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Appendix C

A TWO STEP MODEL FOR GAS PRODUCTION FROM LOW PERMEABILITY SHALES

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Abstract

A model is presented that accounts explicitly for both flow of methane through fractures and diffusion through bulk shale to the fractures. Fracture lengths are calculated which are in the order of several kilometers. A single such fracture .1 mm wide and intersecting a 40 m length of well bore can account for 50 MCF/da after 1 year using a specific degasibility of the rock of 4×10^{-7} $cm^3/cm^2/torr/sec^{\frac{1}{2}}$.

Introduction

In a recent paper (1) we suggested that well production was the net result of a very complex set of phenomena starting with desorption from a site (and/or diffusion to the surface of kerogen), passage (and possible resorption) through the micropores of "bulk" shale to the joint system or other set of macroscopic fractures to the well bore and from there to the surface. Further subdivisions of each of these steps is possible; for example, one may distinguish between major and minor fractures. However by virtue of the concept of "rate limiting step(s)" (1) at least some of the complexities can be simplified to a "pseudo two step" model. The first step is diffusion through bulk shale (presumably via its micro porosity) to a macroscopic fracture which serves as a conduit to the well bore.

In the general case the "resistance" of the fracture and the bulk diffusion constant need to be considered together. However two special cases exist. In the first case the bulk diffusion is rate limiting. This effect will be observed when the fractures are wide enough in comparison to their length so that the pressure drop due to flow along the fracture can be neglected and the diffusion of gas through shale becomes the rate limiting process of production. A well of this type has a characteristic production curve wherein:

$$M_T = Ct^{\frac{1}{2}}$$

M_T = total gas produced at time t

C = constant

or

$$\log M_T = \tfrac{1}{2} \log t + \log C$$

that is to say, a plot of logarithm of production vs. the logarithm of time has a slope of ½. It has been established that at least some wells are of this type (2). This special core might be termed the "finite open fracture" case.

At the other extreme consider the case where a fracture is terminated by a constriction just before entering the well bore. The effect of this constriction is that the pressure in the rest of the fracture is above that of a free flowing fracture. In the limit of a severely constricted fracture this pressure approaches the rock pressure. Within this limit of a "constricted exit" model the flow is thus determined entirely by the (constant) pressure drop across the constriction. This could occur when

1) a uniform natural fracture is constricted by drilling or other fluids.

2) when the well bore intersects a small fracture which in turn accesses a much more conductive fracture system. The small fracture could be either natural or artificial.

For wells of this type

$$\frac{dM_T}{dt} = C \text{ (\underline{i}.\underline{e}., production rate is independent of time)}$$

and $M_T = Ct$.

On a log-log plot the resulting slope would be 1. Lewin Associates (3) have noted that log production versus log time curves do in fact approach unity for some wells.

Ryan and Bagnall (4) have classified wells into 4 different types and have reported averaged production rates on these wells (3). Interestingly the large producers almost exactly fit the "finite open fracture" model while the low producers approached the "constricted exit" model as judged from their production curves. This is consistent with the notion that resistance effects in a fracture system will in any event decrease the productivity as compared to the resistance free "finite open fracture" case.

The "finite open fracture" model thus represents the practical maximum productivity of a definable fracture system. In this case the well productivity is a function only of basic rock parameters (degasibility and rock pressure), the total area associated with the fracture system, and the well head pressure. Since increased technology is not likely to extensively modify these parameters, wells with log productivity versus log time curves of ½ represent a technological "ideal."

On the other hand for the "constricted exit" case, the relevant parameters are the rock pressure, the well head pressure, and the permeability of the constricted exit. Technology can (presumably) be developed to remove or prevent constricted exits with resultant increase in economic value of the well.

Between these extreme cases that reduce the two step model to a "pseudo one step" model there are a large number of intermediate cases wherein both diffusion of gas through the rock and flow through the conduits play roles that need explicit consideration. Important among these is a "constant infinite fracture" case wherein gas flows into the sides of a fracture of indefinite length and hence flows to a low permeability well bore. In this case one must consider simultaneously the pressure profile of the fracture and the concentration profiles of gas in the rock.

Theoretical

Consider a fracture of indefinite length along the y direction (see Figure I). The fracture is assumed to have uniform width h and to lie along the z,y plane in rock with uniform degasibility parameters and equilibrium rock pressure, p_2, at time t = 0. At t < 0

$$1)\quad p(y,t) = p_2$$

$$C(x,y,t) = C_2$$

where C(x,y,t) is concentration of methane in the rock at point x, y at time t and p(y,t) is the pressure in the fracture at time t in position y. It is assumed that the concentration at the fracture surface (at y = ±h/2) is related to the pressure in the fracture by

$$2)\quad C(\pm h/2,y,t) = Sp(y,t)$$

where S is the isotherm (solubility) parameter as has been measured (6). For the purposes of this paper, C is in moles/cc and S is in moles/cc/torr.

At $t = 0$ the well bore pressure decreases to p_0. As has been stated elsewhere (1) the source of production is almost entirely from the bulk rock; that is to say expansion of gas in the fracture system contributes only neglibly to production. Thus the total amount of gas emanating from a fracture (M_T) is given almost entirely by the integrated flux of gas through the walls of the fracture.

$$3)\quad M_T = \int J_0 dA$$

If the fracture has a uniform length, w, in the z dimension this can be simplified to

$$4)\quad M_T = wQ_T = w \int J_0 dy$$

With these boundary conditions established it remains to solve simultaneously the equations of diffusion through the bulk rock in conjunction with the equations of flow along the fracture. The flux within the bulk rock at any point is given by Fick's First Law

$$5)\quad J = -D \text{ grad } C$$

or converting to pressure ($C = Sp$)

$$6)\quad J = -DS \text{ grad } p$$

where p is the partial pressure (or more accurately the fugacity) of gas within the rock. The equation of continuity gives an analog to Fick's Second Law in the usual manner

$$7)\quad \frac{\partial p}{\partial t} = DS\nabla^2 p$$

Because of the relatively high conductance of the fracture $\vec{i} \cdot \text{grad } p \gg \vec{j} \cdot \text{grad } p$ where $\vec{i}$ and $\vec{j}$ are unit vectors in x and y directions respectively. Neglecting edge effects $\vec{k} \cdot$ grad $p = 0$ and equations 6) and 7) reduce to

$$8)\quad J_x = -DS\frac{\partial p}{\partial x}$$

$$9)\quad \frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2}$$

To evaluate J_0 in equation 4) it is necessary to evaluate $(\partial p/\partial x)$ at $x = +h/2$ and $-h/2$. This is done by using equation 9) to develop a concentration profile $p(x,y,t)$. But this requires knowing the appropriate boundary conditions for equation 9), i.e., the pressure profile $p(0,y,t)$ within the fracture as a function of position and time.

The volume flow through the fracture at any position Qy is related to the pressure profile by a solution of the Navier Stokes equation (7)

$$10)\quad Q_v y = + \frac{h^3}{12n}\frac{\partial p(0,y,t)}{\partial y}$$

where n is the viscosity. The molar flow through fracture is related to the volume flow by

$$11)\quad Q_n(y) = Q_v(y)\,\frac{p(0,y,t)}{RT}$$

where R is the gas constant and T the absolute temperature. For large y, $Q_n(y) \approx 0$ but as y decreases $Q_n(y)$ increases due to diffusive flow through the sides of the fracture reaching a maximum at the well bore where $Q_n(y) = Q_T$. In particular

$$12)\quad Q_n(y) = Q_n(y + dy) + 2J_0(y)dy$$

or

$$13)\quad \frac{\partial Q_n}{\partial y} = 2J_0$$

where J_0 is the flux of gas through the 2 fracture surfaces. Combining 13), 11) and 10)

$$14)\quad \frac{\partial}{\partial y}\, p\, \frac{\partial p}{\partial y} = \frac{24\eta RT}{h^3}\, J_0$$

One way of solving this set of simultaneous partial differential equations (8, 9, and 14) is to assume an approximate solution for the flux from the bulk rock and use this solution to solve equation 14). At t = 0 the pressure in the well fracture will drop from its equilibrium value p_2 to a value between p_0 and p_2 (as a function of y) almost discontinuously as the initial gas in the fracture is released. This suggests that (as an approximation) the pressure within the tube is a step function and the appropriate solution is (8)

$$15)\quad J_0 = G\left(p_2 - p(0,y,t)\right)t^{-\frac{1}{2}} .$$

Thus

$$16)\quad \frac{d^2}{dy^2}\,p^2(0,y,t) = \frac{48\pi RTG}{h^3 t^{\frac{1}{2}}}\left(p_2 - p(0,y,t)\right)$$

or by making the substitutions $P = p(y,t)/p_2$ and $v = \left(\frac{48\pi RTG}{h^3 t^{\frac{1}{2}} p_2}\right)^{\frac{1}{2}} y$

$$17)\quad \frac{d^2 P^2}{dv^2} = 1 - P$$

A finite difference method can then be utilized to solve equation 17).

Results

Figure II shows the pressure profile (P) (relative to the rock pressure) and the molar flow rate (relative to the flow at the exit of the fracture) as a function of distance. Several factors are evident. First, there is no definitive fracture length or fracture volume but rather pressure rises smoothly to the rock pressure over a distance of several kilometers with the relative production from the fracture surfaces decreasing in a similar curve. Fifty percent of the production is occurring within 2.8 km of the bore at 1 year. As time increases this distance moves out slowly (e.g., after 16 years the distance of 50% production is doubled and after 100 years the distance is some 8.7 km).

The total productivity of the well as a function of time would be

$$18)\quad \frac{dM_T}{dt} = w \frac{h^{3/2} p_2^{\,3/2} G^{1/2}}{\sqrt{3}\ \eta^{1/2} (RT)^{1/2} t^{1/4}} P_0 \left.\frac{dP_0}{dv}\right|_{p = P_0}$$

where P_0 is the relative pressure (p/p_2) at the well bore. Table I displays several values of $P_0 \frac{dP_0}{dv}$ for a variety of P_0. The slope of a log production versus log time plot is .75. This value is midway between the finite open fracture and constricted exit models of .5 and 1.0 respectively as noted in the introduction.

For a usual case wherein

$$G \approx 4 \times 10^{-7} \frac{cm^3}{torr\ cm^2\ sec^{1/2}}$$

$$\eta_{CH_4} = 102.6 \times 10^{-6} \text{ poise}$$

$$p_2 = 500 \text{ psi}$$

Equation 18) reduces to

$$= 6.22 \frac{h^{3/2} w}{t^{1/4}}$$

where $\frac{dM_T}{dt}$ is in MCF/da, h and w are in cm and t is in years. Thus, for example, a well that produces some 50 MCF/da after 1 year needs 80 meters of fracture intersecting the bore (assuming a fracture width of .01 cm).

Finally Figure III shows the pressure drop at various points in the fracture as a function of time. The solution obtained is fairly self-consistent in that for the most relevant distances the pressure drop occurs very rapidly at first reaching a plateau after a short period of time.

Conclusions

It seems reasonable to suggest that no single model will account for all cases of Devonian shale production beyond the generalized two pseudo kinetic step procedure discussed elsewhere (1). Rather specific models generated from this procedure will be applicable on a base by case basis. The present discussion associates high slopes of log production versus log time plots with fracture constrictions that degrade production from the practical maximum associated with diffusion from the bulk rock as being the only contributing factor. This later case is associated with low slope on a log production versus log time plot; that is to say production is shifted to shorter times.

Since different wells do give different log production versus log t slopes it would be interesting to determine what correlation exists between slope and type of drilling mud, explosive versus foam versus hydraulic fracturing, type of proppants, etc.

The most direct approach to applying technology to the maximizing of production is to drill a lot of wells trying different stimulation and other techniques and measuring the results against production. A major difficulty with this procedure is that the natural fracture pattern utilized by each well is unique and hence one does not know except by drilling a large number of wells for each variation in technique what effect is due to the procedures used and what effect is due to luck. Models should be helpful in resolving these d difficulties in a variety of ways, one of which may be the interpretation of log production versus log time plots suggested. Models that merely fit production curves however are not sufficient. The model must serve as a link between the separately measured characteristics of the shale and associated fracture system and production characteristics. In particular models that ignore the measured sorption characteristics and the diffusive properties of methane through bulk shale cannot account for production in a manner that is (ultimately) self-consistent.

A major shortcoming of the "infinite constant fracture" model at this point is that it ignores the effect of branching fractures. There are of course a variety of possible models, one of which is the effect of two intersecting fractures, one of which is connected to a well bore. Another would be the case where the bore runs through a "field" of highly fragmented, closely spaced fractures that can be treated in the approximation of cylindrical symmetry. Finally, each of the models will have separate implications in regard to the various pressure buildup tests that need to be calculated. Work is continuing.

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Figure I. Schematic drawings indicate the three cases depicted in this report.

(a) The "constricted exit" case.

(b) The "finite open fracture" case.

(c) The "infinite finite fracture" case.

Lengths and widths are schematic only and are not to scale or in their proper proportion.

Figure I

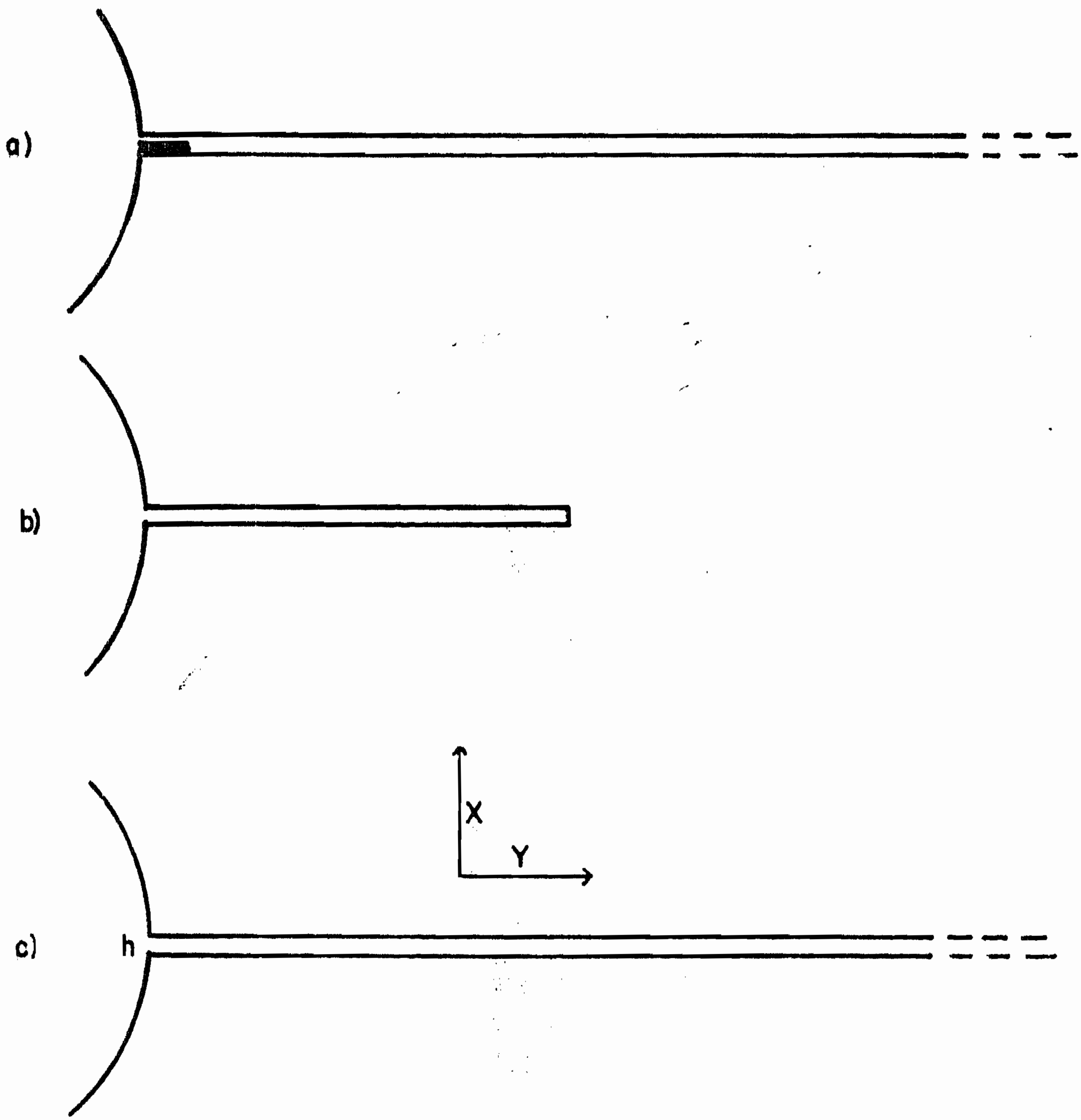

Figure II. Pressure (relative to equilibrium rock pressure) and molar flow rate (relative to exit flow) are plotted as a function of distance from the well bore into the fracture. fracture. Distance is plotted in both dimensionless and in kilometers. The conversion between the two was made assuming:

$$G = 4 \times 10^{-7} \frac{cm^3 \text{ gas}}{cm^2 \text{ surface}} \frac{1}{\text{torr}} \frac{1}{\sec^{1/2}}$$

$$t = 1 \text{ year}$$

$$p_2 = 500 \text{ psi}$$

$$p_0 = 100 \text{ psi}$$

$$h = .01 \text{ cm}$$

Figure II

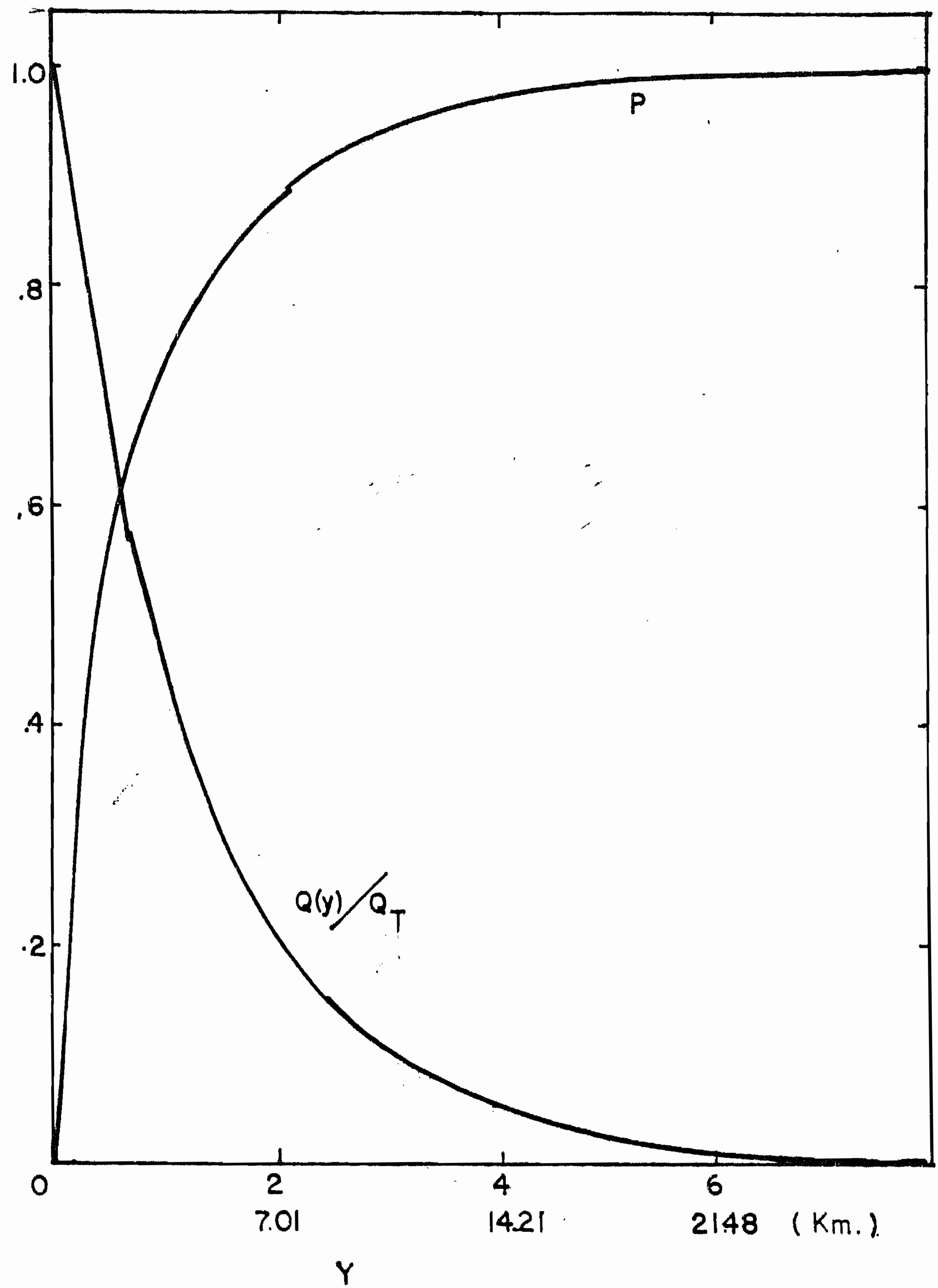

Figure III. Reduced pressure (p/p_2) plotted against time for a variety of distances into the fracture (y)

3×10^6 cm

3×10^5 cm

1×10^5 cm

1×10^4 cm

At time zero all pressures are 1.0.

Figure III

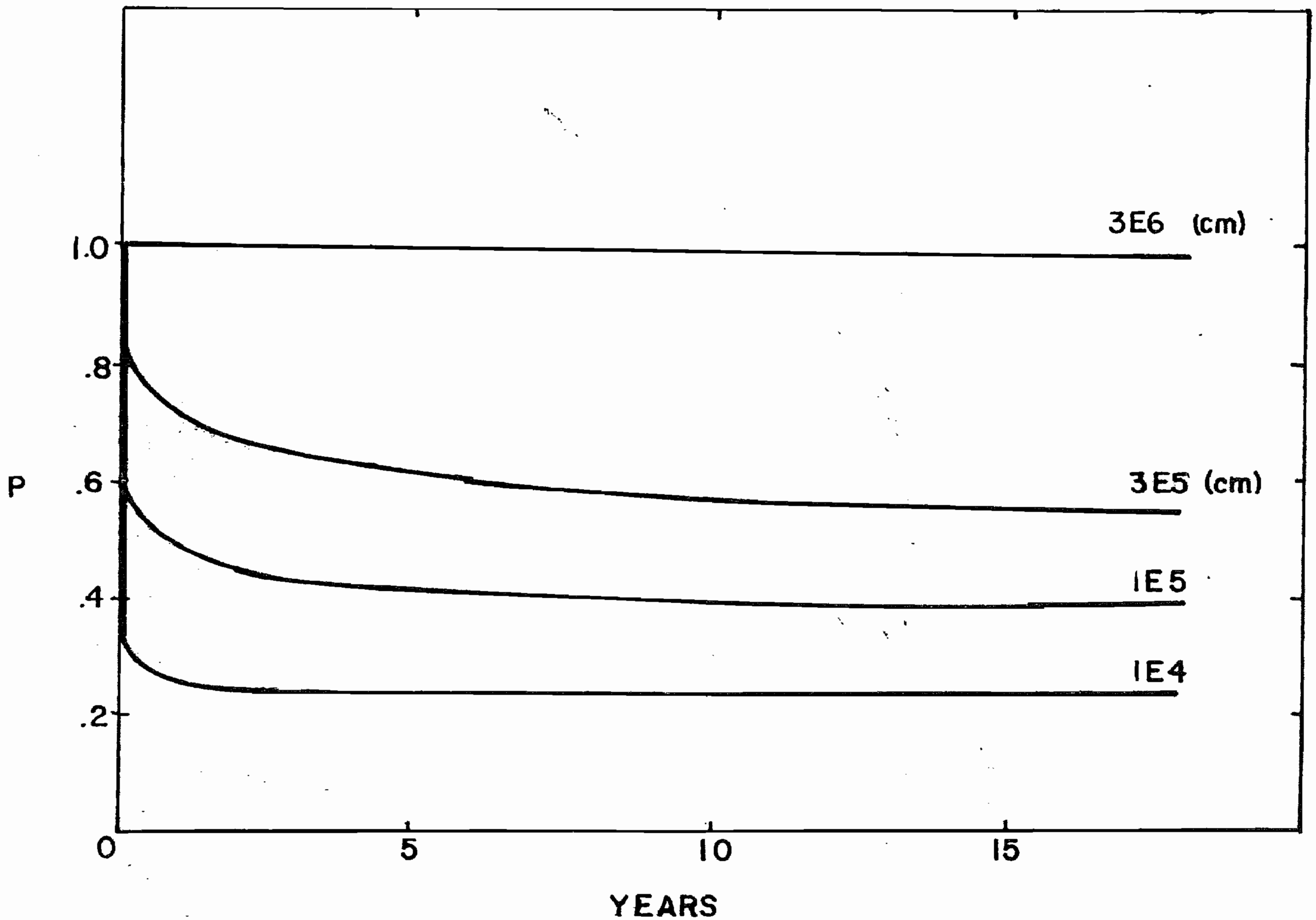

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Table I

P_0	p (bottom hole)(psi)*	$P_0 \frac{dP_0}{dv}$
.11	55	.810
.211	105	.768
.311	155	.670
.381	190.5	.629
.436	218	.535
.545	272	.311
.758	379	

*Based on a rock pressure of 500 psi.