturing process although some pictures have been made of the fractures which intersect the well bore. Some shallow hydraulic fracturing experiments have been performed and later excavated to determine the fracture geometries. It is uncertain whether the information obtained from these shallow experiments can be extrapolated to greater depths.

To create the massive fractures needed to economically produce gas from the tight sands, a better understanding of the fracturing process in the reservoir is required. A knowledge of the current theoretical techniques and their limitations is required so that a relevant program can be developed. To this end, a study of the literature was made and discussions were held with the production research groups in some of the major petroleum companies and service companies in the industry. The results of the literature study and of our many discussions are outlined below.

CURRENT STATE OF THE ART

The hydraulic fracturing method for petroleum well stimulation has been a major development in petroleum well treatment and completion. The technique was introduced to the petroleum industry by Clark[4] in 1949 and since that time has been expanded so that it is a standard well completion method.

The procedures for computing the geometry of hydraulic fractures have been discussed and improved by many people. The list of those who have contributed is long, but some major contributors are Howard and Fast,[5] Perkins and Kern,[6] Christianovich and Zheltov,[7] Haimson and Fairhurst,[8] Geertsma and deKlerk,[9] Williams,[10] Nordgren,[11] and Daneshy.[12] Howard and Fast[13] attempted to summarize the state of the art in a monograph published in 1970.

When hydraulically fracturing an oil or gas well, the liquid pressure in the borehole is increased until the tensile stress created in the surrounding rock exceeds the tensile strength of the rock. Fluid from the borehole penetrates the created tensile fracture, and fracture extension under hydraulic action takes place. The fracturing liquid carries a propping agent to hold the fracture open and ensure a highly permeable flow channel after pressure release. Field results from these stimulations range from failure to outstanding successes.

For several years, the industry has been widely applying analytical techniques to predict the results of the stimulation process. These techniques have been somewhat successful in predicting desirable designs. The techniques assume steady state flow conditions and do not take into account the dynamic effects present during the stimulation process. The application of massive hydraulic fracturing to create massive fractures that extend several thousand feet from the well bore could benefit from an analysis where the dynamic effects, the effects of layering and joints, and in situ stresses are considered.

Current design analyses for hydraulic fractures generally consider three processes which occur simultaneously: the opening of the fracture, the motion and pressurization of the fluid, and the porous flow losses due to leak off into the surrounding medium. The penetrating fracturing fluids establish a pressure distribution inside the fracture, and fracture mechanics determine the shape and orientation of the resulting fracture. The computation of the hydraulic fracture's geometry consists of analytically matching (simultaneously solving) the fluid and fracture mechanics of the process.

To analytically solve the problem of computing the fracture width and extent, a number of simplifying assumptions have to be made (Geertsma and deKlerk[9]). These are: (1) the formation is assumed homogeneous and isotropic with regard to the fracture propagation process; (2) the formation is assumed to be linearly elastic during the fracturing process; (3) the fracturing fluid behaves like a purely viscous liquid and any non-Newtonian flow behavior (due for instance to the addition of gelling agents or other additives) is neglected; (4) fluid flow everywhere in the fracture is assumed to be laminar; and (5) simple geometric fracture extension patterns are assumed – either radial symmetric propagation from a point source or rectilinear propagation originating from a line source. The periphery of the fracture is circular in the first case and in the second case it is rectangular.

The analyses described above rely on previously developed static fracture descriptions. For example, some normally used formulations to describe the geometry of a fracture are based either on the Sneddon[14] description or on the equations obtained by Christianovich and Zheltov.[7] Daneshy[12] discusses the merits of both descriptions.

Hubbert and Willis[15] were among the first to show that hydraulically induced fractures should be formed approximately perpendicular to the least principal stress. Although they did not use their analyses to interpret well fracturing data, they concluded that the state of stress is, in general, not hydrostatic but depends on tectonic conditions. They developed an expression for interpreting the tectonic stress for a nonpenetrating injection fluid using the minimum fracture initiation pressure. Kehle[16] provides a fairly sophisticated analysis of the cylindrical region around a well bore bounded vertically by packers. He obtains essentially the same forms as the previous authors for interpreting the tectonic stress conditions in terms of the fracture initiation pressures. Gretener[17] agrees but concludes that porous flow characteristics and pore pressure effects are also important to the fracturing process. The correctness of these proposals have been verified experimentally by Haimson and Fairhurst.[18] Daneshy[19] has performed several experiments in the laboratory and determines that for both cased and uncased holes the <u>in situ</u> stress field controls the orientation of the produced fractures. In another experimental study, Daneshy[20] found that for the principal stresses inclined with respect to the borehole the created fractures tended to change their orientation after propagation away from the borehole. The new fracture orientation was inclined perpendicular to the least principal stress.

Any tensile fracture introduced into a moderately brittle rock forms normal to the smallest principal stress, σ_3, and contains the intermediate and maximum principal stresses, σ_2 and σ_1, respectively. The relevant equation for a relatively impermeable rock is:

$$P_f = \sigma_T + 3\sigma_3 - \sigma_1 - P_p, \qquad (1)$$

where P_f is the critical formation breakdown pressure, σ_T is the tensile strength of the rock, and P_p is the pore fluid pressure in the formation. The critical assumptions made in the derivation and application of this equation are that the rock is linearly elastic and mechanically isotropic. It is probable that the nearly impermeable gas sands where massive hydraulic fracturing is proposed may be neither linearly elastic or mechanically isotropic.

Most of the fracturing analyses are based on Hubbert and Willis failure criterion that fracturing will occur on a plane perpendicular to the least principal stress when this stress becomes tensile. The dynamic propagation of the fractures has not been analyzed and the dynamic flow characteristics of the fracturing fluid in the fracture have not been considered.

A major assumption in the previous analyses is related to the height of the fracture. This assumption can greatly affect the final fracture volumes and hence the extent. Most analyses assume that the fracture is effectively stopped by some shale layer thickness. Some important questions which need to be addressed in this area are: Do shale or other layers effectively isolate the fracture so that its height can be determined from geologic cross-sections? If so, what thickness of shale layers is required to provide this function? What changes in mechanical properties from the gas sands to the shale impede the fracture propagation?

Daneshy[12] states that of the assumptions made for computing fracture geometry, those concerned with fracture height are the most inaccurate and deserve careful attention and further analysis. The variations in fracture height can influence the fracture width and length to a great extent.

The flow of fracturing fluid within a fracture is normally calculated by some approximation. For example, Perkins and Kern[6] propose the Fanning friction pressure drop equation:

$$\frac{dP}{dx} = \frac{2fv^2\rho}{D_e} \quad , \tag{2}$$

where f is the friction parameter, v is velocity, ρ is density, and D_e is the hydraulic equivalent diameter. In the laminar flow region, the friction factor is a linear function of the Reynolds number:

$$f = \frac{16\mu}{D_e v p} \, , \tag{3}$$

where μ is the viscosity of the fracturing fluid. Using Eqs. (3) and (4), we find

$$\frac{dP}{dx} = \frac{1}{k} v \, . \tag{4}$$

This is essentially a Darcian approximation to fluid flow in a pipe. Daneshy[12] and Geertsma and deKlerk[9] propose a form of Poiseuille's equation to calculate the pressure drop along the fracture:

$$\Delta p(\xi) = \frac{12\mu Q}{h} \int_{xw}^{x} \frac{dx}{w^3} \, , \tag{5}$$

where $\Delta p(\xi)$ is the pressure drop from the wellbore x_w to any point along the fracture x, μ is the viscosity, Q is the assumed constant injection rate, h is the fracture height, and w is the width of the fractures. In both flow descriptions, the pressure profile along the fracture is either implied as having some distinct shape or defined as a function of the coordinate along the cracks. Time variation of momentum is also ignored. Both the Fanning and the Poiseuille equations were initially developed to calculate the pressure drop for steady pipe flows.

The fluid flow from the fracture into the neighboring reservoir rock results in a loss of the fluid available to create fracture volume and results in a buildup of the pore pressure fluids in the surrounding rock. Thus, the formation expands locally causing some change to the fracture geometry. However, this effect is small. In addition, the pore fluid pressure can reduce the effective compressive stress supported by the rock matrix and decrease the effective stress required to extend the fracture. The basic equations governing the loss of fluid into the surrounding media have been derived by Carter and reported by Howard and Fast.[5]

From this literature review, it is evident that analytical techniques have been widely used to analyze the hydraulic fracturing process for design purposes. These analyses have been primarily based on static equilibrium fracture descriptions, on steady flow approximations for the flow within the fracture where variations in the width of the fracture are ignored, and on steady porous flow of fluids out of the fracture. These techniques ignore the time-dependent nonlinear interactions of various physical processes present in hydraulic fracturing and the interactions of the fractures with the <u>in situ</u> stress field.

PROBLEM AREAS DEFINED BY DISCUSSION WITH INDUSTRY

To make the research relevant to current needs in the field, we contacted the petroleum stimulation groups of some of the major petroleum and petroleum service companies. These discussions with both research and application groups resulted in the identification of a list of problem areas which must be addressed to advance the state of the art on the whole. A list of the groups visited is given in Appendix A. The list of problems to be solved and types of research needed, which were identified through discussions with industry, are given below. The research activities proposed by this laboratory follow in Sec. IV.

Fracture Geometry Considerations

Almost everyone visited indicated that this was an important research area. The uncertainty in the fracture height considerations results in the major uncertainty in the analysis and design for fracture length and width. Research needs to be performed to determine the fracture shape and orientation. The question as to whether vertical fractures are limited in vertical extent by shale or other type layers needs to be addressed. Associated with this question is the question of whether it is easier to propagate a fracture in the gas-bearing sandstones than it is in the shale. If the fracture growth is impeded in the shale, the mechanical properties that impede the fracture growth in the shale need to be determined. Can the changes in these mechanical properties be determined by well logging techniques so that the vertical extent of the hydraulic fracture can be predicted by the logs?

a. Static Considerations — How does the fracture change the stress field around the wellbore? A static or quasistatic three-dimensional stress-fracture model needs to be developed to calculate fracturing and the effects of the fracture in the reservoir. The following should be addressed:

- Perform fracture analysis on isotopic elastic media (primarily as a basis for comparison).
- Perform fracture analysis of layered media.
- Analyze regions containing lenses primarily to understand some of the propagation characteristics where the horizontal extent of the reservoir rock is changing.
- Analyze the effects at already existing joints and fractures.
- Determine the effect of statistical inhomogeneities.
- Determine pore pressure effects.
- Determine nonisothermal effects. (Even with the frictional heating of the fluid going down the well base, the fluid can still be much cooler than the reservoir rock.)
- During the relaxation of the sides of the fracture on the proppant after pumping is stopped, at least two mechanisms are thought to occur which allow the fluid trapped in the fracture to get away. These are fluid leakoff into the reservoir and/or further extension of the fracture to displace the fluid. The time required for these processes to occur is important since as soon as the proppant is held by the fracture the back flow or cleanup process can start. The dominant mechanism needs to be determined.
- What is the maximum depth that hydraulic fracturing can be used for stimulation?
- Can some control be exerted on the fracture orientation?

b. Dynamic Considerations — It is evident that hydraulic fractures do not propagate over their total extent at near acoustic velocities. However, the local instantaneous velocity may be high. For example, the fracture may break out in lobes, propagate rapidly for a short distance, and then stop.

- Are dynamic considerations important in determining how fractures are contained between layers?
- Are there pumping techniques which can dynamically enhance certain fracture characteristics?
- Under what conditions, if any, is the dynamic flow of the fracturing fluid important in the understanding of the massive hydraulic fracturing phenomena?

Field Evaluation of Fracture Treatments

Field data which give the size and shape of the fracture are needed both for the verification of fracture geometry predictions and for performance evaluation calculations.

a. Seismic methods, evaluate the passive and active measurement methods. Passive methods include those where an instrument is used to record the signals generated by the disturbance. Active methods are those where both signal generators and listening devices are applied. The generating device generates a signal in the rock and the recording devices record the signal as modified along the path from signal generating device to the recorder. Can the fracturing activity be measured seismically during the process from locations in the observation wells? Are there passive techniques which can give some indication of the fracture geometry; for example, could we conduct seismic transmission surveys to determine if a crack has passed between two observation wells? Are there characteristic signals from the fracturing process which we can measure and interpret?

b. Application of observation wells both during the stimulation process and production. Does measurement of the flow parameters from observation wells indicate something about the fracture orientation and extent?

c. What can well tests and observations from the stimulation well tell us about the fracturing near the stimulation well?

- During stimulation.
- Can we utilize temperature surveys?
- Down hole viewing — both with acoustic and visual techniques.
- What can a well flow test tell us?
- Do well logs indicate parameters and parameteric changes which we can use in design?
- How do pressure buildup tests correlate with created fractures?

d. In situ stress measurements.

Performance

This problem area refers to the use of production data to interpret something about the effects of fractures.

a. Ideally, application of reservoir analysis in three dimensions and time would be desired, but in at least two dimensions the model of the reservoir should include:

- Multiphase flow
- Nonisothermal flow
- Nonadiabatic flow
- Nonhomogeneous reservoirs.

b. Fracture closure due to embedment or crushing of the proppant. Is it possible to interpret fracture closure from some of the performance changes?

c. Proppants (effects of proppants and proppant characteristics on performance and their ability to hold the fracture open). Are there corrosive effects from certain proppants which can cause reduction of the permeability adjacent to the fracture?

d. Reservoir damage from the intruded fluid (capillary unbalance caused by the fracturing fluids).

e. Chemical effects

- Precipitation of salts because of pH changes resulting from the intruding fracturing fluids.
- Phase changes (vapor-fluid effects) as pressures change in the reservoir.

f. Fluid and fluid additive effects (do fluid loss additives plug up proppants).

Pertinent Data

This is data which, when techniques can be developed to make it available, will be required in a complete analysis and design of a massive hydrofracturing job.

a. In situ stress measurement (both local and macro).

b. Rock deformation properties (both reservoir and proppant).

c. Fluid behavior (rheological properties).

d. Fracturing fluid interaction with the reservoir.

- Chemical-acid.
- Effects on clays if present.
- Absorption effects.
- Precipitates.
- Flow in the reservoir.
- Leakoff-erosion of filter cake.

These interaction characteristics may or may not be temperature, rate, and/or pressure dependent.

e. Reservoir properties

- Capillarity (this parameter is normally neglected in the analysis, probably very important in tight gas reservoirs).
- Permeability, K; porosity, ϕ; homogeneity of the reservoir; and saturation of the reservoir.
- Length and shape of the fracture. The K_{eff} and ϕ_{eff} of the individual fractures.

Section IV: Research Activities Required

Although hydraulic fracturing is a standard well completion technique for both gas and petroleum wells, much remains that is unknown about the process. This results primarily because the process takes place deep within the earth and any observations are limited to what can be measured from the borehole. Massive hydraulic fracturing has more uncertainty because of the greater amount of fluids pumped; however, it is being applied in the stimulation of tight gas reservoirs in the Western United States. These massive stimulation jobs, where large amounts of proppant and fluid are displaced down the boreholes and into the subterranean rock, have shown sporadic results. While some applications have been extremely successful, others have been almost total failures. The reasons for the discrepancy in the results have not been established and do not appear to be resolvable within the current state of the art.

To increase the understanding of the process and to facilitate design and interpretation, further research and study are most needed in the fracture geometry area. This includes the study of fluid-driven fractures in a porous, layered, and jointed geologic structure which is under the influence of various in situ stress states. The effects of the stimulation will have to be interpreted both through measurement of the created fracture, if possible, and through an evaluation of the flow characteristics of the reservoir. Because of the complexity of the problems involved, the theoretical analysis of the continuum will have to be pursued by applying numerical techniques.

Based on our discussions with industry, we believe the following research is required. Our well-known capabilities in numerical modeling of continua would be of enormous benefit in this research.

FRACTURE GEOMETRY

Research in this problem area is considered fundamental to advancing the state of the art for the understanding and application of massive hydraulic fracturing. The specific problems outlined on pp. 13-14 will be addressed. Theoretical analyses will be performed by applying numerical techniques for the solution of dynamic equations of the type described in Appendix B. Numerical forms will also be applied for the solution of the equilibrium equations in two and three spatial dimensions. Both dynamic and static analyses of the fracture will be performed in which the dynamic flow of the fracturing fluid in the fracture and in the surrounding reservoir rock is included. The parameters which define the fracturing fluids, such as the viscosity, the effects of the in situ stresses, layering, the joint and flaw patterns of the reservoir, and pore pressure in the reservoir rock, will be included. For the preliminary studies, parametric analysis techniques will be applied to understand the effects of the various parameters and their interactions.

In addition to the theoretical analyses of the fracture geometry, measurement techniques will have to be developed or applied to make in situ stress measurements. Measurement of the size and orientation of the created fracture will also be a requirement and may be the subject of study to determine if a technique can be developed for that purpose. The specific problems to be addressed in this area are outlined primarily on p. 14. In this area, a number of techniques already exist in industry. Our intent in this and in other areas is, when required and where possible, to utilize the techniques developed by industry to obtain the needed information.

Laboratory and small scale experiments will be required for verification of the theory and models. This experimental work will be an integral part of the LLL program. It will provide the various experimental and theoretical comparisons which are essential to obtaining a viable predictive technique. Our rock mechanics facilities will be utilized to obtain laboratory measurements of the mechanical properties of the reservoir rock. Since extensive fluid measurement capabilities exist at a number of the major oil and stimulation companies, we do not propose any research on the fracturing fluids or any measurements of their characteristics.

GEOLOGIC STUDIES AND GEOPHYSICAL DESCRIPTION

Geologic and geophysical studies and interpretation will also be applied to determine the nature of the fractures produced. The questions to be addressed are: (1) are the fractures pervasive or are they localized by preexisting discontinuities, (2) what media properties are most influential in directing fractures (e.g., grain size, preexisting discontinuities such as faults, mineral layering, etc.), given similar external stress fields, and (3) what media and proppant characteristics contribute to the survival of a fracture.

Geophysical studies of actual massive hydraulic fracturing sites will also be applied to determine information as outlined primarily on p. 15. In addition at a particular site, fluid pressure will be measured before the fracturing job for use in both _in situ_ stress determination and porous flow analyses. Anisotropy in the permeability will be measured for porous flow studies. These measurements will also be interpreted to determine if a directional anisotropy can indicate _in situ_ stress anisotropy and fracture direction. Compatibility studies of the reservoir rock will be performed between the fracturing fluid and proppant.

RESERVOIR CHARACTERISTICS

Some reservoir analysis will be performed as a check and comparison of other fracture length predictions and will be compared with data from particular experiments. The specific problems to be addressed in this area are outlined on p. 14 (Performance). In addition, techniques for automating the interpretation of the well test data by comparison of the flow and pressure characteristics with a standard set of characteristics will be developed and studied. Fracture size can be implied from these reservoir analyses; hence, this area of study is related to the field measurements of fracture geometry.

An analysis of proppant transport in horizontal fractures has been performed by Wahl,[21] Lowe and Huitt,[22] and Kern et al.[23] Velocity and viscosity are important fracturing fluid parameters in transporting proppant in a fracture. The use of gelling agents and emulsifiers increases the proppant carrying characteristics of the fluid. Description of the fluids containing these agents will be required in the theoretical models. We hope to obtain these descriptions from the petroleum and stimulation companies. The experience and guidance of these companies will be relied on in these problem areas. However, the literature covering the area of sediment transport will be studied to determine if descriptions developed there can be applied to analyze proppant transport. Yalin[24] has compiled a monograph covering some aspects of sediment transport. Proppant transport is not understood when the proppant particles are moving at different velocities than the flow velocity of the neighboring fluid.

In addition to the proposed research in massive hydraulic fracturing using conventional fluids, some studies will encompass the application of nonideal explosives (slurries) as fracturing fluids. This research will attempt to determine whether the displacement of these non-ideal explosive fluids and their subsequent detonation can result in a greater production enhancement than with conventional fluids. Preliminary analyses of the application of these nonideal explosives will be performed by parametric methods. If these analyses show that recovery of the resource can be enhanced by the application of explosives as fracturing fluids, this area of research will be expanded.

Summary

Current political and economic factors are compelling us to consider every possible method of tapping the extremely tight gas sands of the Western United States. Massive hydraulic fracturing is one technology which could be a significant feature in releasing gas from these sands. At present, this technology is more of an art than a science. The Natural Gas Technology Task Force has prepared an economic analysis addressing the potential of massive hydraulic fracturing in the three major basins in the Western United States — Piceance, Uinta, and Green River. We have made an assessment of the current understanding of the fracturing processes involved with the massive hydraulic fracturing technology. This assessment was based on a literature review as well as on detailed discussions with major production oil company research groups responsible for applying massive hydraulic fracturing. We have summarized the key critical technical issues which industry feels must be resolved to move this technology from an art to a science. Finally, LLL has integrated the economic, industrial, and technical aspects of massive hydraulic fracturing. We have recommended a national program aimed at moving the massive hydraulic fracturing technology to a point where judgments can be made as to whether it may be a viable alternative technology for liberating gas from tight sand structures in the Western United States.

Appendix B

NUMERICAL TECHNIQUES FOR SOLVING DYNAMIC EQUATIONS

The two-dimensional model that will be used to calculate the dynamic response and the dynamic fracturing is based on the numerical techniques developed by Petschek and Hanson.[25] The technique uses a Lagrangian coordinate system to solve the equations of motion in two dimensions:

$$\rho \frac{\partial^2 u}{\partial x^2} = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y},$$

$$\rho \frac{\partial^2 v}{\partial y^2} = \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}, \qquad \text{(B-1)}$$

where ρ is the density, u and v are the displacements in the x and y directions, respectively, σ_x and σ_y are the normal stresses, and τ_{xy} is the shear stress in the x, y frame.

The stresses are related to the strains by Hook's Law:

$$\sigma_x = 2\mu \frac{\partial u}{\partial x} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

$$\sigma_y = 2\mu \frac{\partial v}{\partial y} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

$$\tau_{xy} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \qquad \text{(B-2)}$$

where (λ, μ) are the Lamé constants for the elastic material. The Lagrangian coordinate system is tied into the material and undergoes motion with the material. The mass is discretized at the nodal points of the Lagrangian frame and remains constant throughout the calculation.

The strain energy in the elastic material can be calculated with the integral form,

$$E_v = \frac{1}{2}\int_V (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy}\, \gamma_{xy})\, dV, \qquad \text{(B-3)}$$

where ϵ_x, ϵ_y, and γ_{xy} are the normal strains and the shear strain in the x, y frame. This integration is performed numerically over the domain included in the simulation.

This numerical code permits a time dependent fracture to occur in the grid. In this form, it has been applied in several analyses of earthquake source functions, Hanson and Sanford,[26] Hanson et al.,[27] Hanson et al.,[28] and Hanson et al.[29] Dynamic fracturing processes were analyzed using the techniques and have been reported in Hanson and Sanford,[30] and Hanson et al.[31] Comparison of the results from this model with experiments has shown significant correlation with the experimental results. The application of the current model requires the use of a fracture criterion to calculate fracture initiation and propagation.

Others have tried to apply the analytical fracture techniques outlined by Griffith,[32] which is similar to the theory proposed by Sneddon,[14] to the problems of fractures in compressed materials. In particular, Lajtai[33] found that the stress level for crack initiation was dependent on flaw direction and, thus, has no resemblance to the predictions of the Griffith theory. Hahn,[34] in some of

Appendix A

COMPANIES AND PETROLEUM
STIMULATION GROUPS VISITED

El Paso Natural Gas, Texas
Leo Rogers and Dean Power
May 16, 1974

Amoco Production Research, Tulsa, Oklahoma
Bob Fast and George Holman
May 20, 1974

Dowel, Tulsa, Oklahoma
A. R. Hendrickson, C. L. Wendorff, and Phil Warembourg
May 21, 1974

Mobile, Dallas, Texas
Paul Berry, John Fitch, and Louis Madlin
May 30, 1974

Atlantic Richfield, Plano, Texas
Glenn Martin and Lloyd Kern
May 29, 1974

Shell Development Company, Houston, Texas
Michael Prats, Roger Rolke, and Randy Saucier
June 13, 1974

Esso Production Research Laboratory, Houston, Texas
Bert Williams, Dale Nierode, Don Kehn, Claud Cook, and David Tannic
June 14, 1974

Halliburton Services Research Center, Duncan, Oklahoma
Ed Stahl, Gerald R. Coulter, J. D. Williams, Abbas Ali Daneshy, and David Lord
June 18, 1974

Conoco, Ponca City, Oklahoma
Harry Wahl
June 19, 1974

his experimental work, has indicated that the fracture energies he has measured are three orders of magnitude greater than the energies predicted by the Griffith theory. These works provide evidence that the Griffith description cannot be used to predict fracture in compressed materials or to interpret dynamic fracturing phenomena.

The type model that will be used to calculate the fluid flow in the fracture is similar to the numerical model developed by Crowley.[35] Glenn and Crowley[36] and Crowley et al.[37] have used the technique to calculate the phenomena in high-explosive-driven and nuclear-explosive-driven shock tubes. The code was also applied in the analysis of the containment for an underground nuclear event, Crowley et al.[38]

The one-dimensional code has been linked with another two-dimensional rock mechanics code in a manner similar to the method proposed for this research, Crowley and Barr.[39] These codes were linked through the boundary conditions at the solid-fluid interface. The two-dimensional code which simulated the solid material provides the displacement vs time and position boundary condition and the one-dimensional fluid code provides the pressure vs time and position boundary condition. It is anticipated that a similar linking process will be used between the fluid code and the code described herein.

The technique uses a modified Lagrangian formulation to solve the conservation equation of mass, momentum, and energy. The modified Lagrangian formulation was developed to permit the mass, energy, or momentum losses or additions from the boundaries. The mass is assumed to be concentrated on the one-dimensional zone boundaries and can change according to the relation:

$$\frac{D\rho}{Dt} = \frac{1}{V}\left\{-\rho\,\frac{Dv}{Dt} + \dot{m}\,S\right\}, \qquad \text{(B-4)}$$

where ρ is the local density, V is the volume of the zone, t is the time, and $\dot{m}$ is the local mass change occurring through the external boundary of surface area S.

The equation of motion includes the momentum terms as well as the fluid shearing stress along the boundary of the flow:

$$\frac{Du}{Dt} = \frac{1}{m}\left\{\dot{m}\,S\,(U_\omega - U) - V\,\frac{\partial p}{\partial x} - \tau_\omega S\right\}, \qquad \text{(B-5)}$$

where U is the local velocity of the fluid, U_ω is the velocity of the mass entering the zone, m is the zonal mass, p is the local pressure, τ_ω is the shearing stress along the wall, and x is the flow axis coordinate. Finally, the energy relationship is written to include the energy transported by the mass addition or deletion as well as energy transport from the external boundaries:

$$\frac{De}{Dt} = \frac{1}{m}\left\{\dot{m}\,S\,[(U-U_\omega)^2/2 + (e_\omega - e)] - p\,\frac{DV}{Dt} + \tau_\omega\,S\,|u| + \dot{H}\right\}, \qquad \text{(B-6)}$$

where e is the specific internal energy of the material in a zone, e_ω is the specific internal energy of the mass entering the zone, and $\dot{H}$ is the rate of energy supplied to or lost from a zone. The last term in the energy equation will probably be ignorable for the application to the massive hydraulic fracturing problem.

An equation of state (EOS) of the fracturing fluid is required. In its simplest form, a compressibility or bulk modulus might be used which is a constant throughout the problem. Or, it may be a complex pressure vs density relationship. That is, the compressibility is determined from the density at each time step from the EOS description provided. However, much more complex forms are also possible. For instance, the pressure in each zone might be a function of the zone's current density, temperature or internal energy, flow velocity, the past history of some flow variable, and/or time since the fracture fluid entered the fracture.

Viscosity of the fracturing fluid can also be considered, and it may well be the most important parameter to consider for massive hydraulic fracturing. For instance, in massive hydraulic fracturing jobs, the viscosity of the fracturing fluid may decrease dramatically due to its being exposed to downhole temperatures for a period of time that is longer than that which is common for "non-massive" jobs. Such a decrease in viscosity with respect to its temperature history could cause the proppant to drop out of the fracture fluid at a location considerably before it would if the viscosity had remained relatively constant.

References

1. Report of the Natural Gas Technology Task Force for the Technical Advisory Committee of the Natural Gas Survey by the Federal Power Commission (April, 1973).
2. S. A. Holditch and R. A. Morse, Oil and Gas J. 69, 57 (1971). S. A. Holditch and R. A. Morse, Oil and Gas J. 69, 85 (1971).
3. L. E. Elkins, Amoco Production Laboratory, Tulsa, Oklahoma, private communication (1973).
4. J. B. Clark, Pet. Trans. AIME 186, 1 (1949).
5. G. C. Howard and C. R. Fast, Drill. and Prod. Prac., p. 261 (1957).
6. T. K. Perkins and L. R. Kern, J. Pet. Tech. 13, 937 (1961).
7. S. T. Christianovich and Yu. P. Zheltov, "Formation of Vertical Fractures by Means of Highly Viscous Liquid," Proc. Fourth World Pet. Cong., Rome, 1955, II, 579-586.
8. B. Haimson and C. Fairhurst, Soc. Pet. Eng. J., pp. 310-318 (Sept. 1967).
9. J. Geertsma and F. DeKlerk, J. Pet. Tech. 21, 1571 (1969).
10. B. B. Williams, J. Pet. Tech. 22, 882 (1970).
11. R. P. Nordgren, Soc. Pet. Eng. S., pp. 306-314 (August 1972).
12. A. A. Daneshy, J. Pet. Tech. 25, 83 (1973).
13. G. C. Howard and C. R. Fast, Hydraulic Fracturing, Monograph, vol. 2, H. L. Doherty Series (Society of Petroleum Engineers of AIME, New York, 1970).
14. I. N. Sneddon, Proc. Roy. Soc. London Series A 187, 229 (1946).
15. M. K. Hubbert and D. G. Willis, Trans. AIME 210, 153 (1957).
16. R. O. Kehle, J. Geophys. Res. 69, 259 (1964).
17. P. E. Gretener, J. Geophys. Res. 70 (24), 6205 (1965).
18. B. Haimson and C. Fairhurst, J. Pet. Tech. 21, 811 (1969).
19. D. A. Daneshy, J. Pet. Tech. 25, 1201 (1973).
20. A. A. Daneshy, Soc. Pet. Eng. J., 13, 61 (1973).
21. H. A. Wahl, J. Pet. Tech. 17, 723 (1965).
22. D. K. Lowe and J. L. Huitt, J. Pet. Tech. 18, 733 (1966).
23. L. R. Kern, T. K. Perkins, and R. E. Wyant, Trans. AIME 216, 403 (1959).
24. I. M. S. Yalin, Mechanics of Sediment Transport (Pergamon Press, New York, 1972).
25. A. G. Petschek and M. E. Hanson, J. Comp. Phys. 3, 307 (1968).
26. M. E. Hanson and R. Sanford, Bull. Seis. Soc. Am. 60 (4), 1209.
27. M. E. Hanson, A. Sanford, and R. J. Shaffer, Jour. Geophys. Res. 76, 3375 (1971).
28. M. E. Hanson, A. G. Petschek, A. R. Sanford, and R. J. Shaffer, Jour. Geophys. Res. 78, 8746 (1973).
29. M. E. Hanson, A. R. Sanford, and R. J. Shaffer, Geophys. Jour. P. Astr. Soc. 38, 365 (1974).

30. M. E. Hanson and A. R. Sanford, Geophys. Jour. R. Astr. Soc. 24, 231 (1971).

31. M. E. Hanson, A. R. Sanford, and R. J. Shaffer, Pressure Driven Fractures Near Flaws and Some Frictional Effects on Shear Fracture, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, Rept. AFWL-TR-73-123 (1973).

32. A. A. Griffith, "The Theory of Rupture," Proc. 1st Intern. Congr. Appl. Mech., Delft, Netherlands, 1924, pp. 55-73.

33. E. Z. Lajtai, Tectonophysics 11, 129 (1971).

34. G. T. Hahn, Battelle Memorial Institute, Ohio, private communication (1973).

35. B. K. Crowley, J. Comp. Phys. 2, 61 (1967).

36. H. D. Glenn and B. K. Crowley, Journ. Appl. Phys. 42, 2099 (1971).

37. B. K. Crowley, H. D. Glenn, and R. E. Marks, Journ. Geophys. Vis. 76, 3356 (1971).

38. B. K. Crowley, H. D. Glenn, J. S. Kahn, and J. M. Thomsen, Int. J. Rock Mech. Am. Sci. 8, 77 (1971).

39. B. K. Crowley and L. K. Barr, TENSORpPUFL; Boundary Conditions Linked Codes, Lawrence Livermore Laboratory, Rept. UCRL-72120 (1967).

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