Optimal Sizing and Placement of Energy Storage in a Power System
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Objectives
1) To determine the optimal locations and capacity of energy storage devices in a power system by using a 2-stage Stochastic optimization approach
2) Determine the optimal number of storage devices by implementing mixed-integer programming
3) Employ a scenario reduction technique to make the Stochastic optimization problem more computationally feasible

Two-Stage Stochastic Optimization
- Use Stochastic Optimization to represent different states of the power system
- Each scenario in the optimization is a 24-hour period of load/wind
- Optimize each scenario simultaneously

Example of a single scenario:

Perform Economic Dispatch on a 5-minute scale

Variables common to each scenario:
- $E_{\text{max}}$: Storage Capacity
- $P_{\text{max}}$: Power converter rating
- $E_0$: Initial stored energy at the beginning of the day (must be equal to energy at the end of the day)

Variables specific to each scenario:
- $P_{G_i}(s,t)$: Generation value at bus $i$ time $t$ for scenario $s$
- $E_i(s,t)$: Energy level for storage at bus $i$ at time $t$ for scenario $s$
- $P_{\text{in}}(s,t)$: Power into storage at bus $i$ at time $t$ for scenario $s$
- $P_{\text{out}}(s,t)$: Power out of storage at bus $i$ at time $t$ for scenario $s$

Mixed-Integer Programming:
- Introduce binary variables $\text{bin}E_i \in [0,1]$ at each bus that are 0 if there is no storage at that bus and 1 if there is storage at that bus
- Introducing a capital cost to place a storage at a bus will minimize the number of total storage devices in the system while still reducing overall system cost

Steady-State Storage Device Model

$$E(t+T) = E(t) + \alpha * T * P_{\text{in}}(t) - T * P_{\text{out}}(t)/\alpha$$

- $\alpha = \text{efficiency}$
- $T = \text{time scale}$

Objective Function and Constraints

$$\sum_{i=1}^{N} a_i P_{G_i}^2 + b_i P_{G_i} + c_i P_{\text{max},i} + d_i E_{\text{max},i} + e_i \text{bin}E_i$$

- Generation
- Capacity and Power Converter Rating
- Capital Cost of Storage

$$0 \leq E_{\text{max},i} \leq 10000 * \text{bin}E_i$$
$$0 \leq P_{\text{max},i} \leq 10000 * \text{bin}E_i$$
- If there is no storage at bus $i$, then $\text{bin}E = 0$ and $E_{\text{max}}$ and $P_{\text{max}}$ are forced to 0

Simulation Results
9-bus system with 3 generators, 2 wind generators, 3 loads
50 Scenarios : $3^2 \times 288 \times 50 + 9 \times 3 = 43,227$ Variables

How do we perform Scenario Reduction?
Notice that optimal capacity at a bus (without system congestion) is correlated with the variance in Locational Marginal Price (LMP) at that bus

Group similar scenarios together, weight the probability of each scenario by the number of scenarios in that cluster
Reduce the number of scenarios while preserving their diversity

Each new scenario is weighted by how many scenarios were clustered into that scenario
50 scenarios reduced down to 10
Stochastic Optimization with 50 scenarios produces very similar results as stochastic optimization with 10 scenarios. Accuracy increases as the number of clusters increases.